The Quantum Critical Higgs

John Terning

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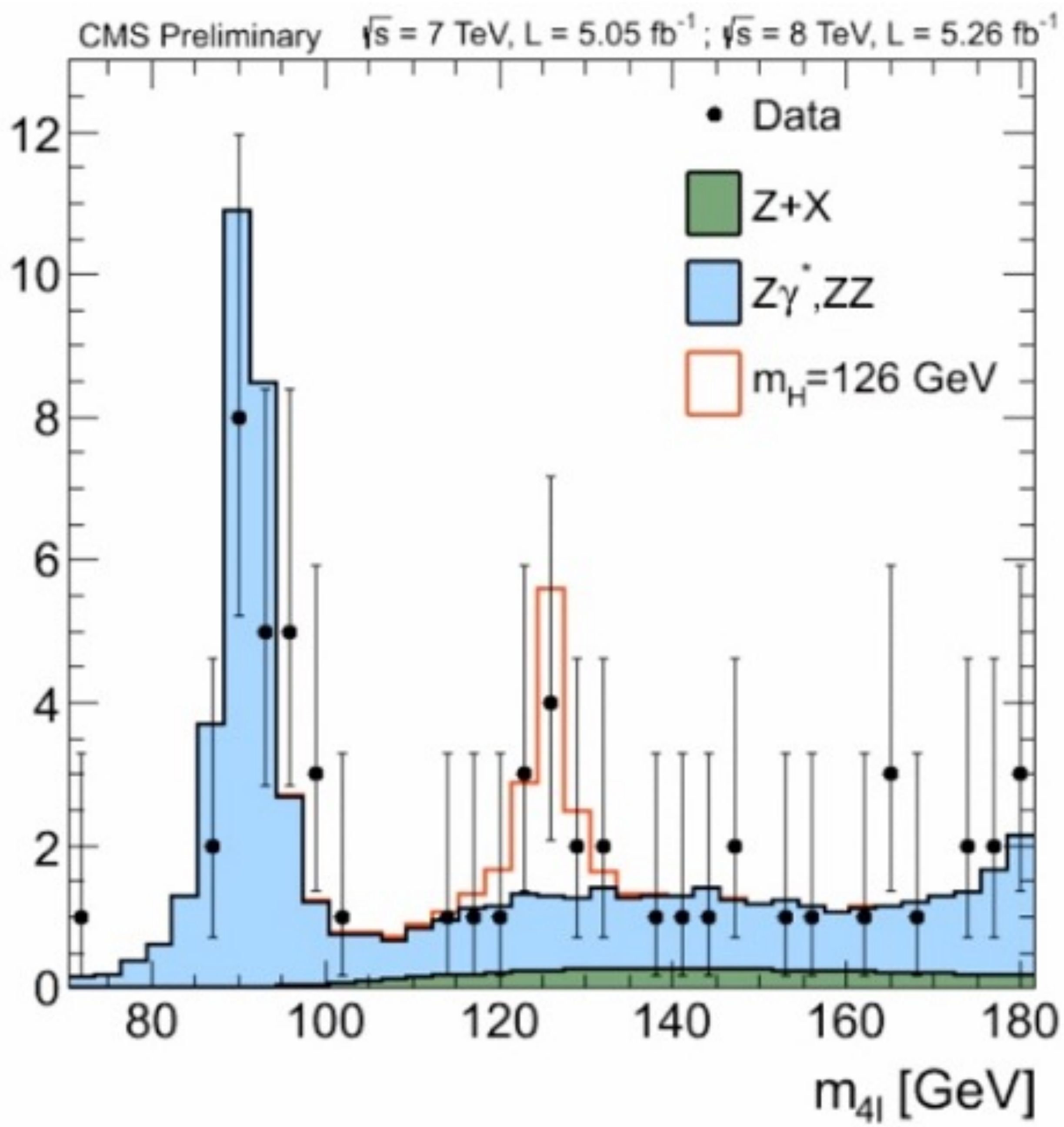
Jay Hubisz, Seung J. Lee, Javi Serra

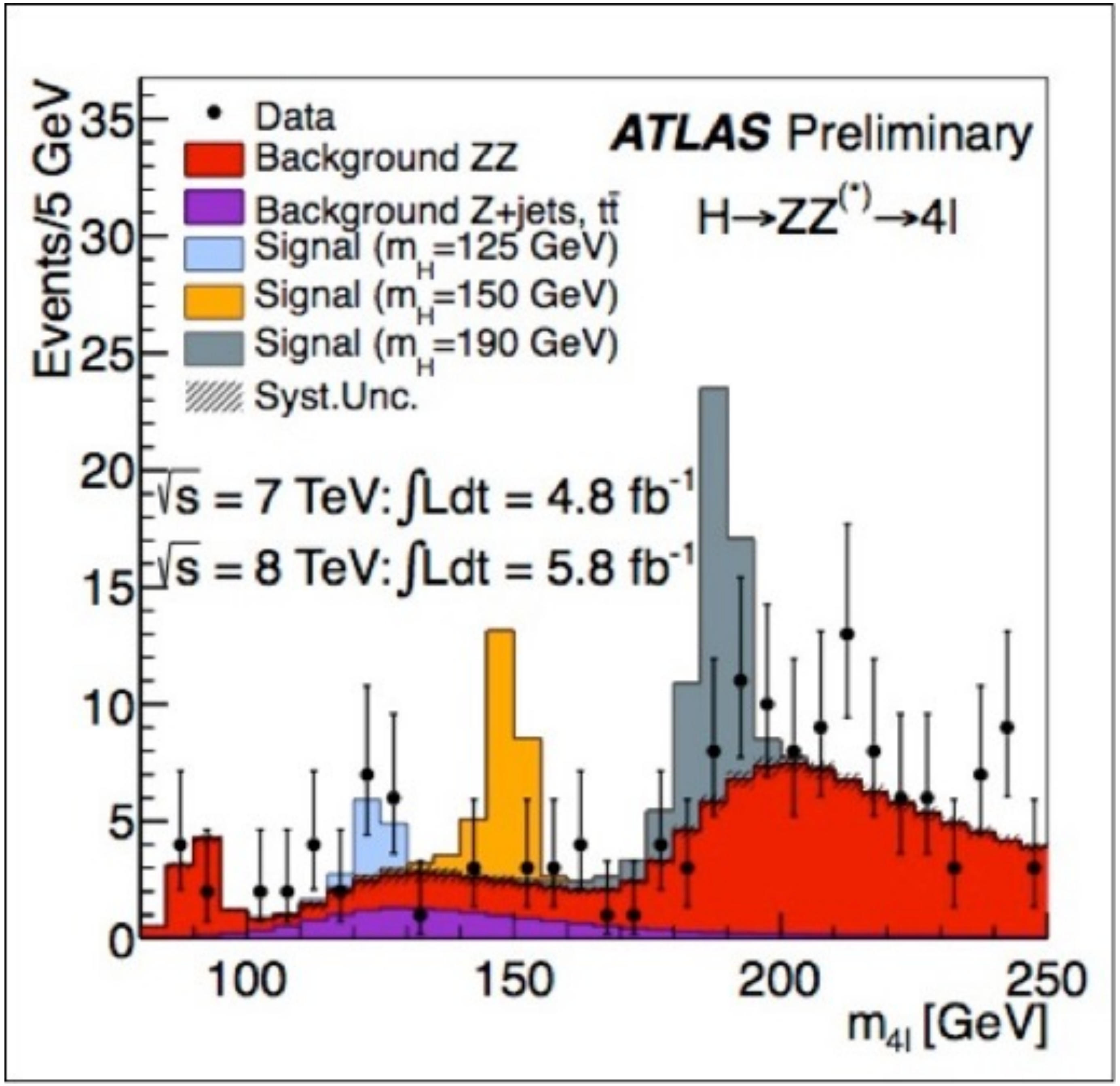
hep-ph/1502.????

Outline

* Motivation
* AdS/CFT/unparticle correspondence
* effective action for quantum critical Higgs
* gauge interactions
* LHC measurements

Higgs-like Resonance



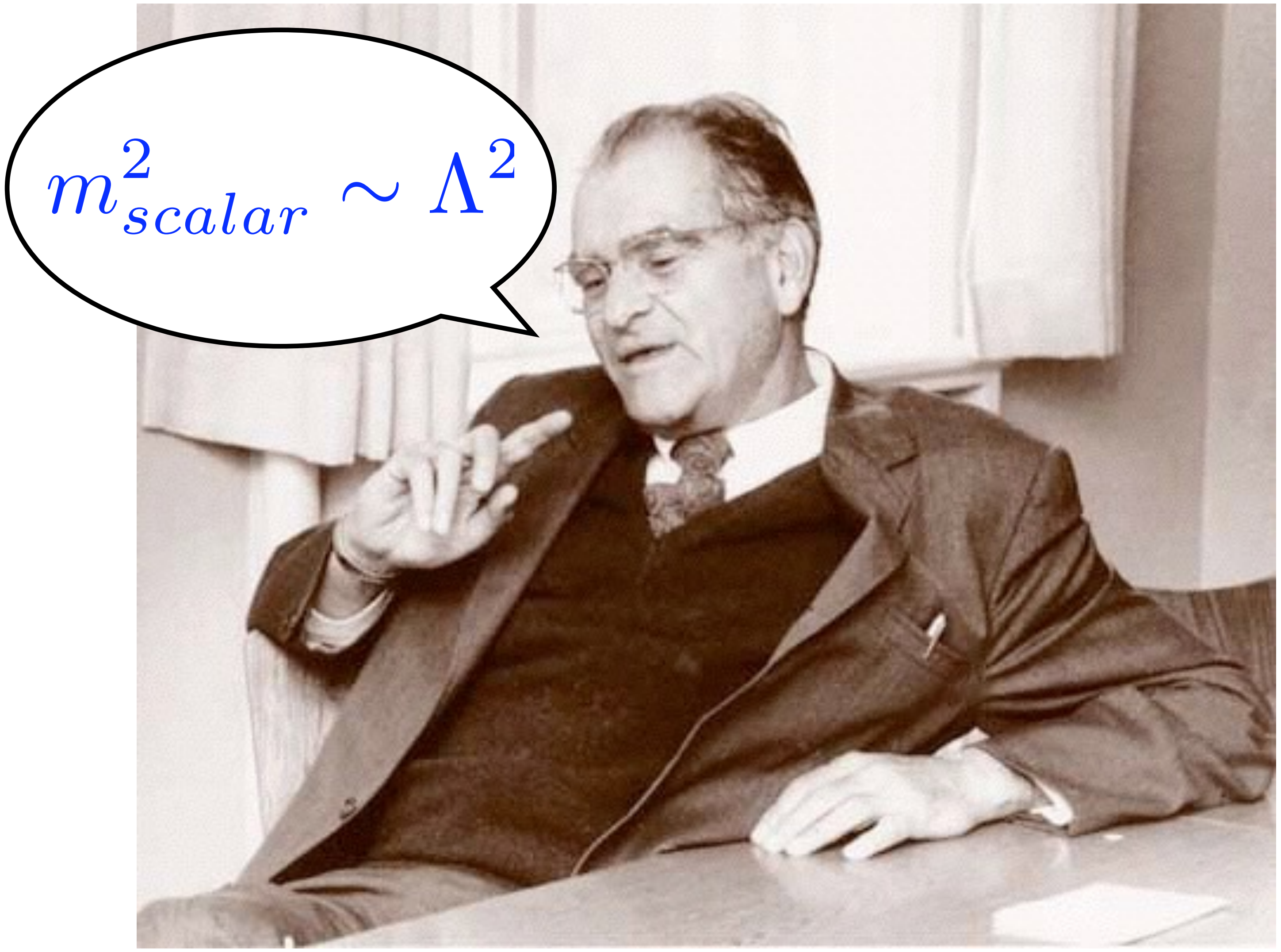


finally something really new!

What's the problem?

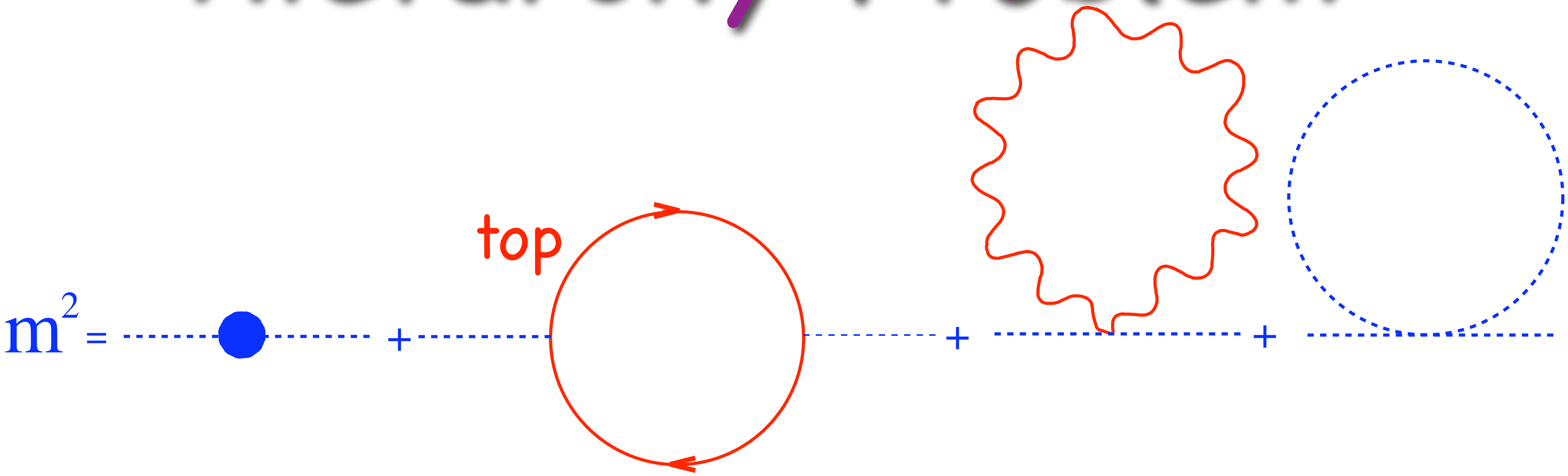
Weisskopf Phys. Rev.56 (1939) 72

What's the problem?

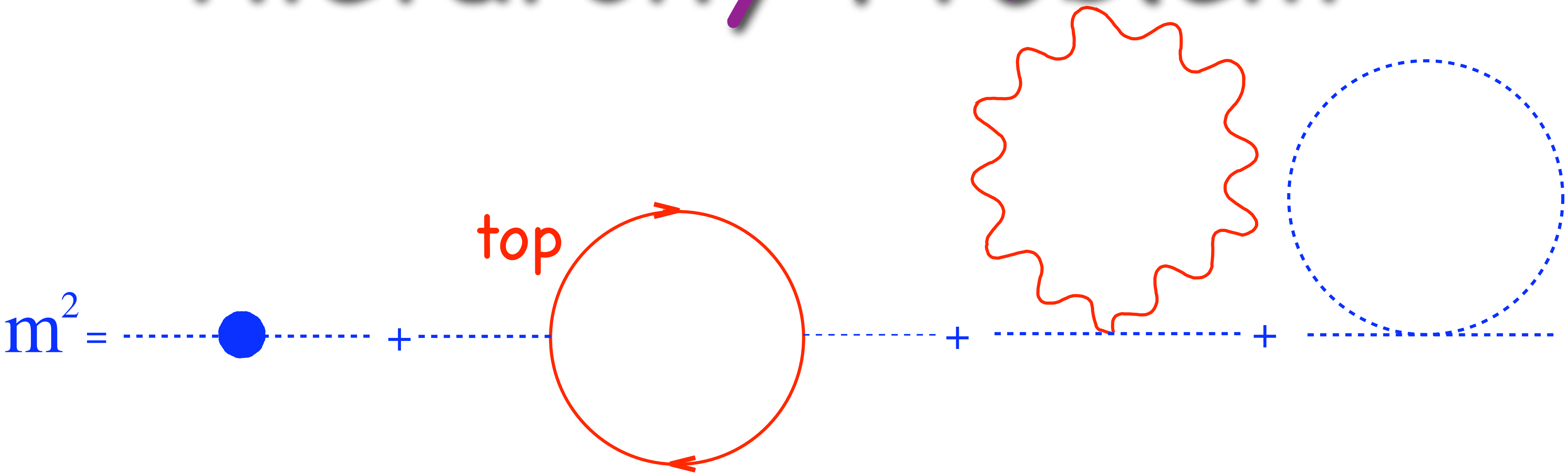


Weisskopf Phys. Rev.56 (1939) 72

Hierarchy Problem

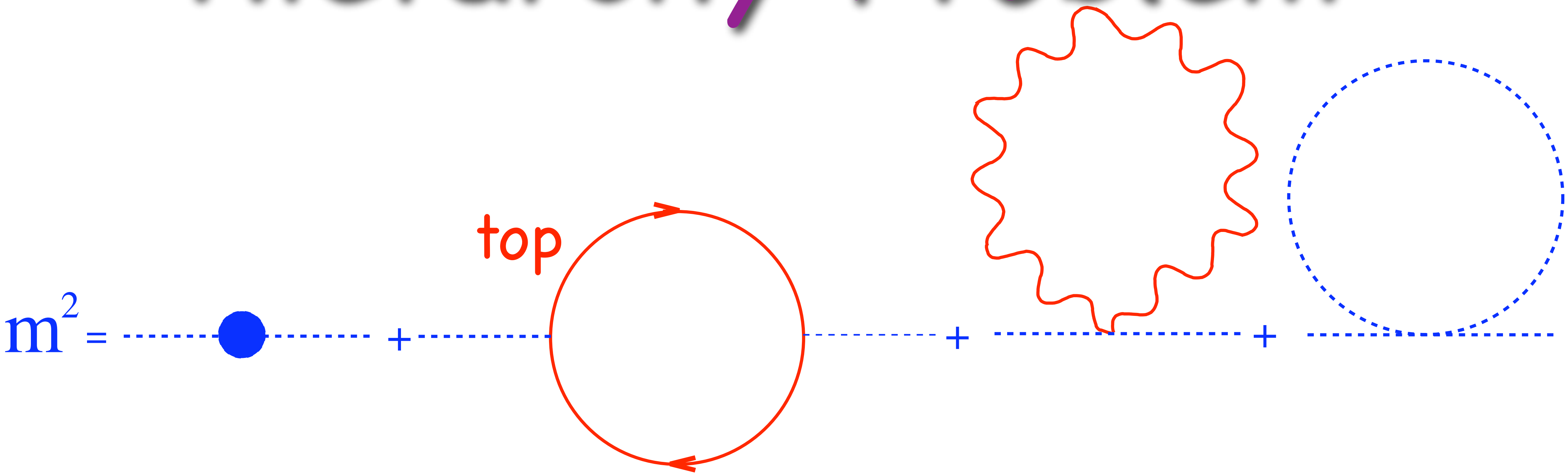


Hierarchy Problem



$$
\left(\frac{125}{\sqrt{2}}\right)^2 = \begin{array}{l} 164199715127639936078810934470380 89115 \\ -194020311600080166772778861799914 76752 \\ +244128109906655995494381822573963 7142 \\ +540778548177463114452974507213751 495 \end{array}
$$

Hierarchy Problem

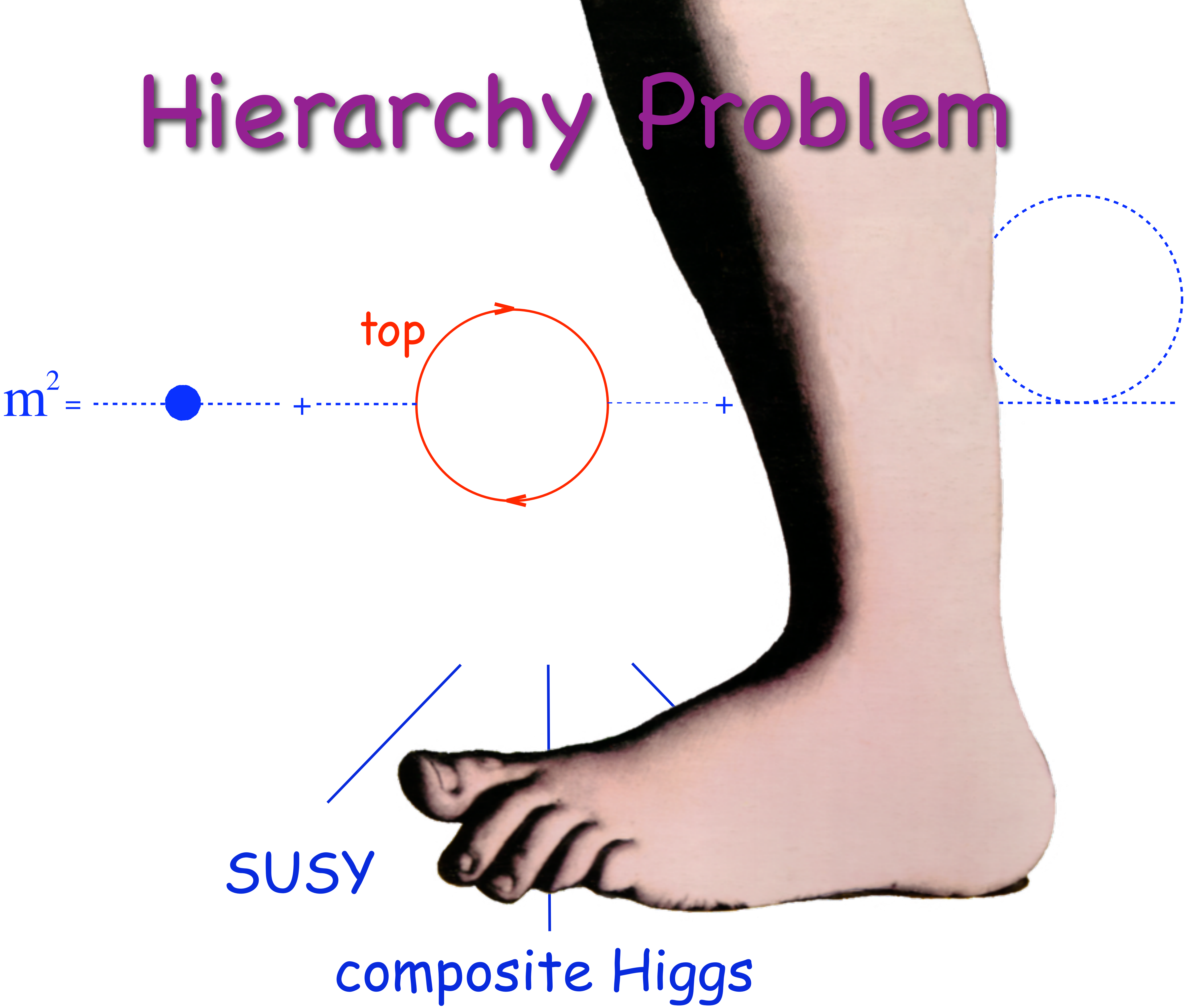


SUSY

composite Higgs

Technicolor

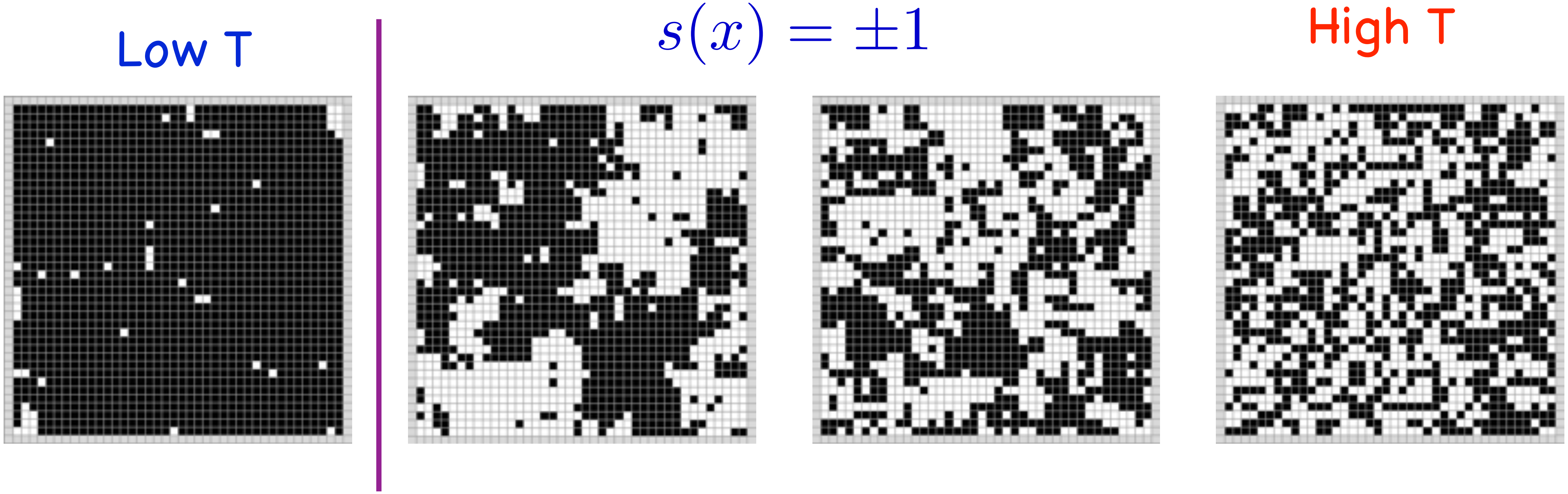
Hierarchy Problem



Ising Model

$$H = -J \sum s(x)s(x+n)$$

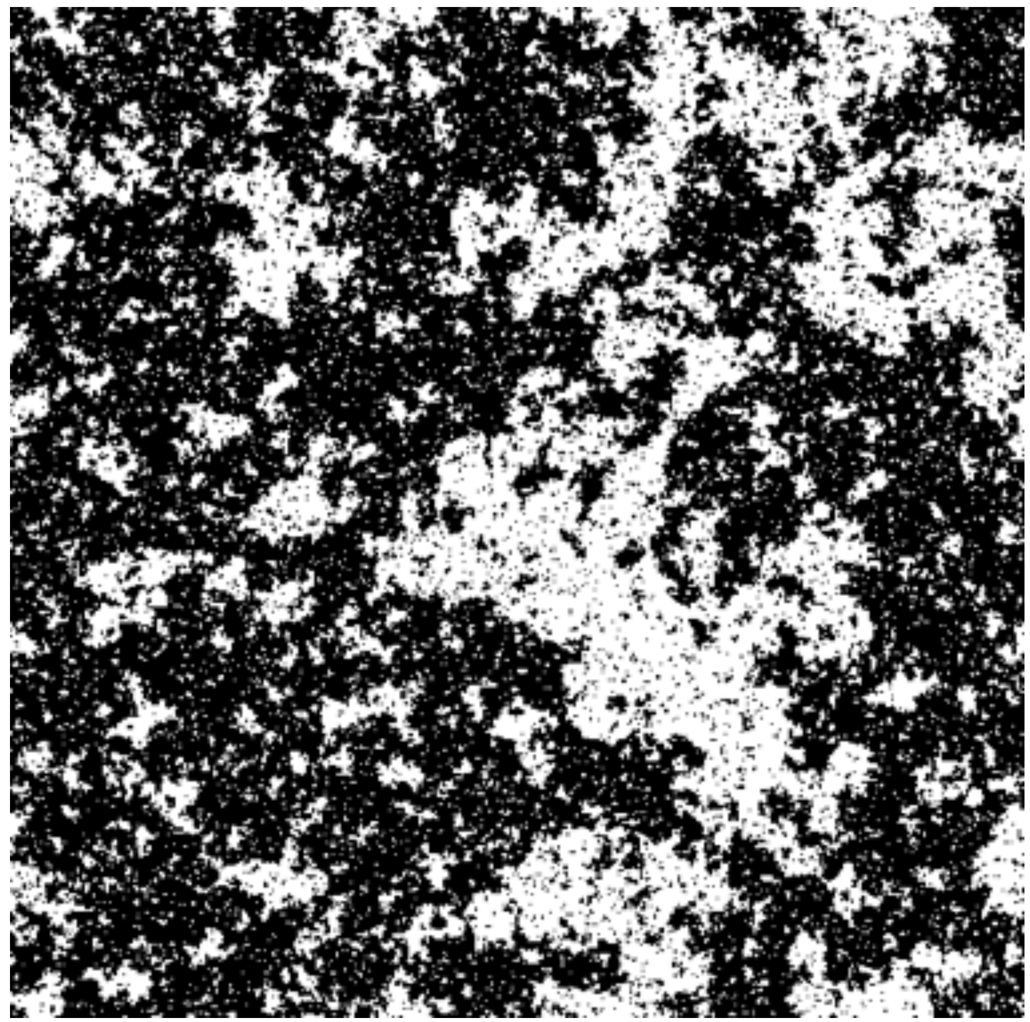
$$s(x) = \pm 1$$



$$\langle s(0)s(x)\rangle = e^{-|x|/\xi}$$

at $T=T_c$ $\quad \xi \to \infty$

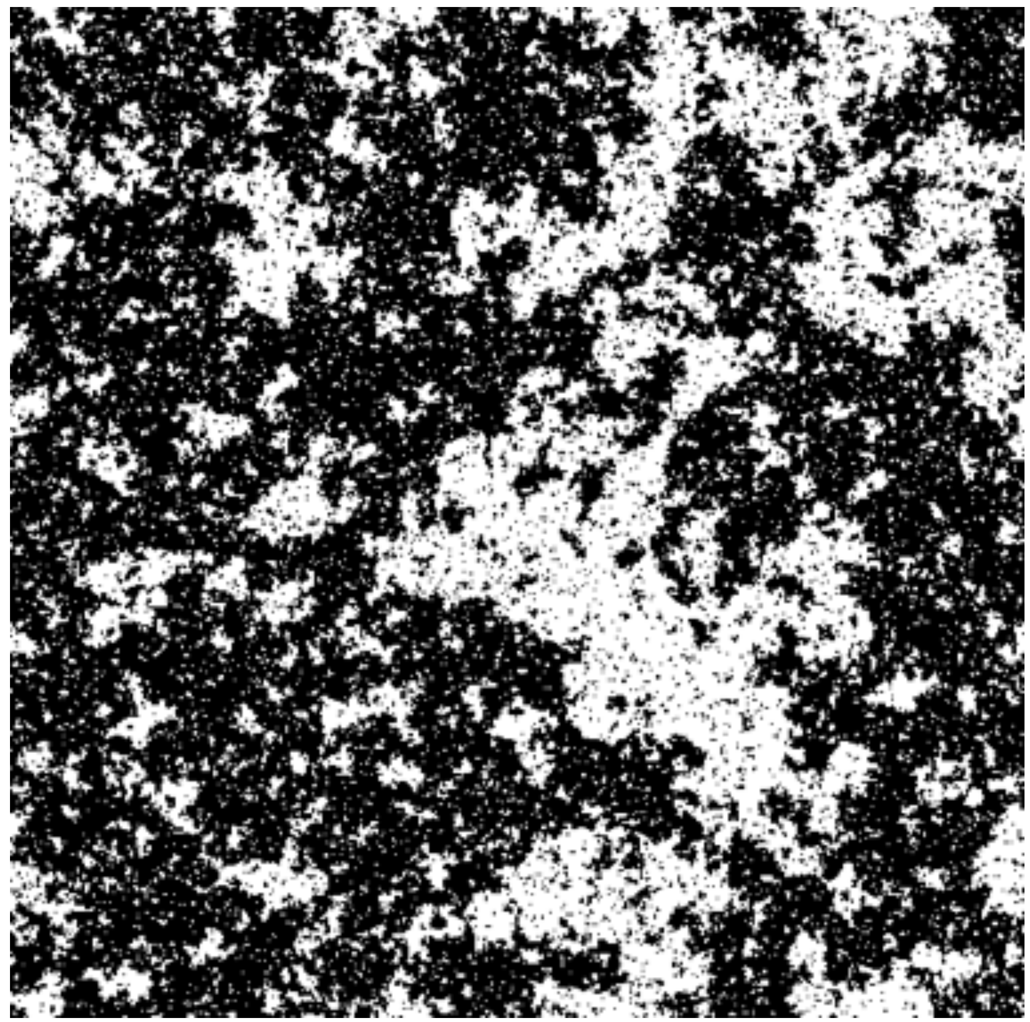
Critical Ising Model is Scale Invariant



http://bit.ly/2Dcrit

at T=T$_c$ $$\langle s(0)s(x)\rangle \propto \frac{1}{|x|^{2\Delta-1}}$$

Critical Ising Model is Scale Invariant

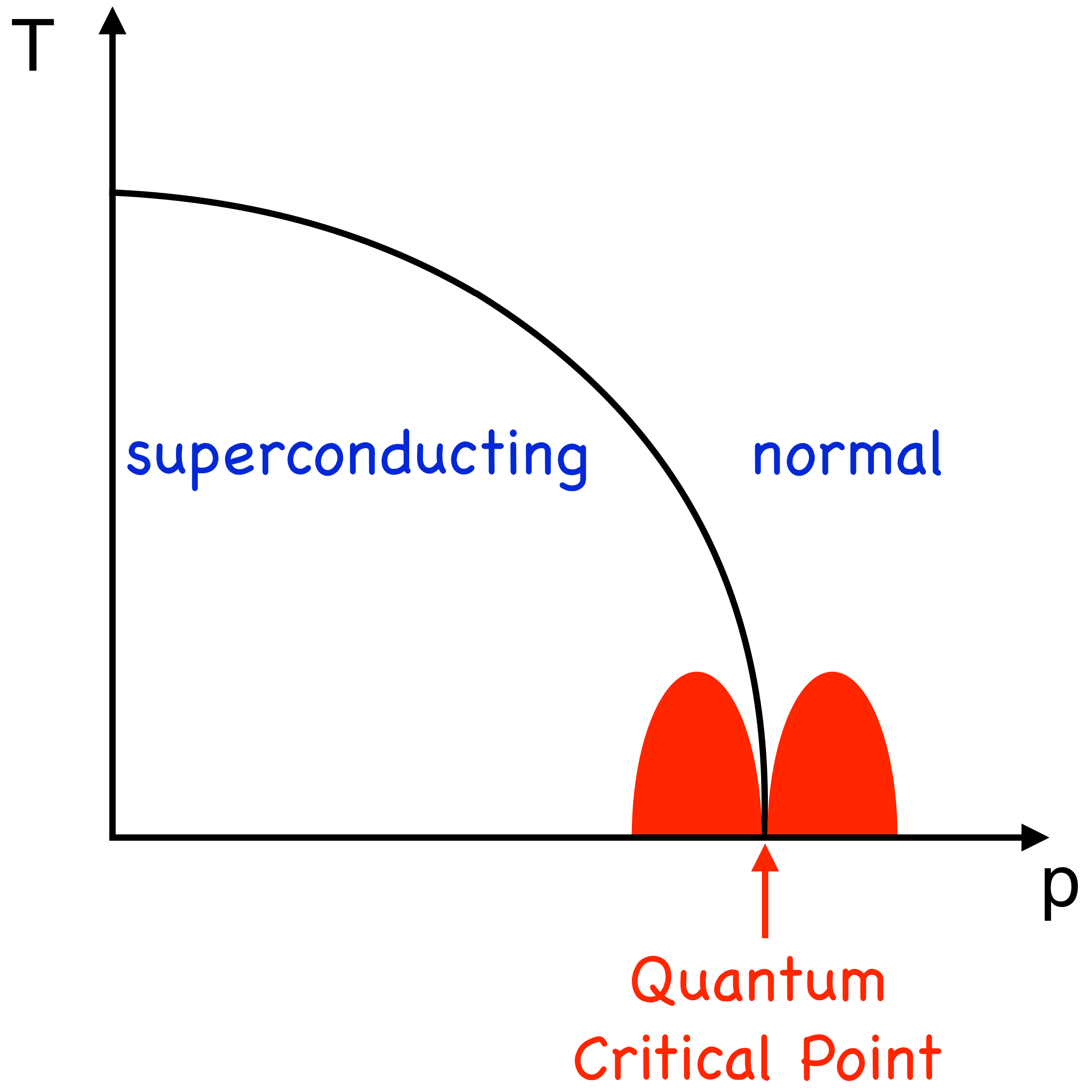


http://bit.ly/2Dcrit

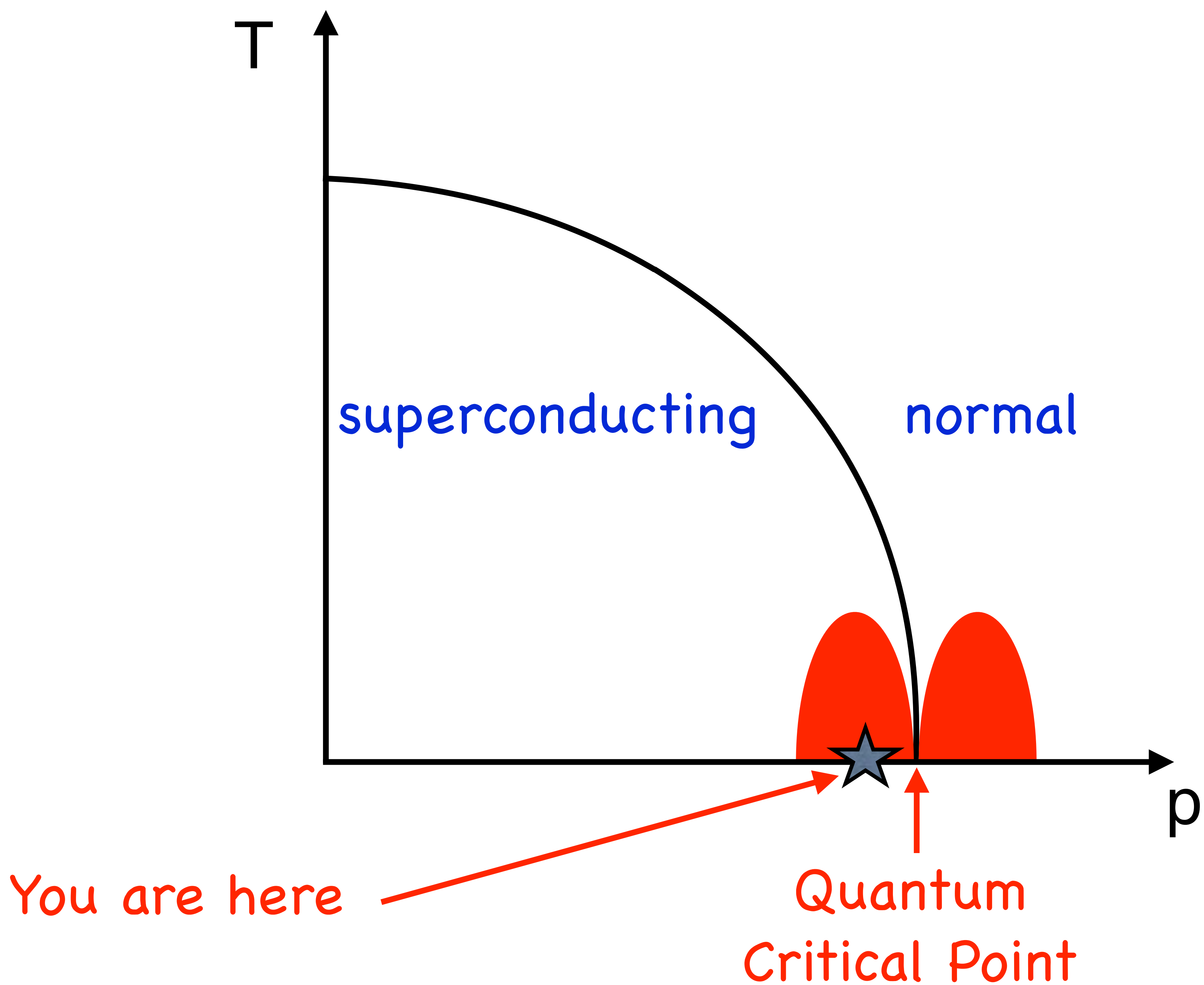
at T=T$_c$ $\langle s(0)s(x)\rangle \propto \frac{1}{|x|^{2\Delta-1}} = \int d^3p \, \frac{e^{ip\cdot x}}{|p|^{4-2\Delta}}$

critical exponent

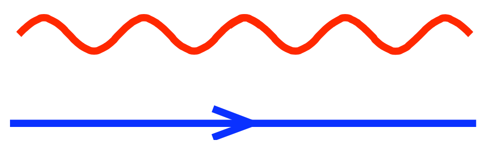
Quantum Phase Transition



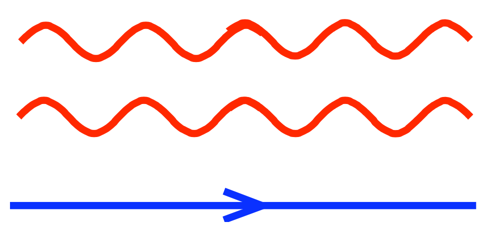
Quantum Phase Transition



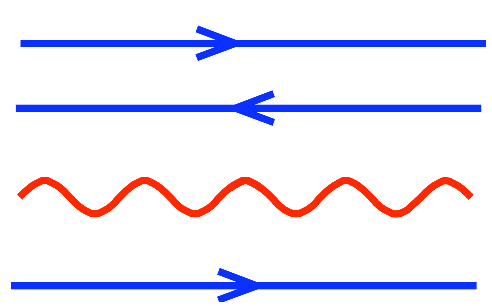
CFT 101



$$p_1^2 = 0$$
$$p_2^2 = 0$$
$$p^2 = (p_1 + p_2)^2 \neq 0$$

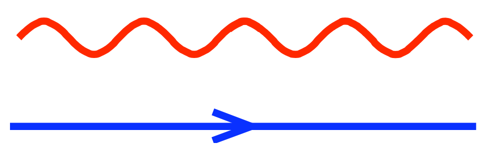


$$p_i^2 = 0$$



$$p^2 = \left(\sum_i p_i \right)^2 \neq 0$$

CFT 101



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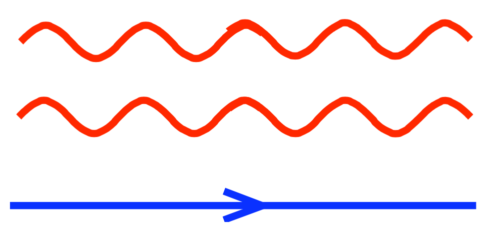

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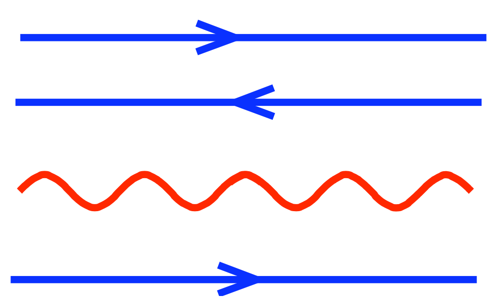

jets!

AdS/CFT

$$\langle\, e^{\int d^4x\, \phi_0(x)\mathcal{O}(x)} \rangle_{\rm CFT} \approx e^{-S_{\rm 5Dgrav}[\phi(x,z)|_{z=0}=\phi_0(x)]}$$

$$ds^2 = \frac{R^2}{z^2}\left(dx^2 - dz^2\right)$$

$\mathcal{O} \subset$ CFT $\leftrightarrow \phi$ AdS$_5$ field, $\phi_0(x)$ is boundary value

AdS/CFT

$$ds^2 = \frac{R^2}{z^2}\left(dx_\mu^2 - dz^2\right)$$

$$z > \epsilon$$

$$S_{bulk} = \frac{1}{2}\int d^4x\, dz \sqrt{g}(g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + m^2\phi^2)$$

$$\phi(p,z) = az^2 J_\nu(pz) + bz^2 J_{-\nu}(pz)$$

$$\Delta[\mathcal{O}] = 2 \pm \nu = 2 \pm \sqrt{4 + m^2R^2}$$

Unparticles

Georgi:

a different way to calculate in CFT's

phase space looks like a fractional number of particles

Georgi hep-ph/0703260, 0704.2457

unparticle propagator

$$G(p) \equiv \int d^4x\, e^{ipx} \langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle$$

unparticle propagator

$$
\begin{aligned}
G(p) &\equiv \int d^4x\, e^{ipx} \langle 0 | T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle \\
&= \frac{A_d}{2\pi}\int_0^\infty (M^2)^{\Delta-2}\frac{i}{p^2 - M^2 + i\epsilon}dM^2
\end{aligned}
$$

spectral density

unparticle propagator

$$
\begin{aligned}
G(p) &\equiv \int d^4x\, e^{ipx} \langle 0|T\mathcal{O}(x)\mathcal{O}^\dagger(0)|0\rangle \\
&= \frac{A_d}{2\pi}\int_0^\infty (M^2)^{\Delta-2} \frac{i}{p^2 - M^2 + i\epsilon} dM^2 \\
&= i\frac{A_d}{2}\frac{\left(-p^2 - i\epsilon\right)^{\Delta-2}}{\sin d\pi}
\end{aligned}
$$

spectral density

$$
A_d = \frac{16\pi^{5/2}}{(2\pi)^{2\Delta}} \frac{\Gamma(\Delta + 1/2)}{\Gamma(\Delta - 1)\Gamma(2\Delta)}
$$

AdS/CFT/Unparticles

$$\phi(p,\epsilon) = \epsilon^{-\nu} R^{-3/2}\, \phi_0(p)$$

$$S = \frac{1}{2}\int d^4x\, dz\, \partial_z \left(\frac{R^3}{z^3}\phi\partial_z\phi\right)$$

AdS/CFT/Unparticles

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$$S = \frac{1}{2}\int d^4x\, dz\, \partial_z \left(\frac{R^3}{z^3} \phi \partial_z \phi \right)$$

← boundary term

AdS/CFT/Unparticles

$$\phi(p,\epsilon) = \epsilon^{-\nu} R^{-3/2}\, \phi_0(p)$$

$$S = \frac{1}{2}\int d^4x\, dz\, \partial_z \left(\frac{R^3}{z^3} \phi \partial_z \phi \right)$$

surface term

$$S = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} \phi_0(-p)\phi_0(p) K(p)$$

$$K(p) = (2-\nu)\epsilon^{-2\nu} + b\, p^{2\nu} + c\, p^2 \epsilon^{2-2\nu} + \ldots$$

$$K(p) = G(p) \qquad\qquad \Delta = 2 + \nu$$

Legendre Transform

$$\Delta = 2 - \nu$$

$$S' = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\phi_0(-p)K\phi_0(p) + \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\phi_0(p)A(p)$$

$$S' = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}A(-p)\,K^{-1}A(p)$$

A is the source

$K(p)^{-1} = G(p)$ ϕ_0 is the field

Klebanov, Witten hep-th/9905104

Legendre Transform

$$\Delta = 2 - \nu$$

$$S' = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\phi_0(-p)K\phi_0(p) + \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\phi_0(p)A(p)$$

$$S' = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}A(-p)\,K^{-1}A(p)$$

$$\langle \mathcal{O}(p')\mathcal{O}(p)\rangle \quad \propto \quad \frac{\delta^2 S'}{\delta A(p')\,\delta A(p)} \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^4}\,(p^2)^{\Delta-2}$$

Klebanov, Witten hep-th/9905104

AdS/CFT/Un Dictionary

“Georgi” “string theorist”

AdS/CFT/Un Dictionary

“Georgi”	“string theorist”
moose	

AdS/CFT/Un Dictionary

“Georgi”	“string theorist”
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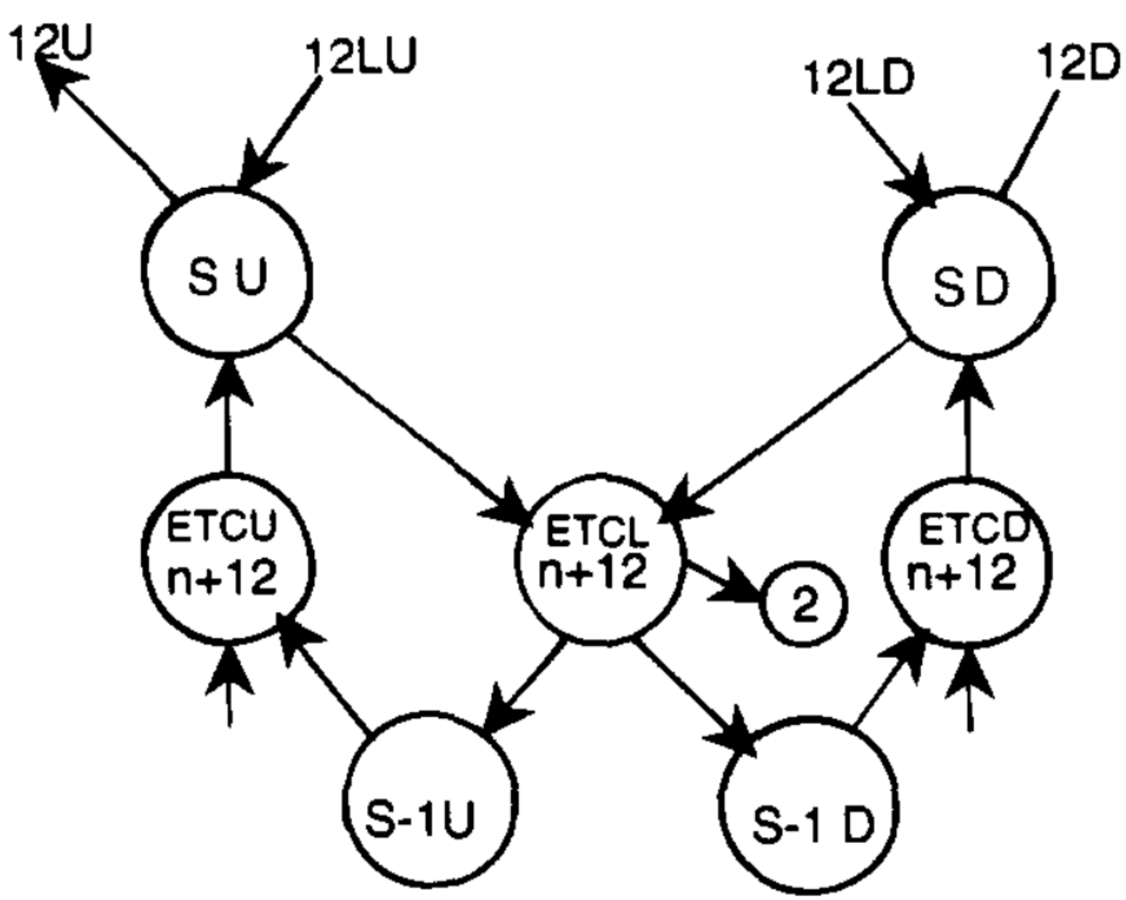
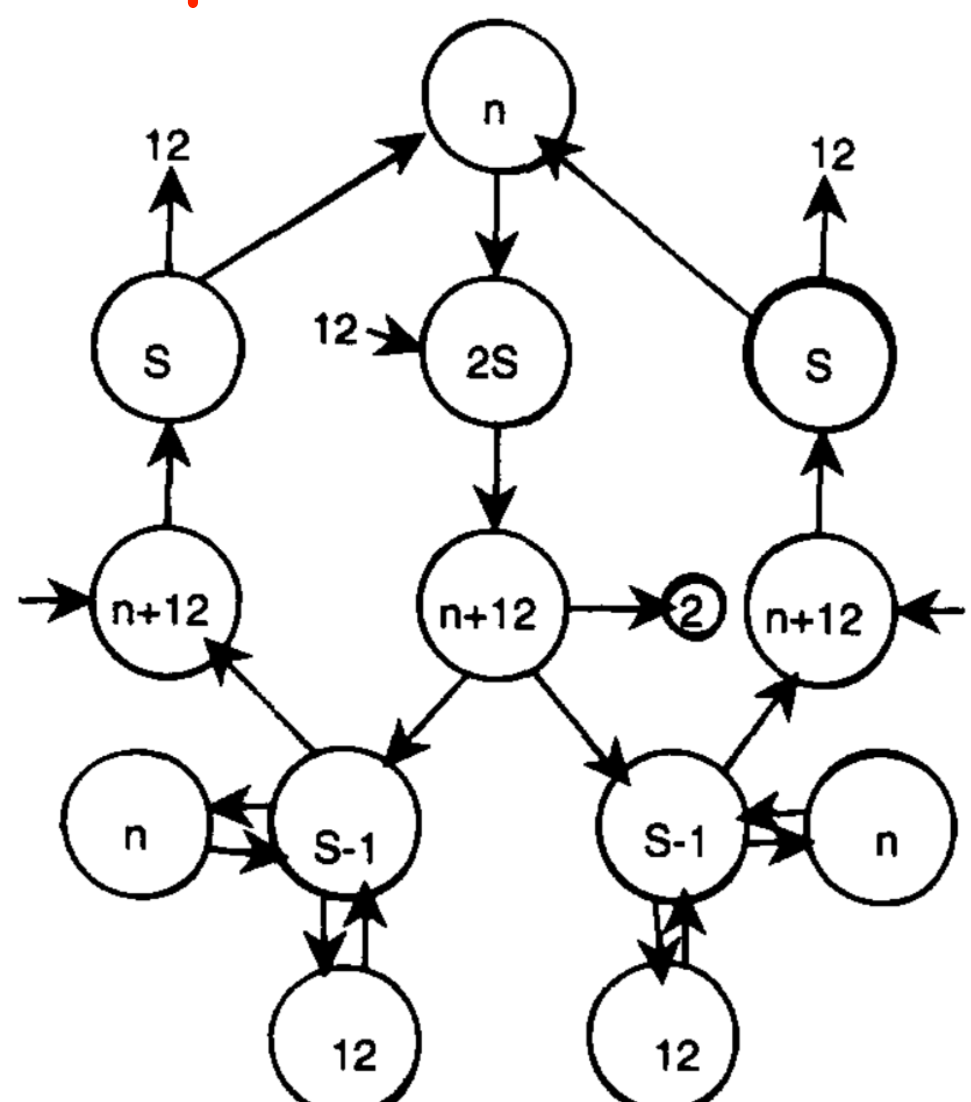
AdS/CFT/Un Dictionary

"Georgi" — "string theorist"

moose — quiver model





AdS/CFT/Un Dictionary

"Georgi"	"string theorist"
moose	quiver model
	double trace perturbation

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AdS/CFT/Un Dictionary

“Georgi”	“string theorist”
moose	quiver model
mass term	double trace perturbation
unparticle	state created by a CFT operator
unparticle action	Legendre transform of a holographic boundary action

Why (broken) CFT's are Interesting

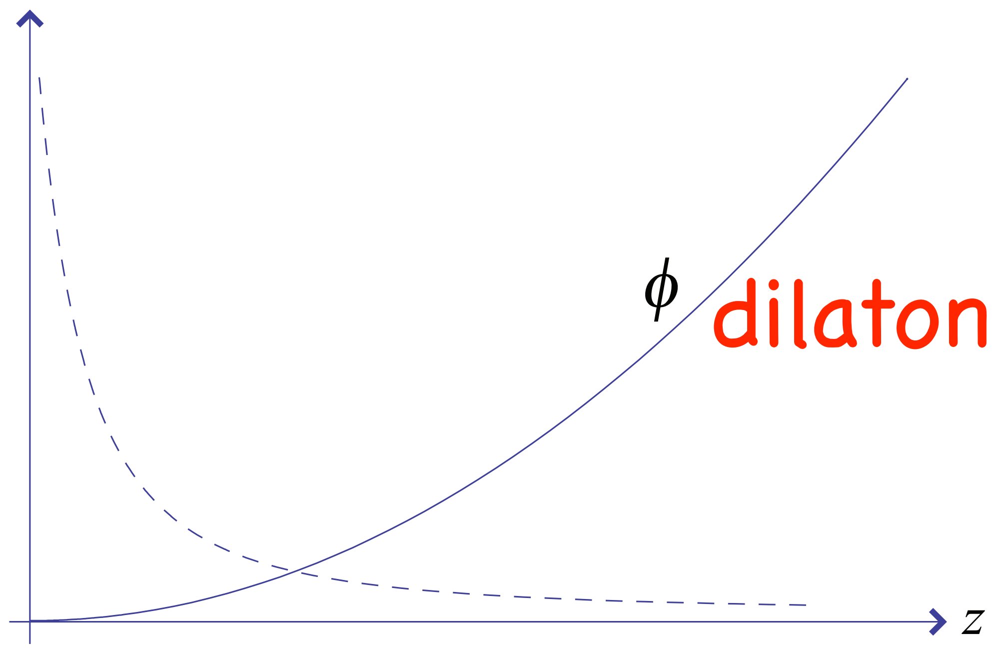
pure unparticles are equivalent to RS2

IR cutoff at TeV turns RS2 into RS1

IR brane cutoff is one type of scale breaking

a new type of IR cutoff will lead to new LHC phenomenology

Soft-Wall



Karch, Katz, Son, Stephanov hep-ph/0602229
Gherghetta, Batell hep-th/0801.4383

AdS/CFT/Unparticles IR Cutoff

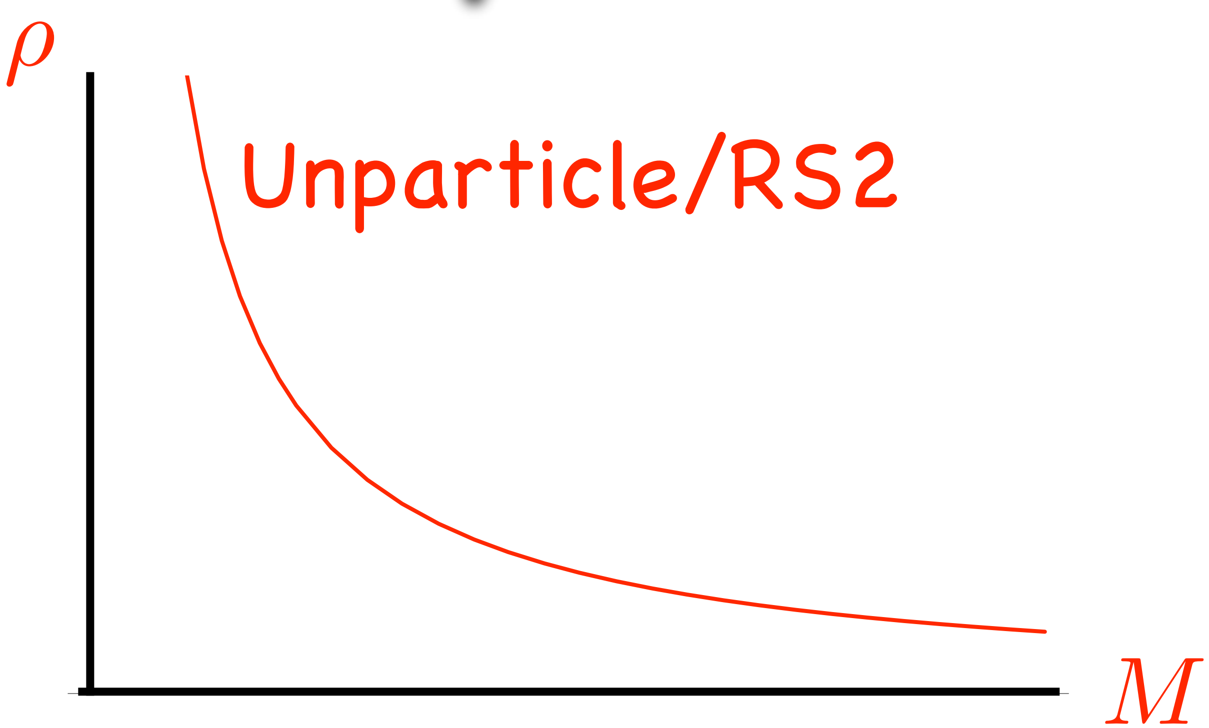
$$S_{int} = \frac{1}{2}\int d^4x\, dz\, \sqrt{g}\phi\mathcal{H}^\dagger\mathcal{H}$$

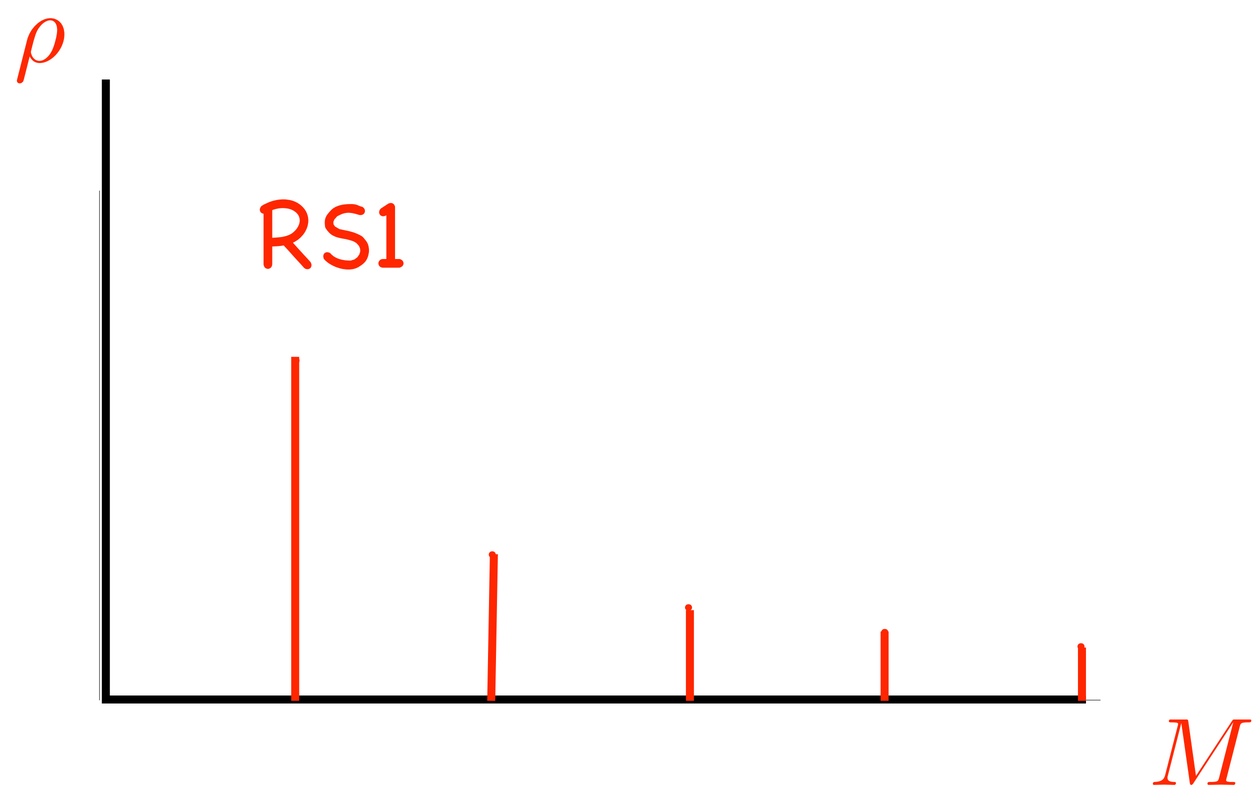
$$\phi = \mu z^2$$

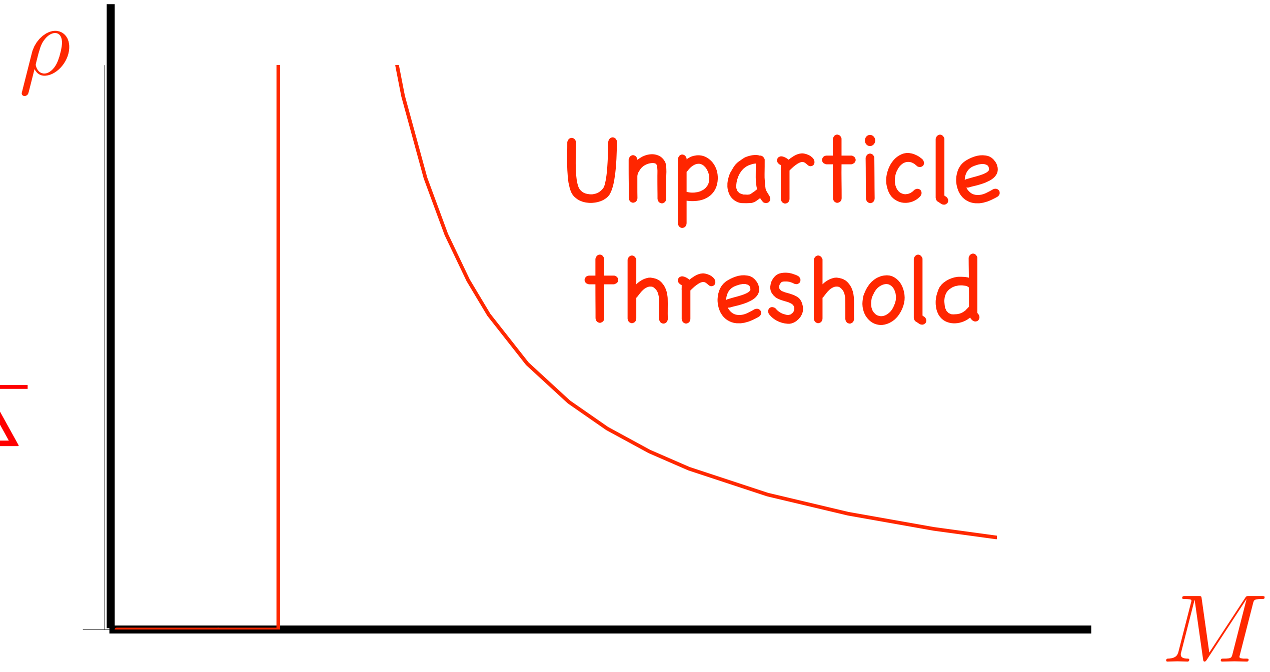
$$z^5\partial_z\left(\frac{1}{z^3}\partial_z\mathcal{H}\right) - z^2(p^2-\mu^2)\mathcal{H} - m^2R^2\mathcal{H} = 0$$

$$\langle\mathcal{O}(p')\mathcal{O}(p)\rangle \propto \frac{\delta^{(4)}(p+p')}{(2\pi)^4}(p^2-\mu^2)^{\Delta-2}$$

Spectral Densities







$$G(p) = -\frac{i}{(\mu^2 - p^2 - i\epsilon)^{2-\Delta}}$$

Effective Action

$$S = \int \frac{d^4p}{(2\pi)^4} \mathcal{H}^\dagger(p) \left[\mu^2 - p^2\right]^{2-\Delta} \mathcal{H}(p)$$

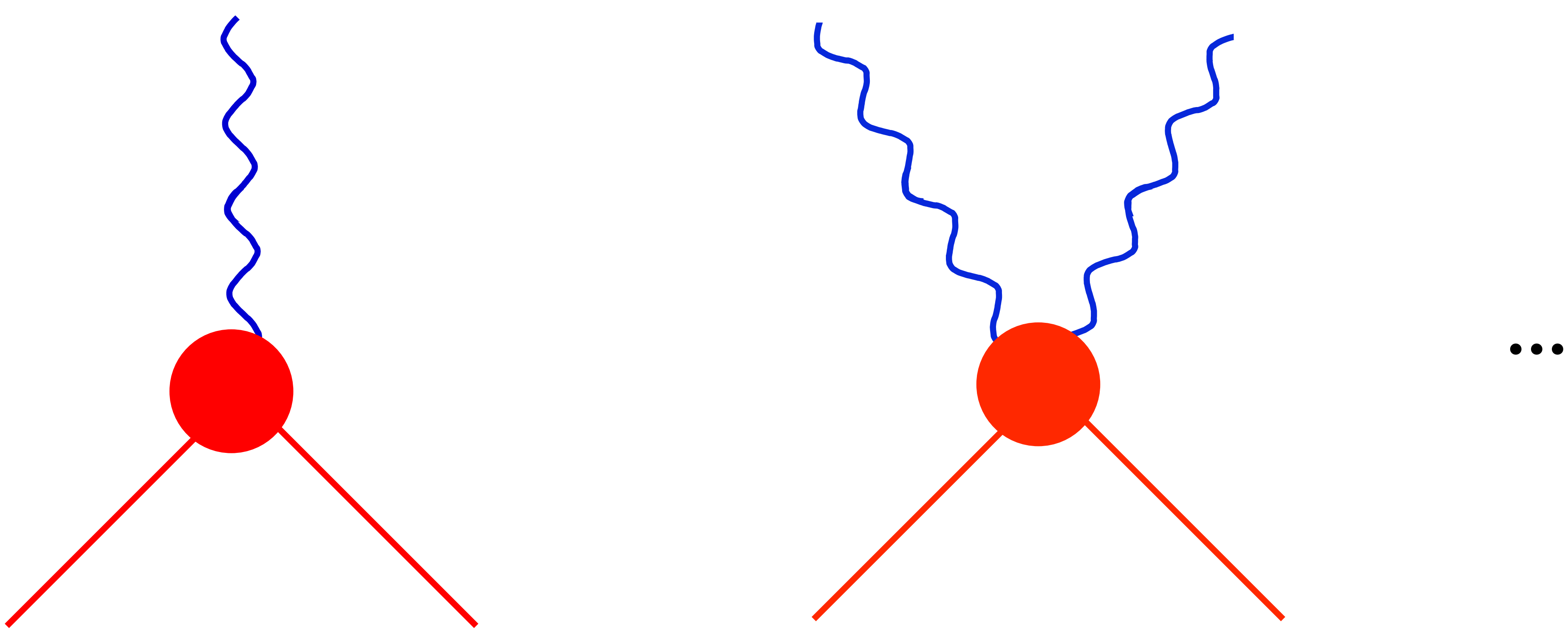
$$S = \int d^4x d^4y \, \mathcal{H}^\dagger(x) F(x-y) \mathcal{H}(y)$$

$$F(x-y) = \left[\partial^2 - \mu^2\right]^{2-\Delta} \delta(x-y)$$

Minimal Gauge Coupling

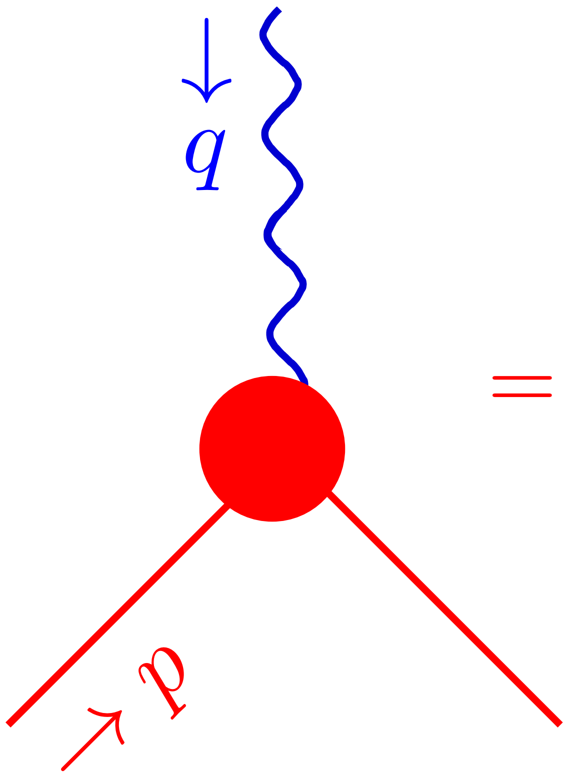
$$F(x-y) \to F(x-y)W(x,y)$$

$$W(x,y) = P\exp\left[-igT^a \int_x^y A_\mu^a dw^\mu\right]$$



cf Mandelstam Ann Phys **19** (1962) 1

Gauge Vertex



$$= \frac{2p^\alpha + q^\alpha}{2p\cdot q + q^2}\left[\left(\mu^2 - (p+q)^2\right)^{2-\Delta} - \left(\mu^2 - p^2\right)^{2-\Delta}\right]$$

Ward-Takahashi Identity

$$ig\Gamma^{a\alpha}(p,q) \;=\; \frac{2p^\alpha + q^\alpha}{2p\cdot q + q^2}\left[\left(\mu^2 - (p+q)^2\right)^{2-\Delta} - \left(\mu^2 - p^2\right)^{2-\Delta}\right]$$

$$iq_\mu \Gamma^{a\mu} = G^{-1}(p+q)T^a - T^a G^{-1}(p)$$

Quantum Critical Higgs Model

$$\mathcal{L} = -\mathcal{H}^\dagger \left[D^2 + \mu^2\right]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta}\mathcal{H}^\dagger\mathcal{H} - V(|\mathcal{H}|)$$

$$-\frac{Y}{\Lambda_F^{\Delta-1}}\bar{\psi}_L \mathcal{H} \psi_R + h.c$$

$$\langle \mathcal{H} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v^\Delta \end{pmatrix}$$

QC Higgs Model

$$\mathcal{L} = -\mathcal{H}^\dagger \left[D^2 + \mu^2\right]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta}\mathcal{H}^\dagger\mathcal{H} - V(|\mathcal{H}|)$$

$$-\frac{Y}{\Lambda_F^{\Delta-1}}\bar{\psi}_L\mathcal{H}\psi_R + h.c$$

$$G = \frac{-i}{(\mu^2 - p^2)^{2-\Delta} + m^{4-2\Delta}}$$

minimal parameterization requires
two mass scales: pole and cut threshold

QC Higgs Model

$$\mathcal{L} = -\mathcal{H}^\dagger \left[D^2 + \mu^2\right]^{2-\Delta} \mathcal{H} + \mu^{4-2\Delta} \mathcal{H}^\dagger \mathcal{H} - V(|\mathcal{H}|)$$

$$-\frac{Y}{\Lambda_F^{\Delta-1}} \bar{\psi}_L \mathcal{H} \psi_R + h.c$$

$$G(p) = \frac{i\, Z_h}{p^2 - m_h^2} + i \int_{\mu^2}^{\infty} \frac{\rho_h(M^2) dM^2}{p^2 - M^2}$$

minimal parameterization requires
two mass scales: pole and cut threshold

QC Higgs Model

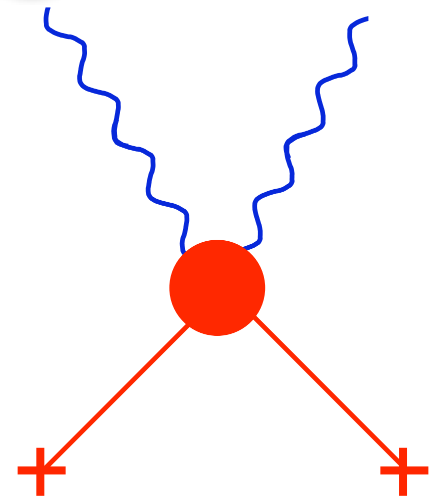
$$G(p) = \frac{i\,Z_h}{p^2 - m_h^2} + i\int_{\mu^2}^{\infty} \frac{\rho_h(M^2)dM^2}{p^2 - M^2}$$

$$\mathcal{H} \to \frac{1}{\sqrt{2-\Delta}}\mu^{\Delta-1}H$$

$$Z_h = \left(\frac{\mu^2}{\mu^2 - m_h^2}\right)^{1-\Delta} = 1 - (\Delta - 1)\frac{m_h^2}{\mu^2} + \mathcal{O}\left(\frac{m_h^4}{\mu^4}\right)$$

approach the SM in two limits: $\Delta \to 1$ or $\mu \to \infty$

QC Higgs and M_W

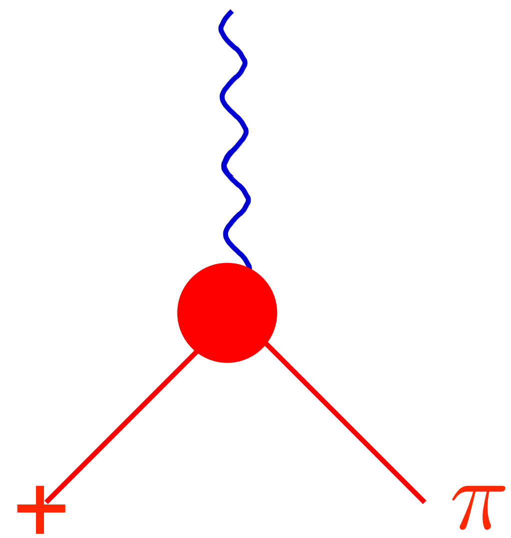


$$-g^2 A^a_\alpha A^b_\beta \langle \mathcal{H}^\dagger \rangle T^a T^b \langle \mathcal{H} \rangle \left\{ \; g^{\alpha\beta}(\Delta - 2)\mu^{2-2\Delta} \right.$$

$$\left. - \frac{q^\alpha q^\beta}{q^2} \left[(\Delta - 2)\mu^{2-2\Delta} - \frac{\left(\mu^2 - q^2\right)^{2-\Delta} - \left(\mu^2\right)^{2-\Delta}}{q^2} \right] \right\}$$

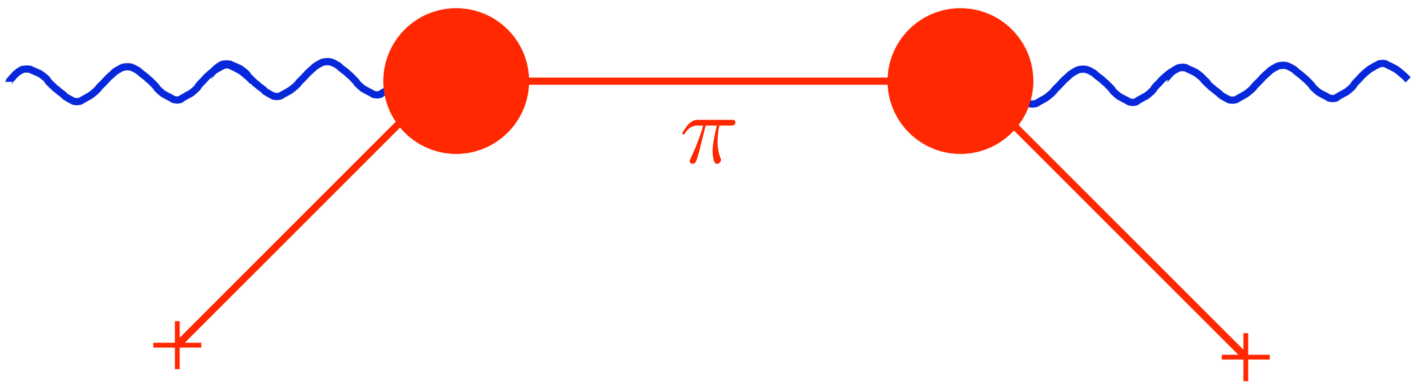
$$M_W^2 \quad = \quad \frac{g^2(2-\Delta)\mu^{2-2\Delta} v^{2\Delta}}{4}$$

GB mixing



$$g\left(\langle\mathcal{H}^\dagger\rangle A^a_\alpha T^a \Pi - \Pi^\dagger A^a_\alpha T^a \langle\mathcal{H}\rangle\right)\left[\left(\mu^2 - q^2\right)^{2-\Delta} - \left(\mu^2\right)^{2-\Delta}\right] q^\alpha / q^2$$

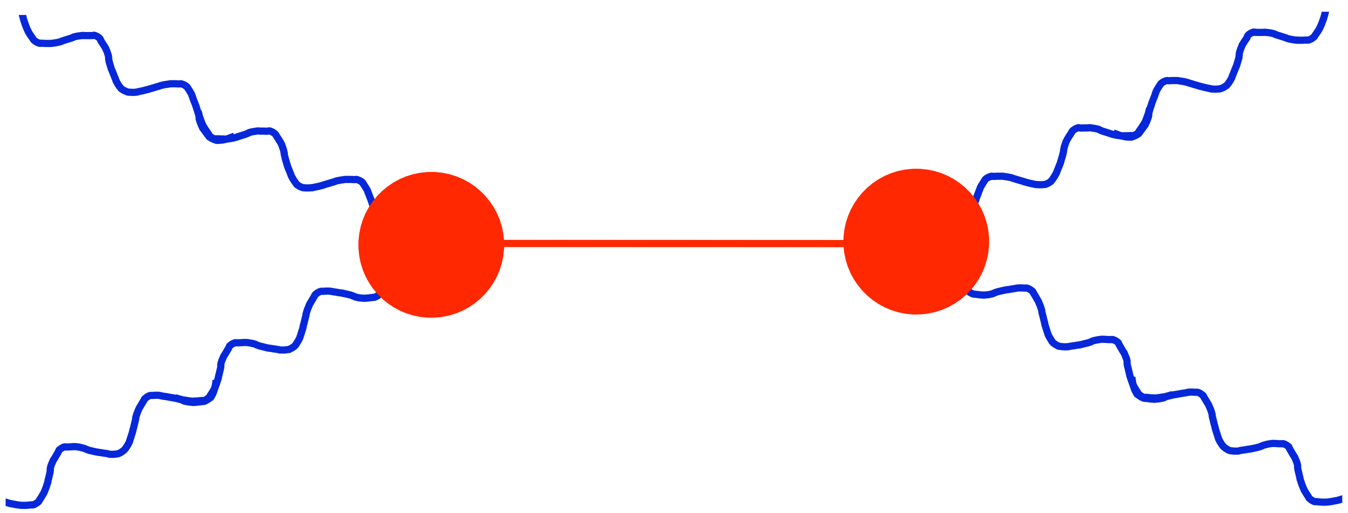
Gauge invariance is maintained



$$\Pi^{ab\alpha\beta}(q) = -g^2 \langle \mathcal{H}^\dagger \rangle T^a T^b \langle \mathcal{H} \rangle \frac{q^\alpha q^\beta}{q^4} \times \left[\left(\mu^2 - q^2\right)^{2-\Delta} - \left(\mu^2\right)^{2-\Delta} \right]^2 G_{GB}(q)$$

$$G_{GB}(q) = -\frac{i}{\left(\mu^2 - q^2 - i\epsilon\right)^{2-\Delta} - \mu^{4-2\Delta}}$$

WW Scattering

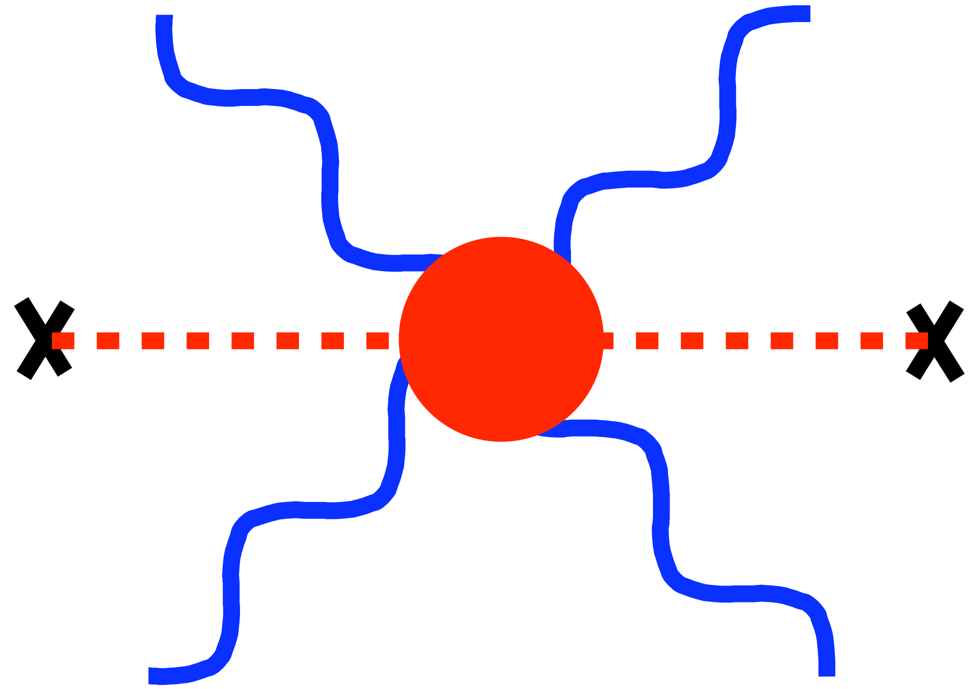


at large s

$$\mathcal{M}_h = -i\frac{g^4}{4M_W^2(2-\Delta)\mu^{2-2\Delta}}(-s)^{2-\Delta}$$

QC Higgs exchange is insufficient
to unitarize WW scattering

WW Scattering

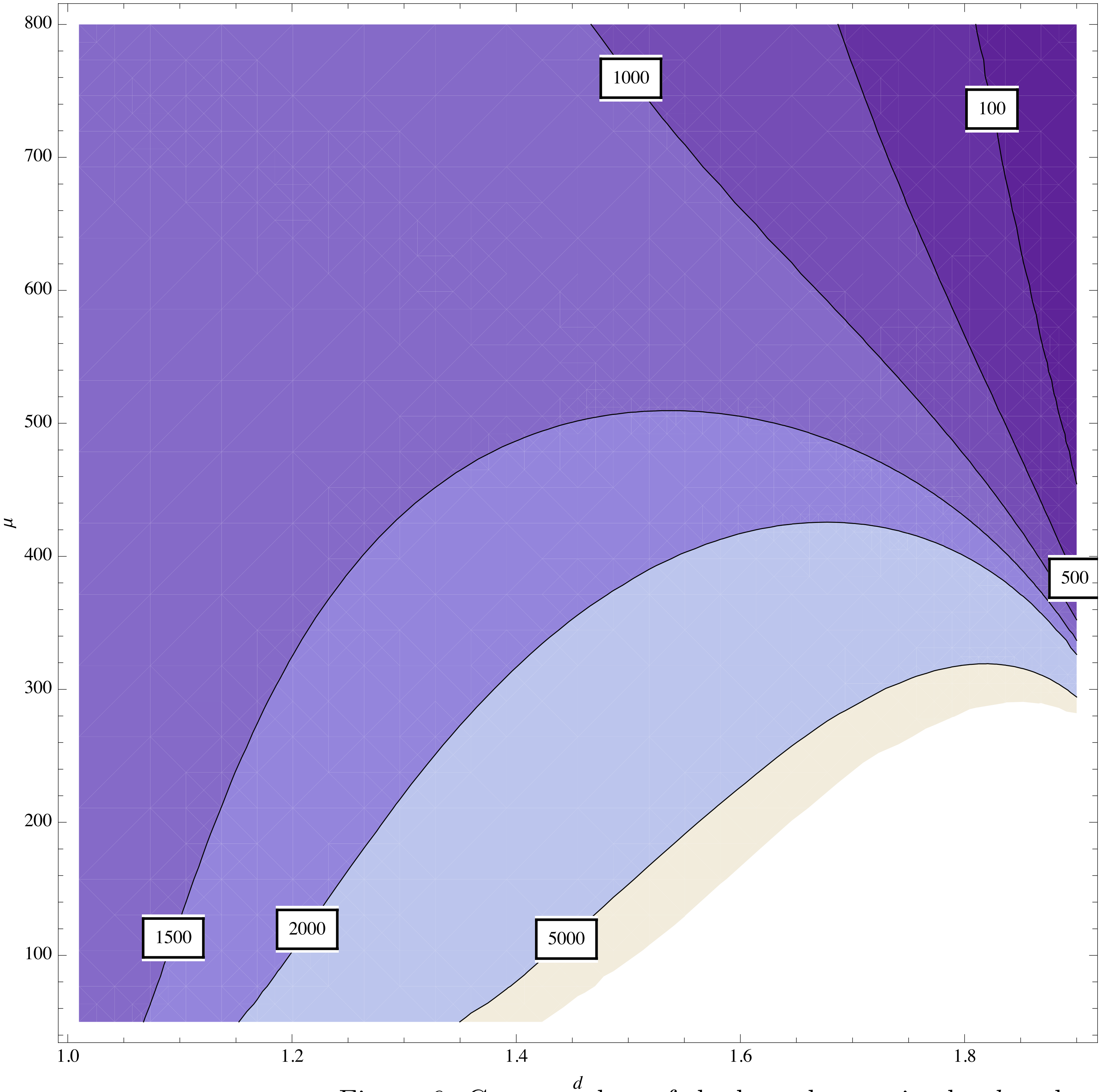


$$\mathcal{M}_{hh} \quad = \quad -i\frac{g^2}{4M_W^2}\left[s + \frac{(-s)^{2-\Delta}}{(2-\Delta)\mu^{2-2\Delta}}\right]$$

QC Higgs 6 point vertex does unitarize WW scattering

Stancato JT, hep-ph/0807.3961

Partial Wave Bound



$$|a_0| \leq 1$$

$$G_F\, m^{4-2\Delta} < 4\pi\sqrt{2}(2-\Delta)\mu^{2-2\Delta}$$

Figure 6: Contour plots of the bound on m in the d-μ plane. The darkest regions have the lowest upper bound on m, Contour lines are shown for 100, 500, 1000, 1500, 2000, and 5000 GeV.

Mass Divergence

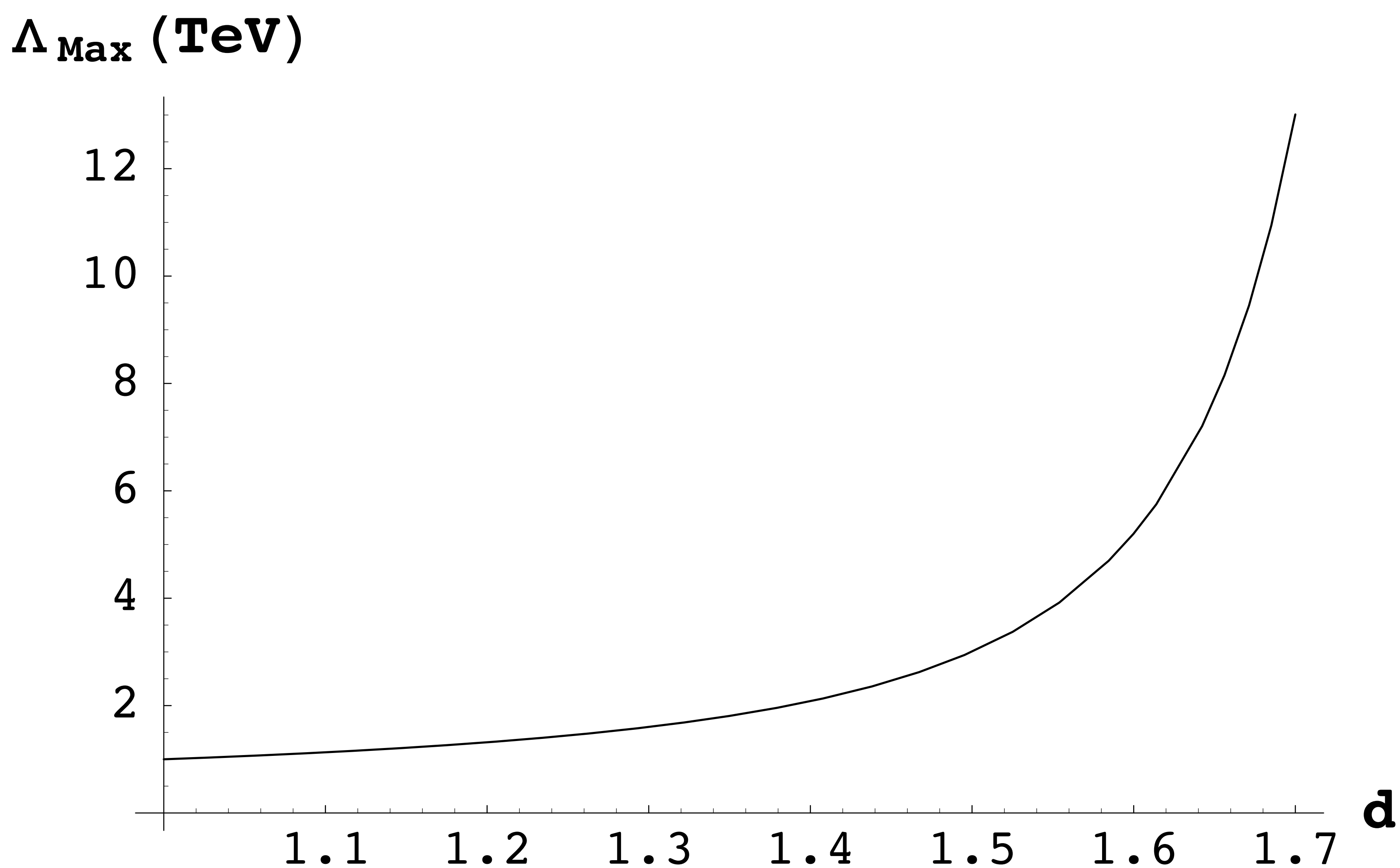
$$m_h^2 \sim \frac{\Lambda^{4-2\Delta}}{16\pi^2}$$

$$\mathcal{L}_Y = -\frac{Y}{\Lambda^{\Delta-1}}\bar{\psi}_L \mathcal{H} \psi_R + h.c$$

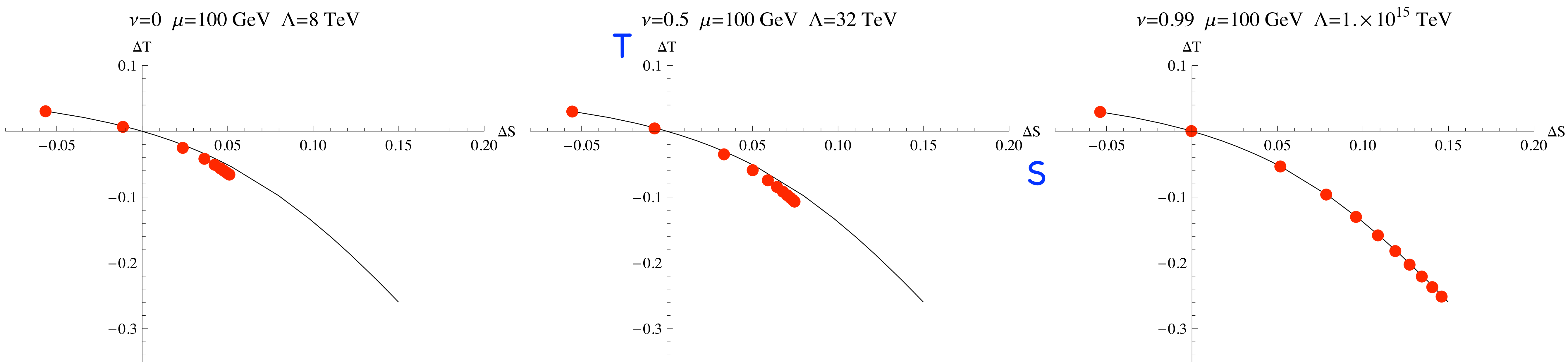
$$m_h^2 = 3\left(\frac{Y}{\Lambda^{\Delta-1}}\right)^2 \frac{\Lambda^2}{16\pi^2} = 3\,Y^2\,\frac{\Lambda^{4-2\Delta}}{16\pi^2}$$

Solve the little hierarchy problem?

loop < tree



Precision Measurements



$\Delta = 2$ $\Delta = 1.5$ $\Delta = 1.01$

Falkowski & Perez-Victoria, hep-ph/0901.3777

Quantum Critical Higgs

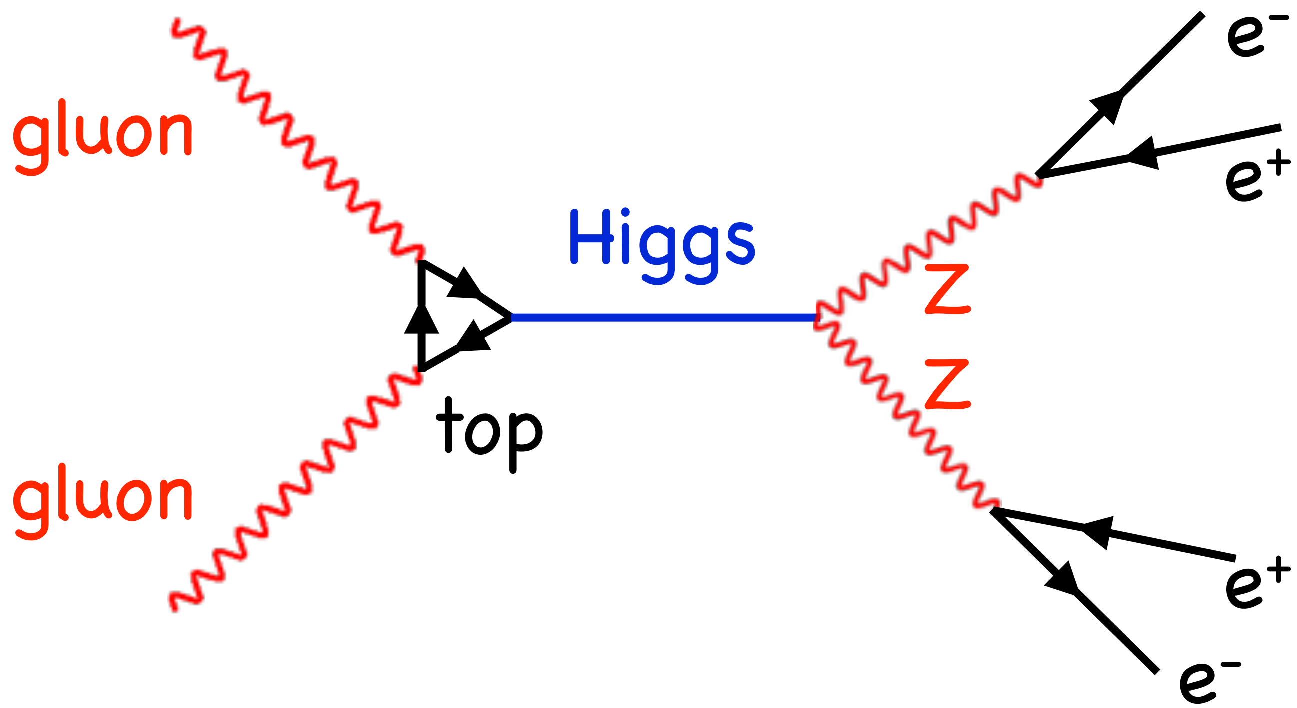
$$G = \frac{-i}{(\mu^2 - p^2)^{2-\Delta} + m^{4-2\Delta}}$$

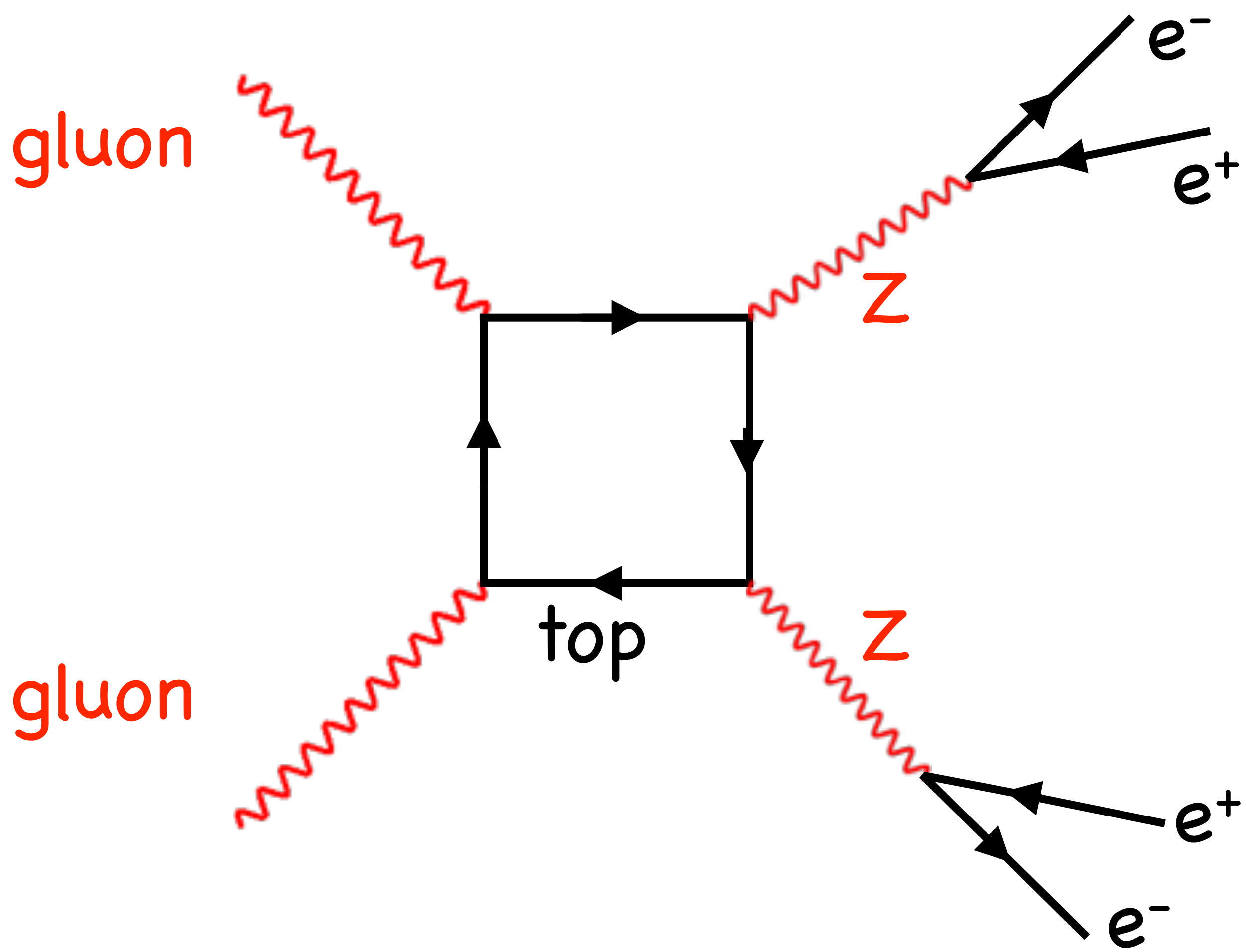
compare to Standard Model:

$$G = \frac{i}{p^2 - m_h^2}$$

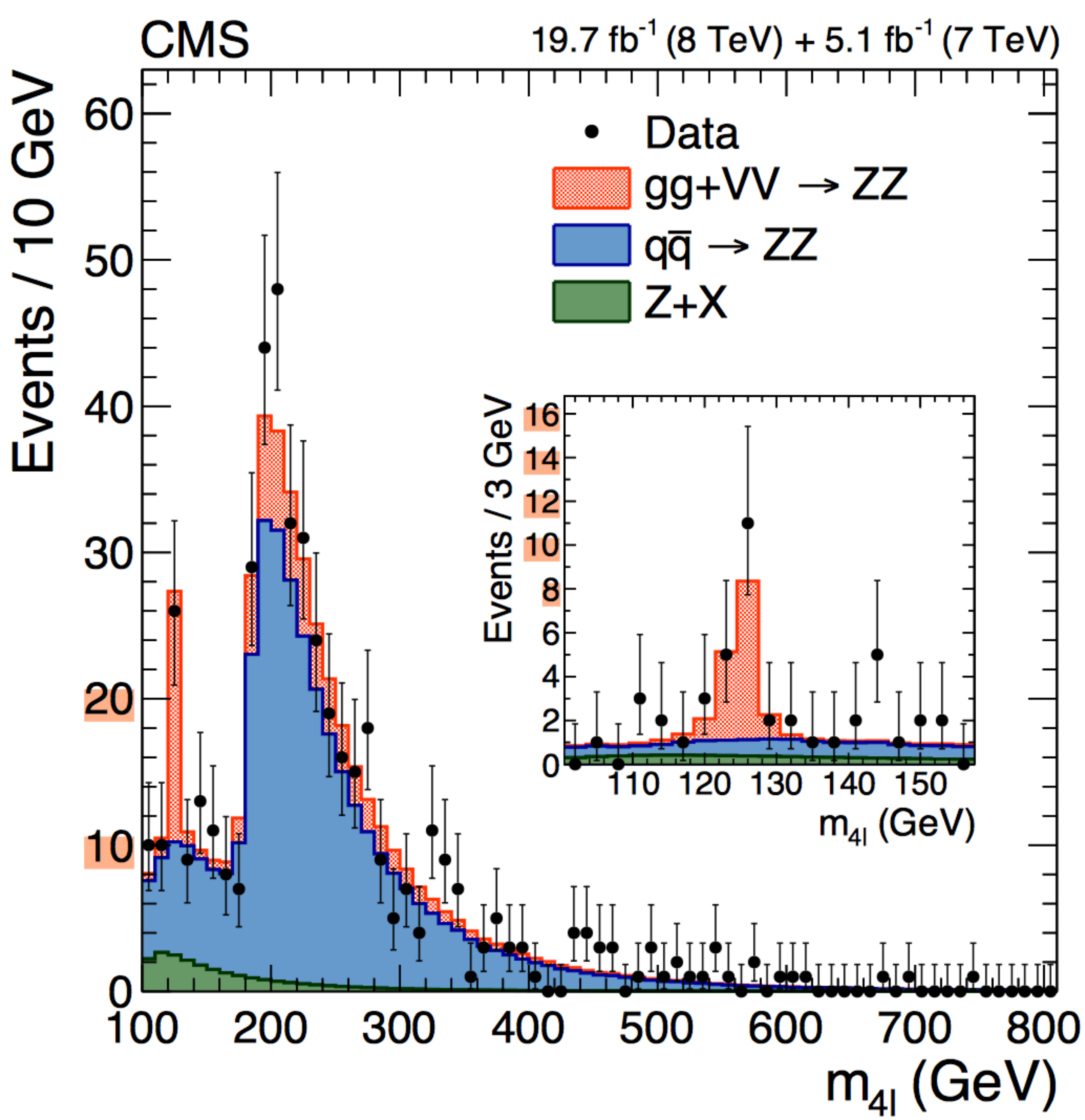
How do we test this?

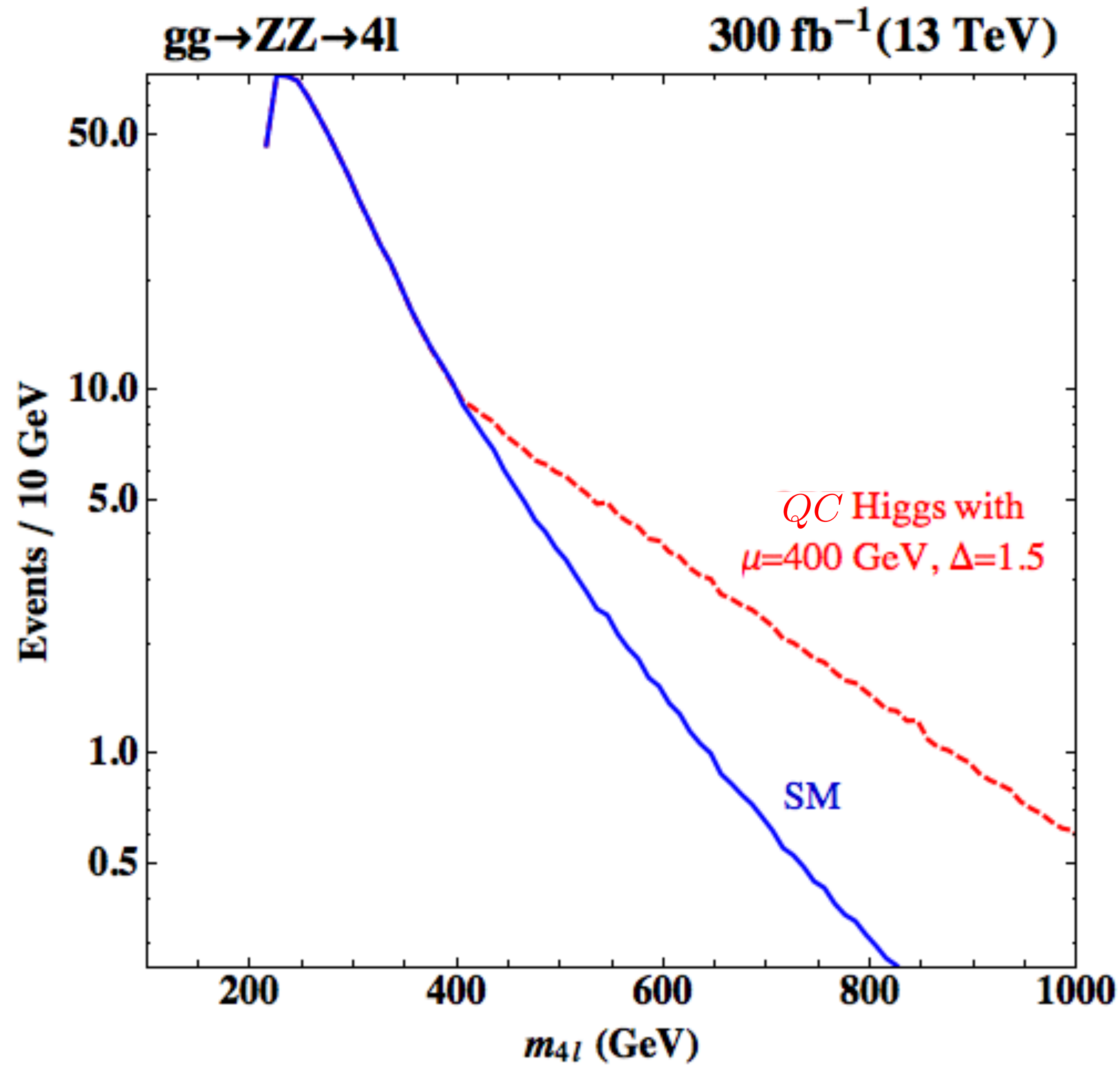
LHC Interference





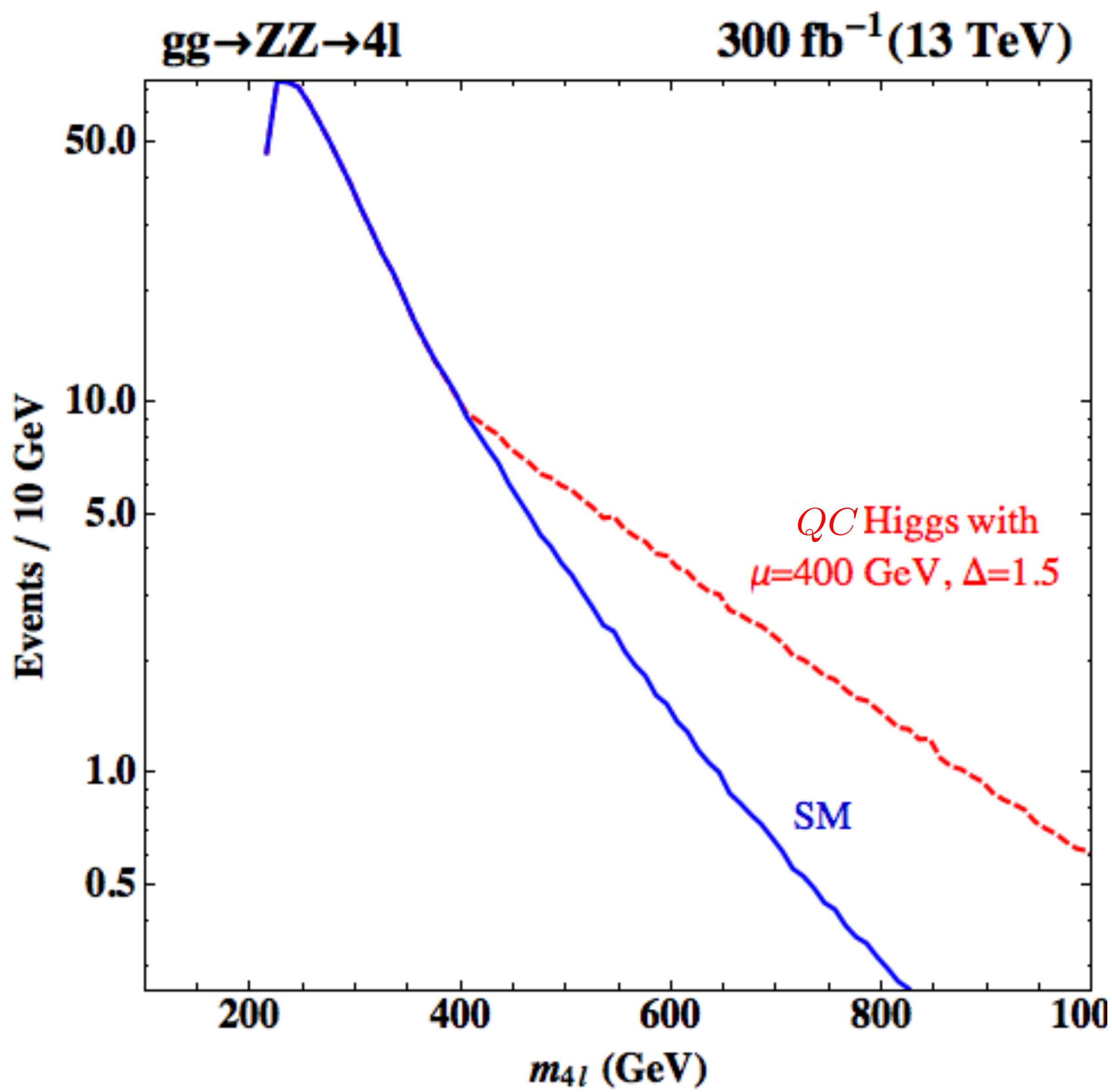
LHC Experiment

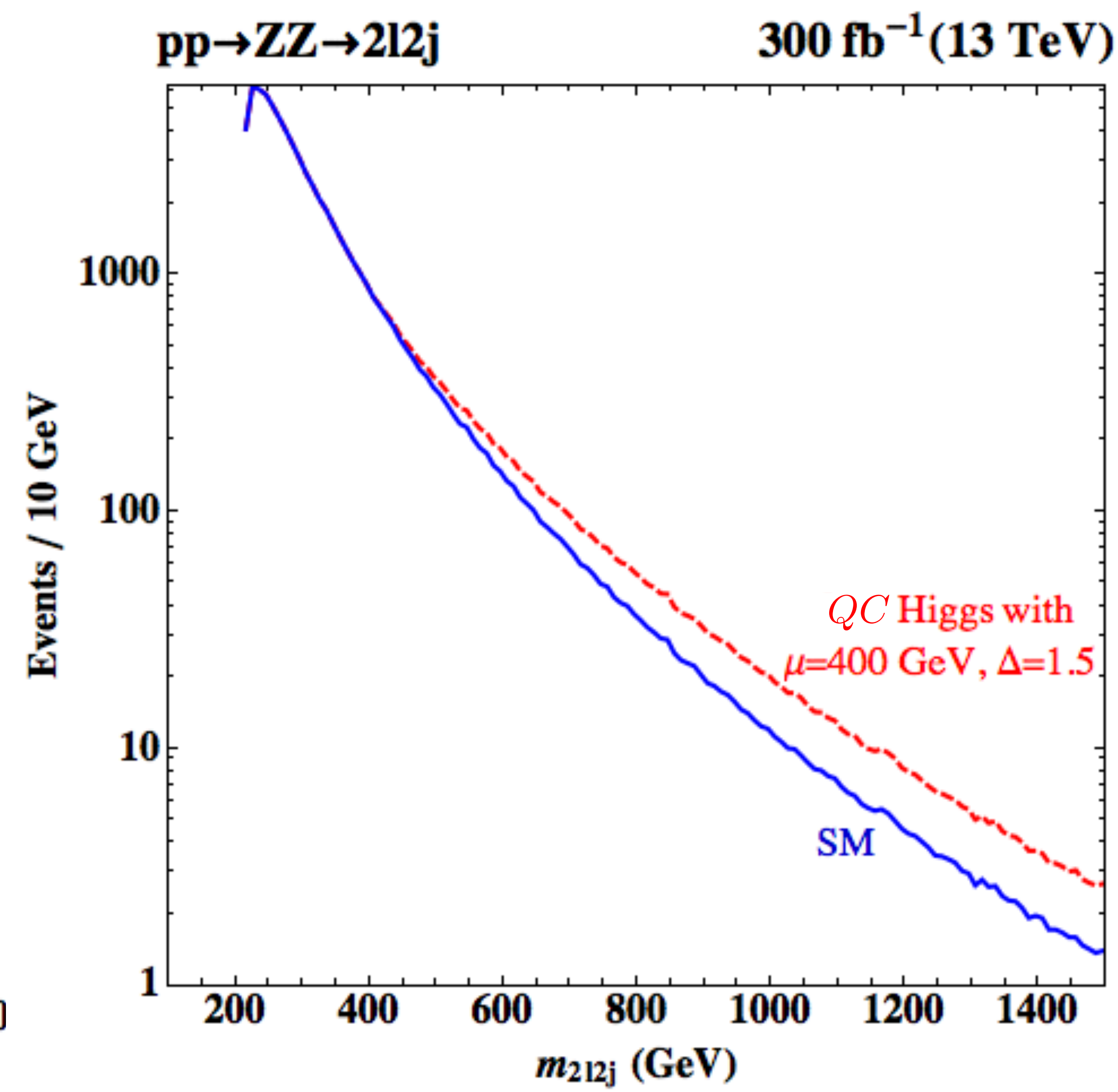




Csáki, Hubisz, Lee, Serra, Bellazzini, JT

LHC Experiment





Conclusions

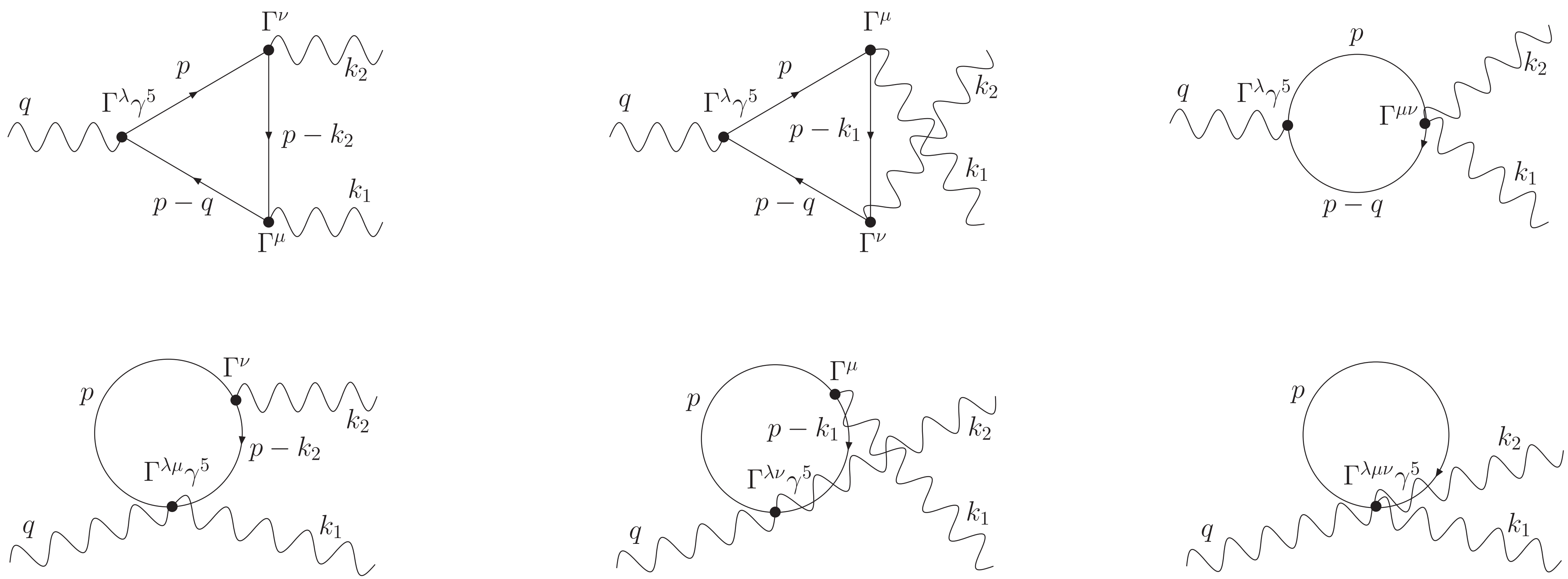
The Electroweak Phase Transition seems

to be close to a Quantum Critical Point

The LHC can test whether the Higgs

has a non-trivial critical exponent

Unfermion Anomalies



independent of Δ

Galloway, McRaven, JT hep-th/0805.079

AdS Gauge Fields

$$-\frac{1}{4g_5^2}\int_\epsilon^\infty d^4x\,dz\left(\frac{R}{z}\right)\Phi(z)F^{aMN}F^a_{MN}$$

dilaton: $\Phi(z) = e^{-mz}$

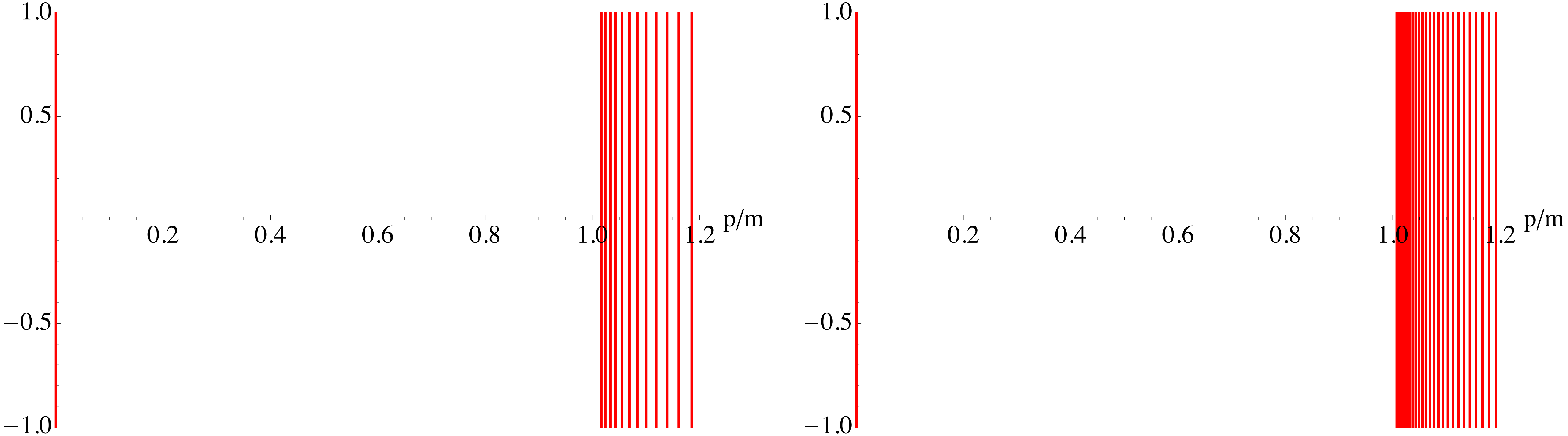
$$\frac{1}{g_4^2} = \frac{R}{g_5^2}\int_\epsilon^\infty \frac{e^{-mz}}{z} \approx \frac{R}{g_5^2}[-\gamma_E - \log(m\epsilon)]$$

$$\frac{1}{g_4^2} \approx \frac{R}{g_5^2}\log\left(\frac{\Lambda_{UV}}{m}\right)$$

Gauge KK Modes

$$f''(z) - \left(m + \frac{1}{z}\right) f'(z) + p^2 f(z) = 0$$

spectrum



$z_{IR} = 100/m$ $z_{IR} = 200/m$

Massive Unparticle phase space

$$d\Phi(p,\mu,d) = A_d\,\theta\left(p^0\right)\,\theta\left(p^2-\mu^2\right)\,\left(p^2-\mu^2\right)^{d-2}$$

$$d\Phi(p,\mu,1) = 2\pi\,\theta\left(p^0\right)\,\delta(p^2-\mu^2)$$

SUSY AdS/CFT

$$S = \int d^4x\,dz \left\{ \int d^4\theta \left(\frac{R}{z}\right)^3 [\Phi^*\,\Phi + \Phi_{\,c}\,\Phi_c^*] + \right.$$

$$\left. + \int d^2\theta \left(\frac{R}{z}\right)^3 \left[\frac{1}{2}\,\Phi_c\,\partial_z\Phi - \frac{1}{2}\,\partial_z\Phi_c\,\Phi + m(z)\frac{R}{z}\,\Phi_c\,\Phi\right] + h.c.\right.$$

$$\Phi = \{\phi, \chi, F\}$$

$$\Phi_c = \{\phi_c, \psi, F_c\}$$

$$m(z) = c$$

$$d_s = \frac{3}{2} - c$$

$$d_f = 2 - c$$

$$c < \frac{1}{2}$$

Cacciapaglia, Marandella, JT hep-th/0802.2946

SUSY
AdS/CFT/Unparticles

$$S = \int d^4x\, dz \left\{ \int d^4\theta \left(\frac{R}{z}\right)^3 [\Phi^* \Phi + \Phi_c \Phi_c^*] + \right.$$

$$\left. + \int d^2\theta \left(\frac{R}{z}\right)^3 \left[\frac{1}{2} \Phi_c \partial_z \Phi - \frac{1}{2} \partial_z \Phi_c \Phi + m(z) \frac{R}{z} \Phi_c \Phi\right] + h.c. \right.$$

$$m(z) = c + \mu z$$

Bulk Profiles

$$\chi(p,z) = \chi_4(p)\left(\frac{z}{z_{UV}}\right)^2 f_L(p,z) \qquad \phi(p,z) = \phi_4(p)\left(\frac{z}{z_{UV}}\right)^{3/2} f_L(p,z)$$

$$\psi(p,z) = \psi_4(p)\left(\frac{z}{z_{UV}}\right)^2 f_R(p,z) \qquad \phi_c(p,z) = \phi_{c4}(p)\left(\frac{z}{z_{UV}}\right)^{3/2} f_R(p,z)$$

Effective Potential

$$\mu > 0, \;\; c < 0$$

$$\frac{\partial^2}{\partial z^2} f_R + \left(p^2 - \mu^2 - 2\frac{\mu c}{z} - \frac{c(c-1)}{z^2} \right) f_R \quad = \quad 0$$

$$\frac{\partial^2}{\partial r^2} u + \left(2\mu E - 2\frac{\mu \alpha}{z} - \frac{\ell(\ell+1)}{z^2} \right) u \quad = \quad 0$$

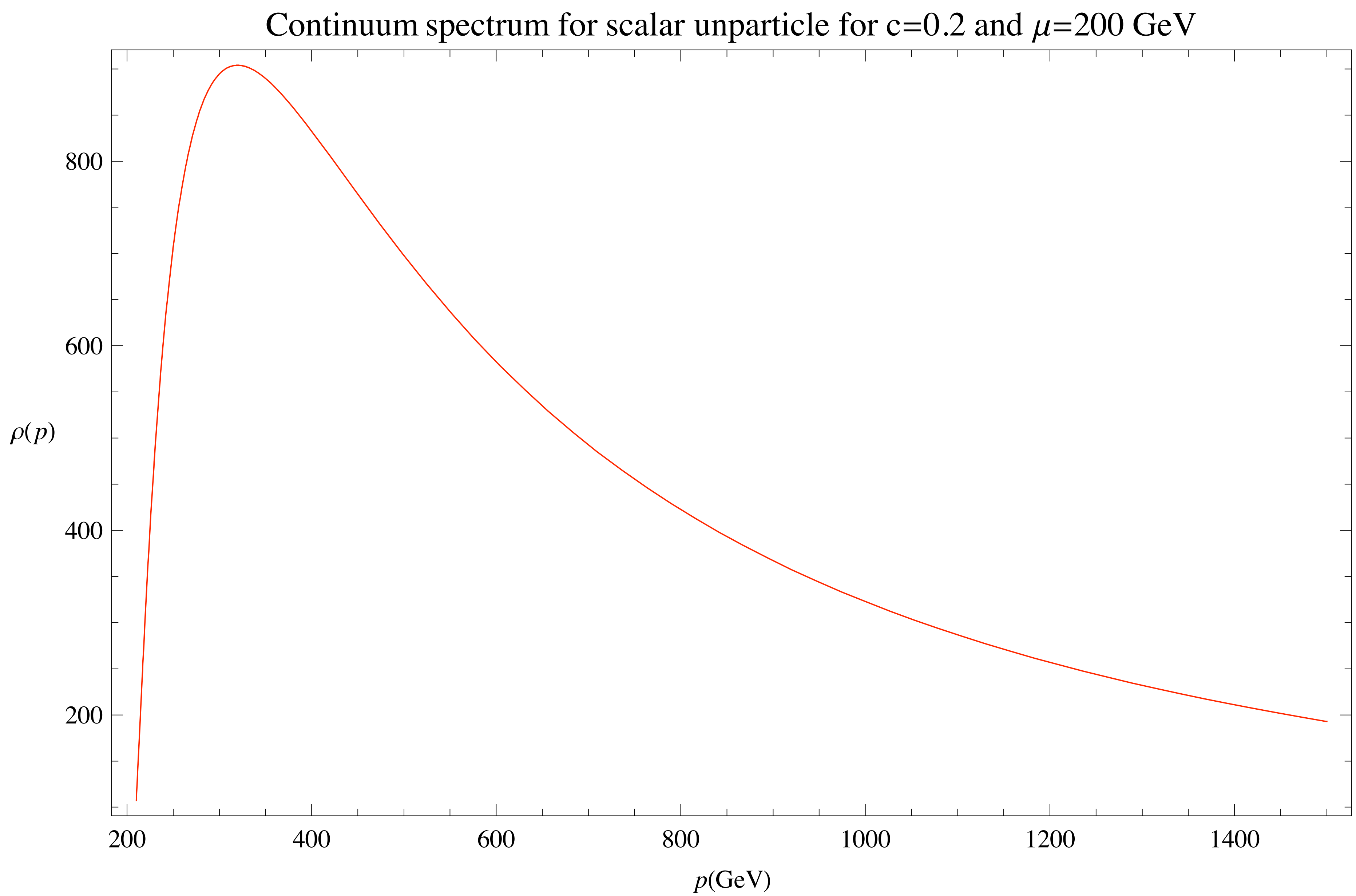
$$\alpha = \ell = |c|$$

Two-Point Function

$$\Delta(p) \approx -\frac{2^{1-2c}(-p^2+\mu^2)^{1/2-c}\epsilon^{1-2c}\Gamma(2c)\Gamma\left(1-c+\frac{c\mu}{\sqrt{-p^2+\mu^2}}\right)}{p^2\Gamma(1-2c)\Gamma\left(c+\frac{c\mu}{\sqrt{-p^2+\mu^2}}\right)}$$

discrete resonances below threshold
for c<0

Spectral Density



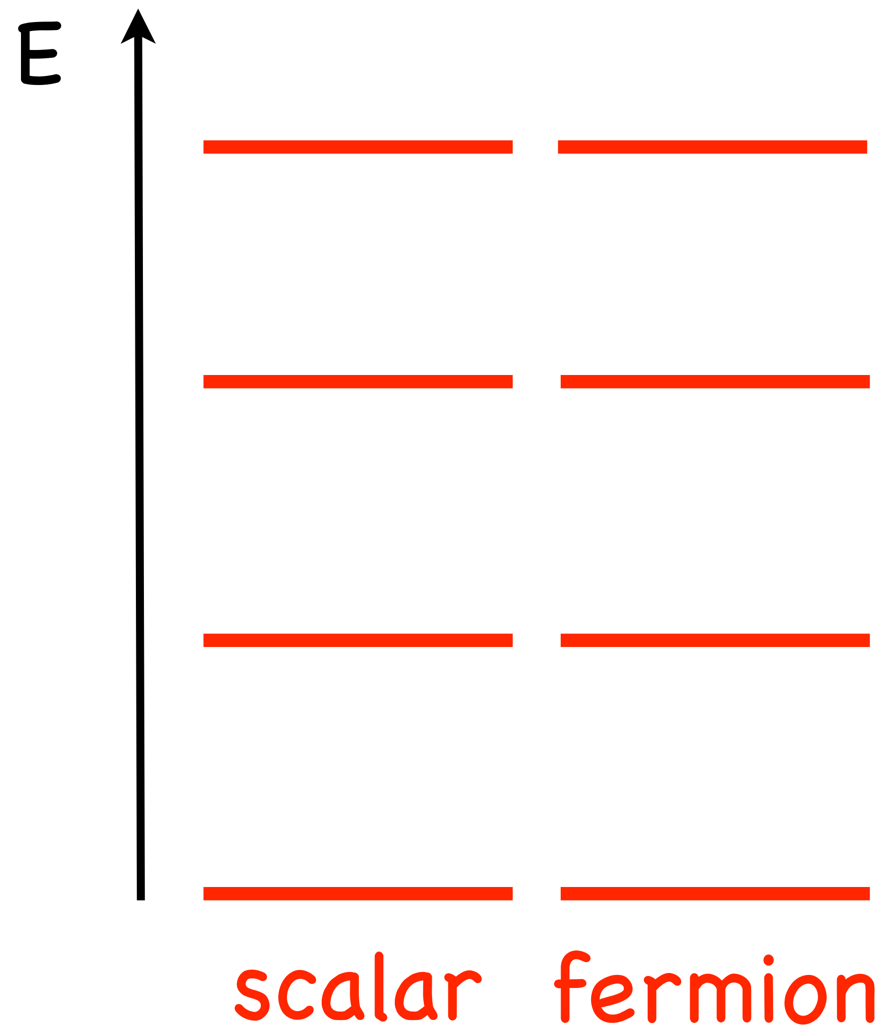
SUSY Breaking

$$\delta S = \frac{1}{2}\int d^4x \int dz \,(m^2/z_{UV} \cdot \phi^* \phi + h.c.)\delta(z - z_{UV})$$

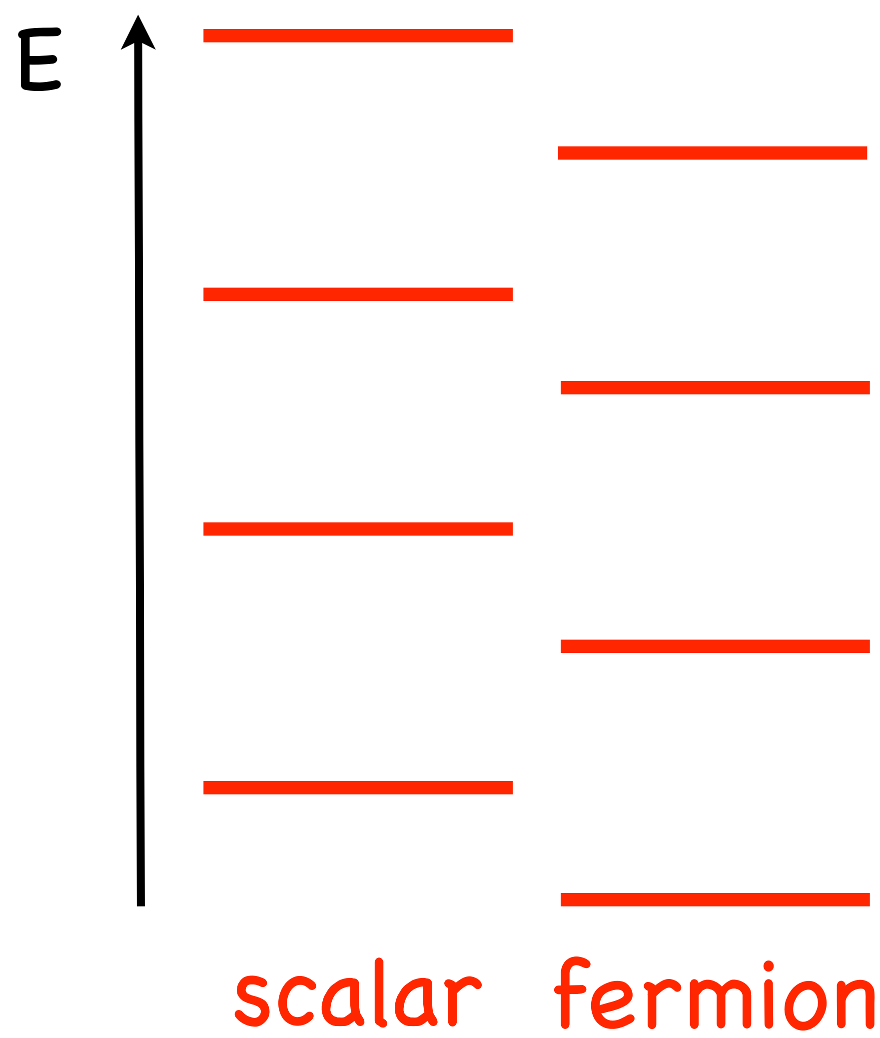
$$\Delta(p^2) = \left(\frac{R}{z_{UV}}\right)^3 \frac{f_L}{p f_R - (m^2/z_{UV}) f_L}$$

Scalar spectrum shifted up

KK Modes

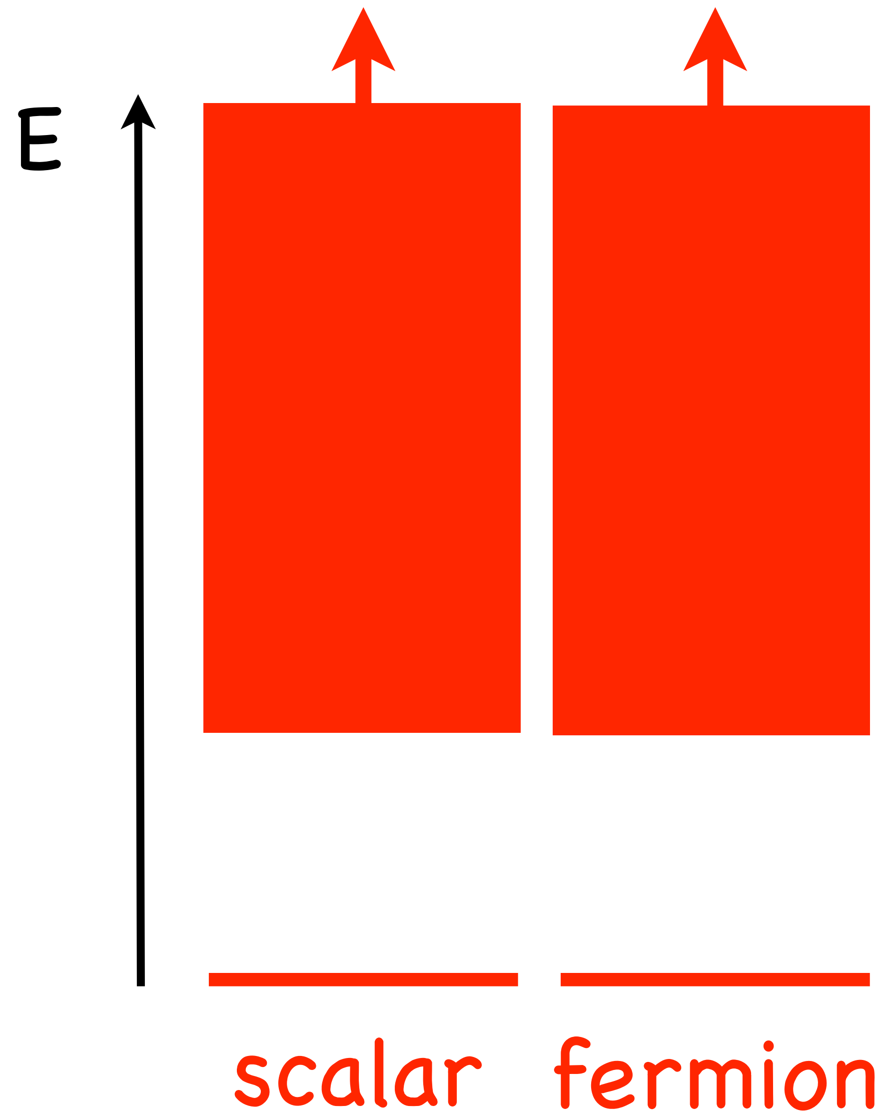


Scherk-Schwarz

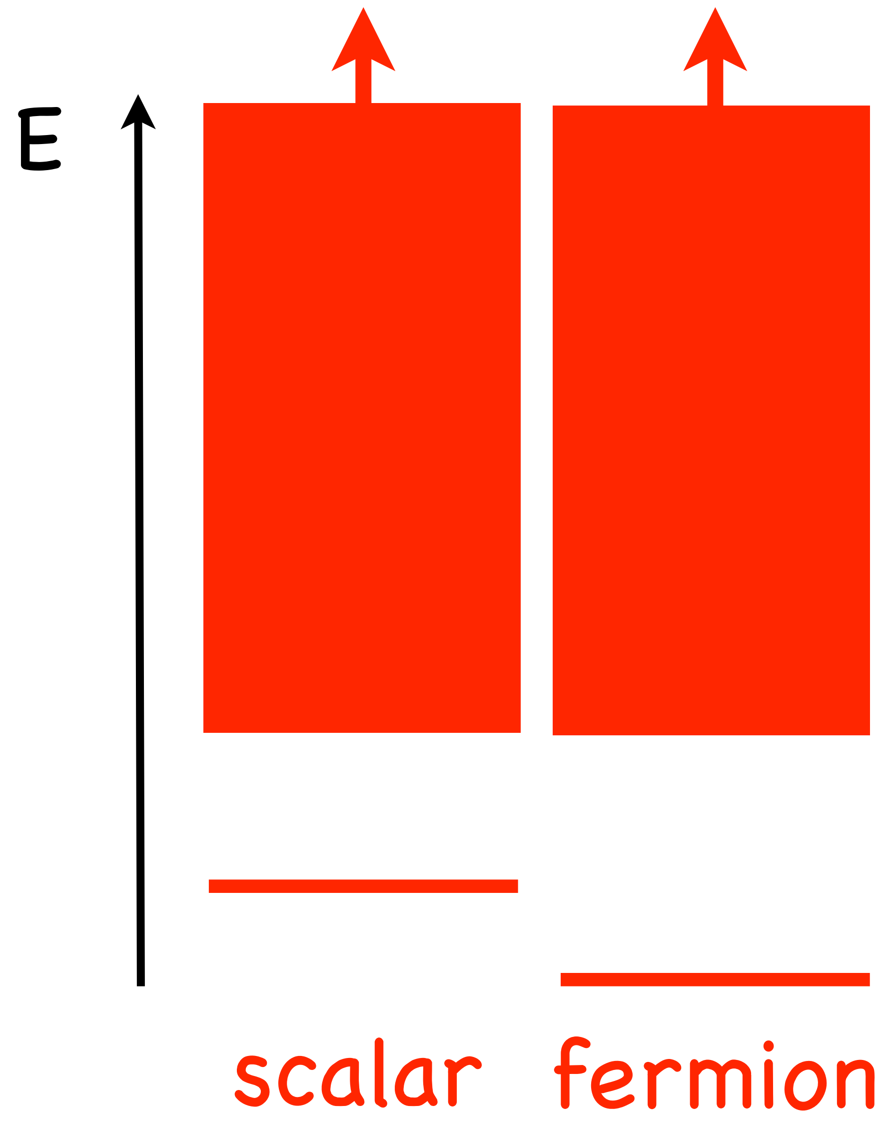


Phys. Lett. B82, 60 (1979)

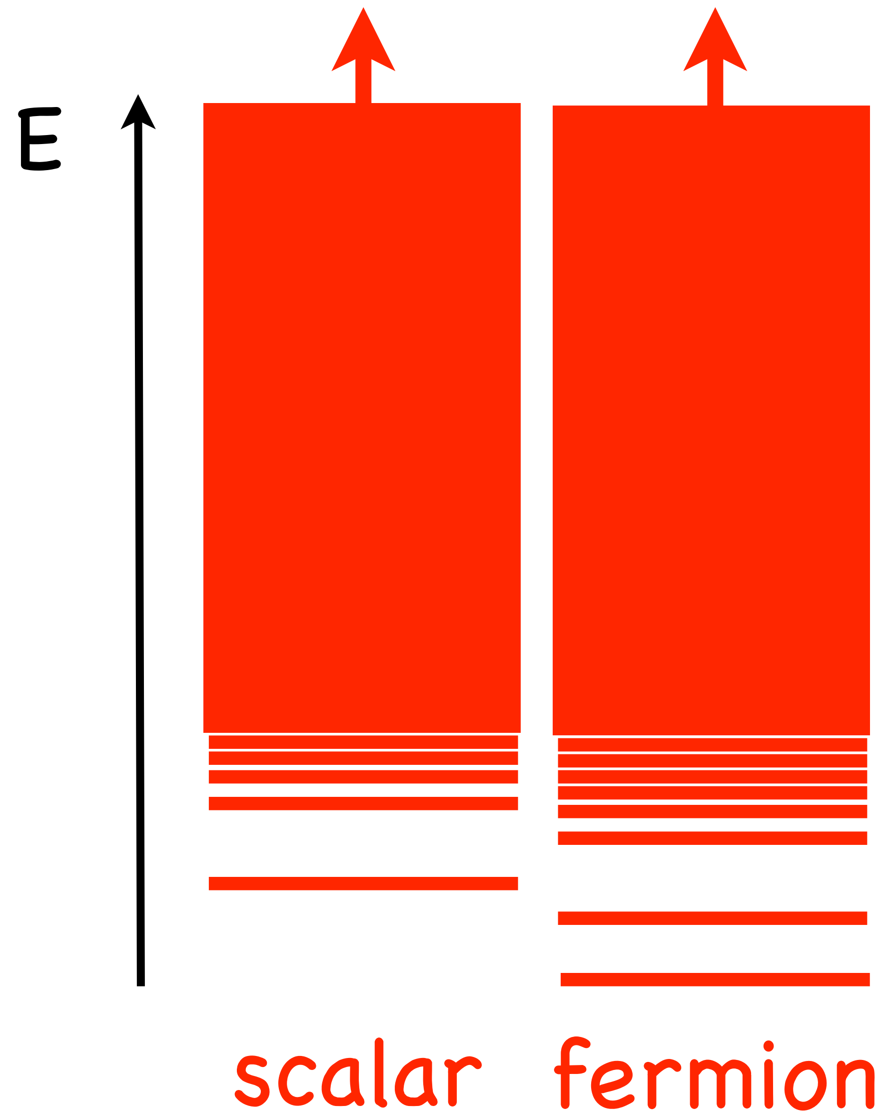
SUSY-CFT



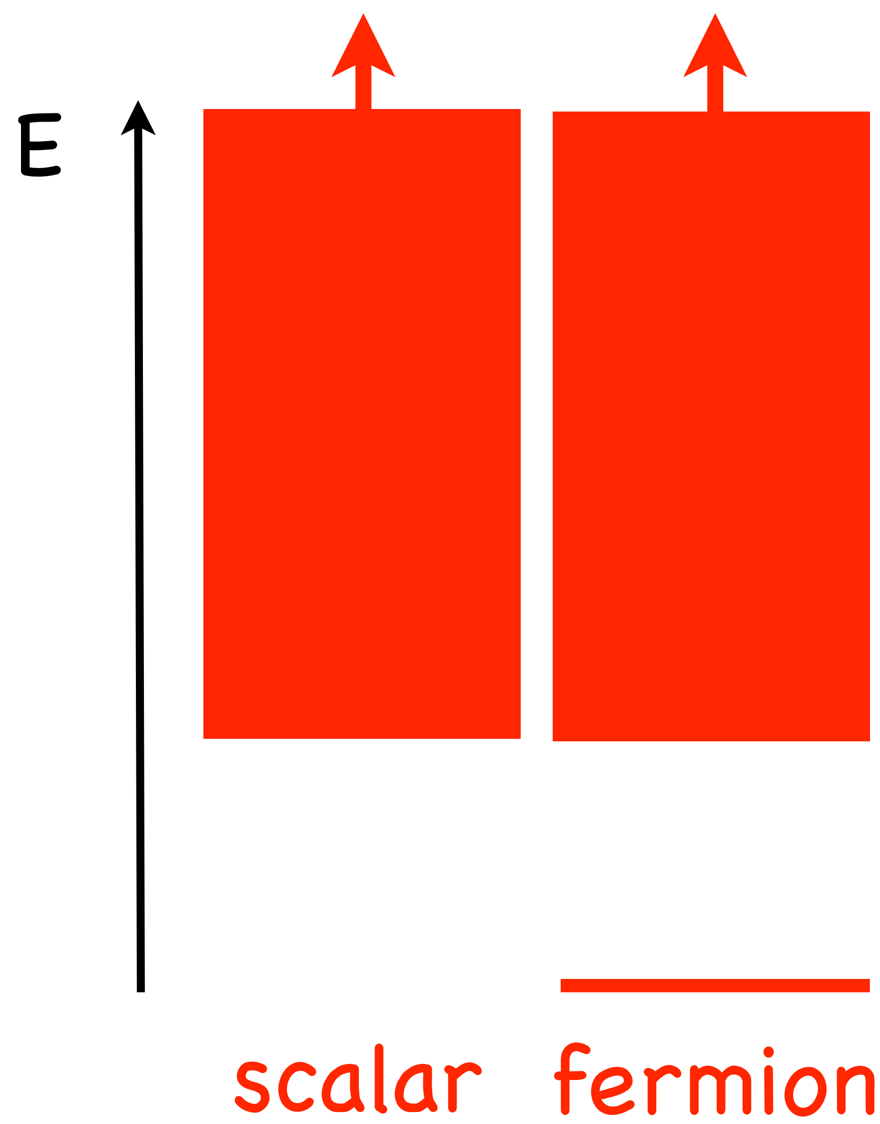
Broken SUSY-CFT



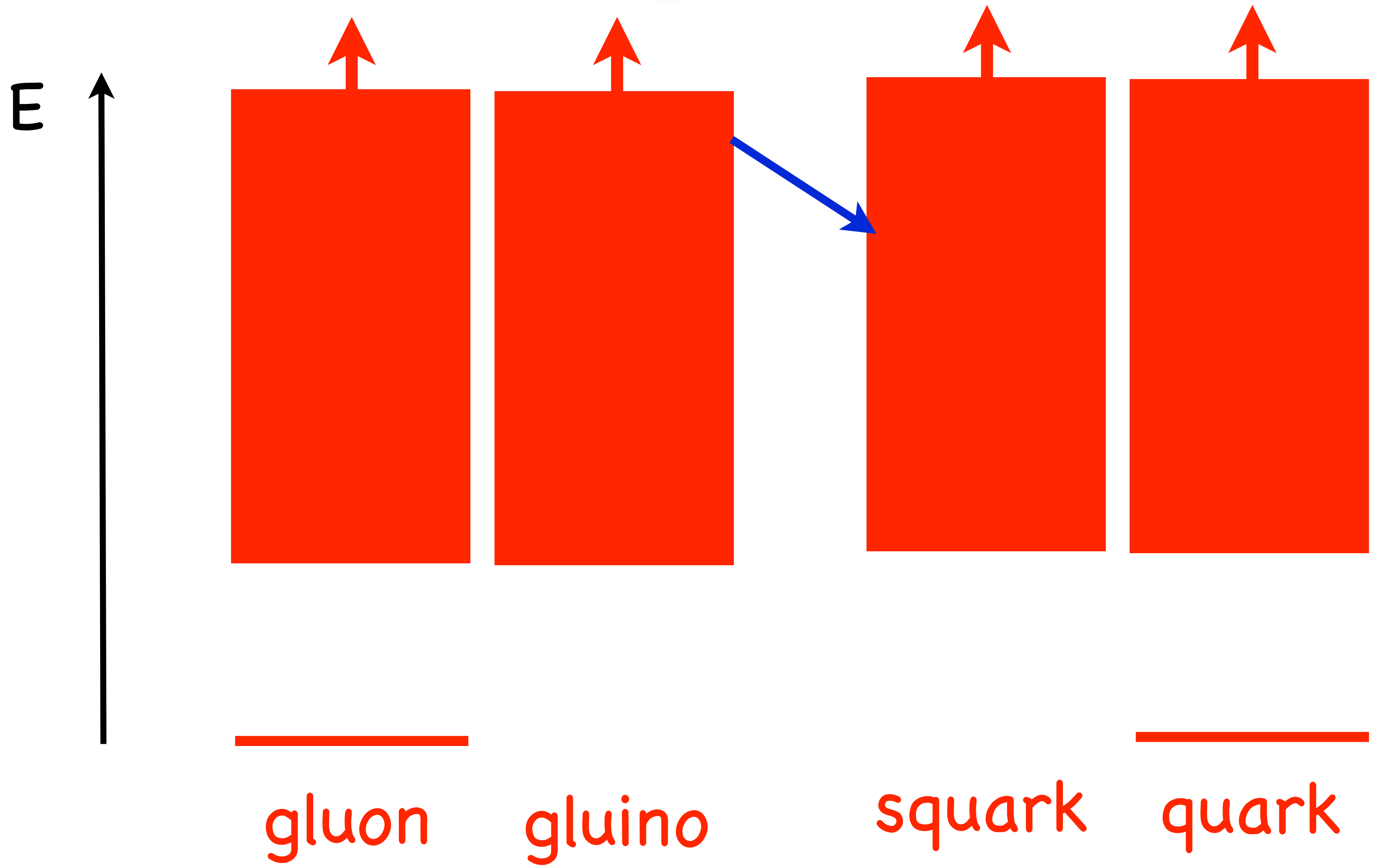
SUSY-Un-Partners



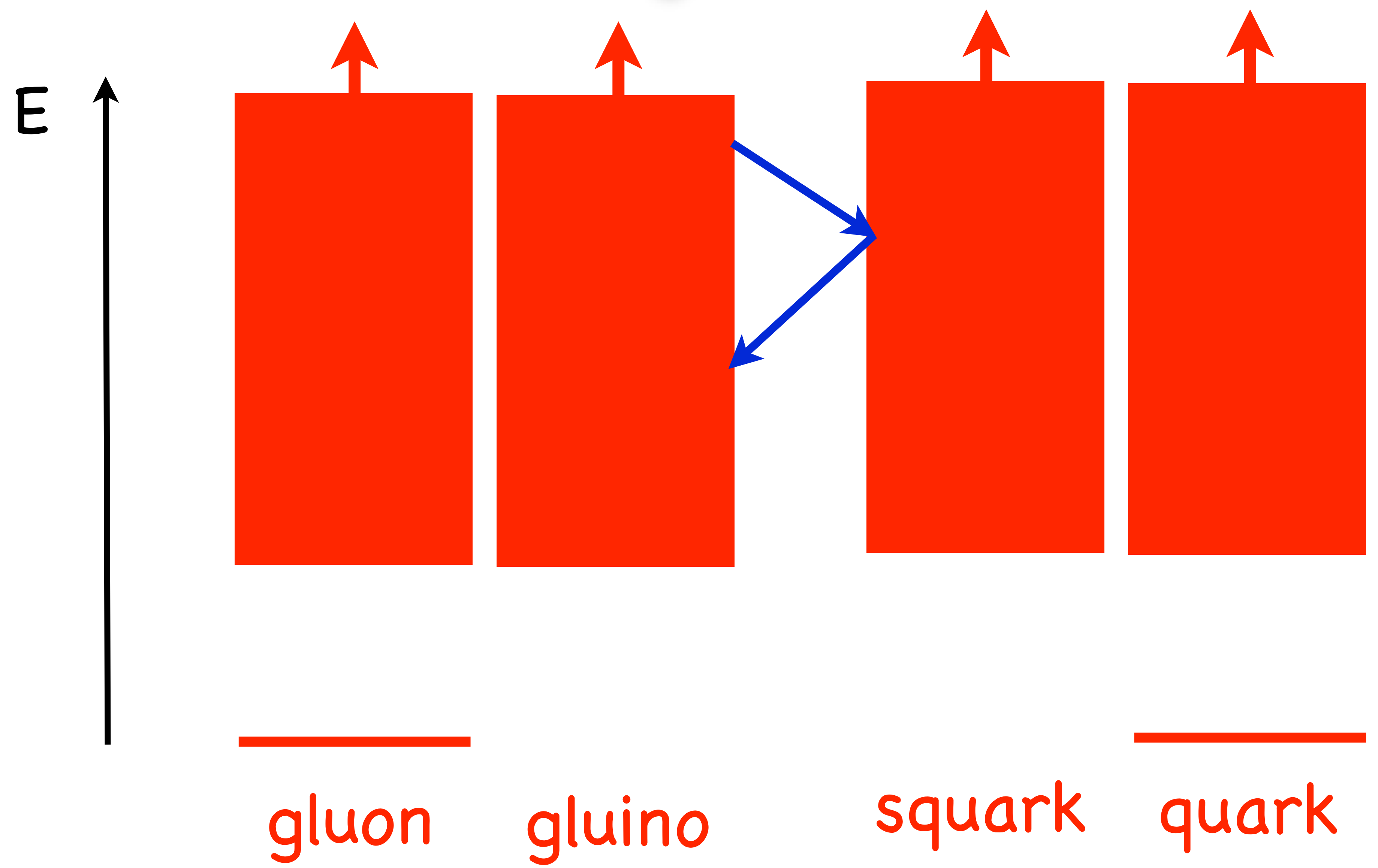
SUSY-Un-Partner



Decay Chains



Decay Chains



Decay Chains