Natural Inflation and Quantum Gravity

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Based on arXiv:1412.3457 (to appear in PRL)

with Anton de la Fuente and Prashant Saraswat

$\approx$ PLAGIARIZED FROM

Natural Inflation and Quantum Gravity

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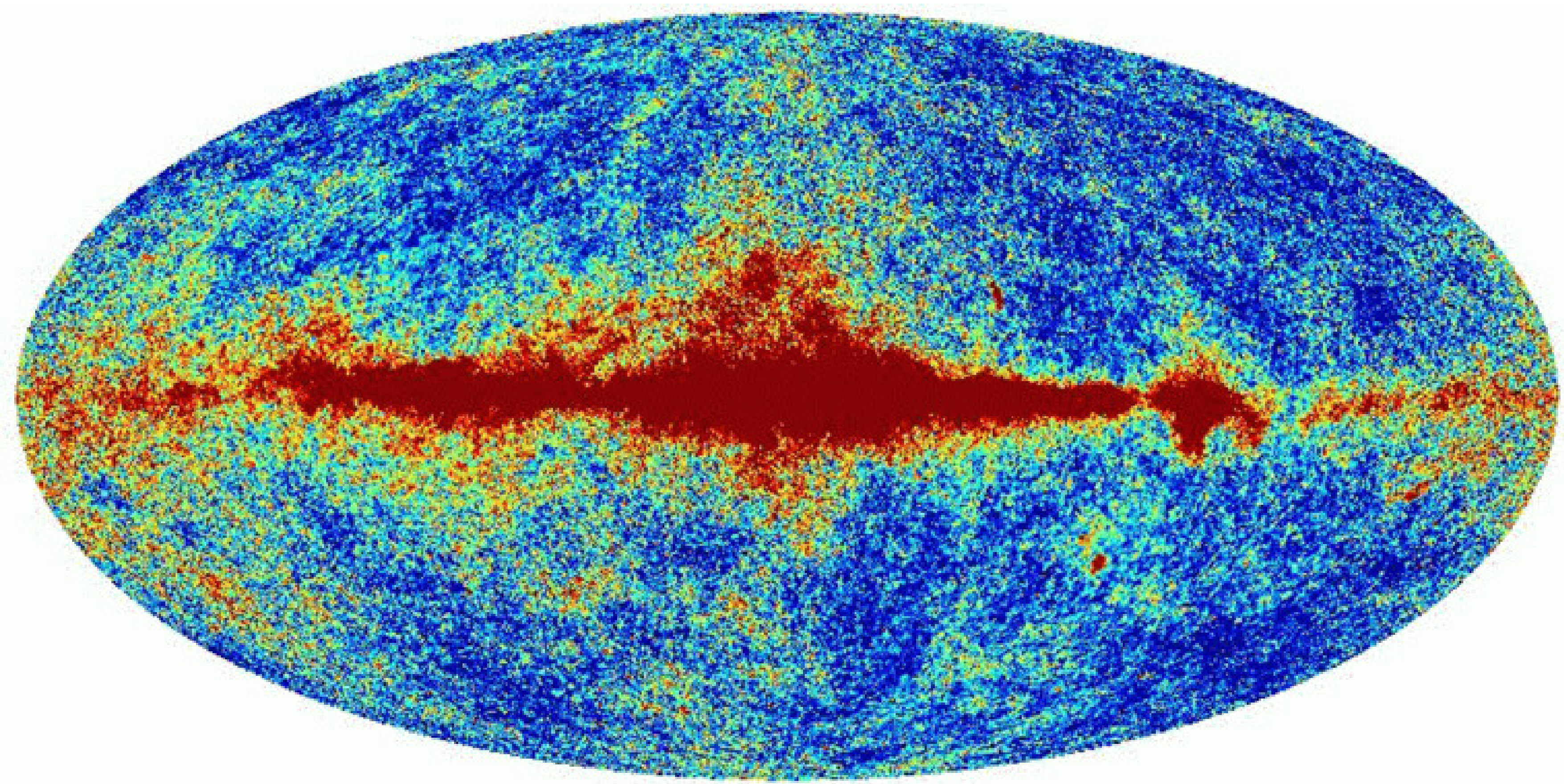
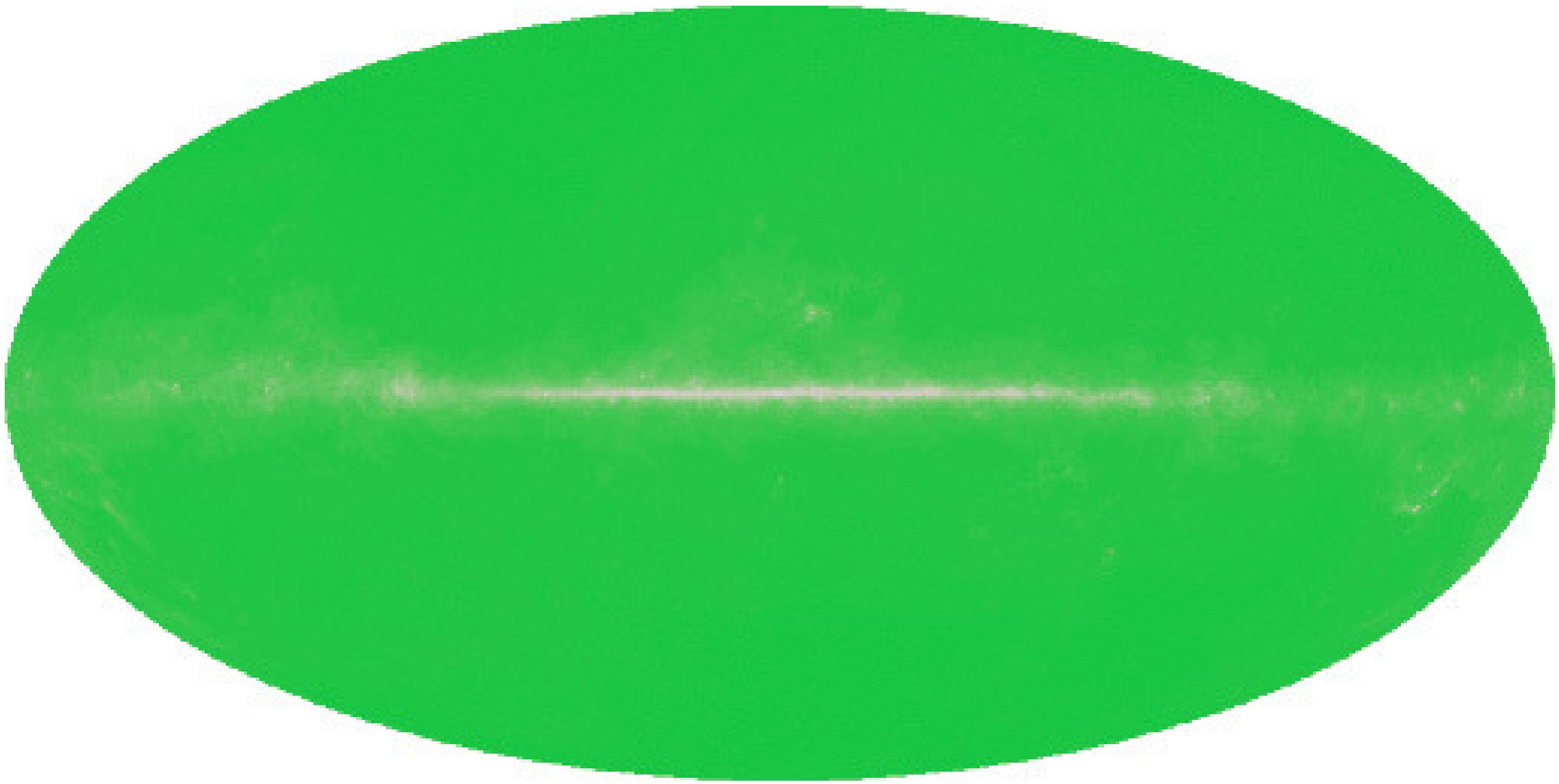
Outline

- Why inflation?
- Why inflation may be sensitive to quantum gravity
- Attempts to protect the inflaton from UV physics and why they may fail:
  - Global Symmetries: Black hole quantum mechanics
  - Gauge Symmetries: Weak Gravity Conjecture
- Controlled models: “winding” in field space
  - Long inflation at the expense of low Hubble scale
  - Small UV corrections could lead to CMB observables

Why Inflation?



Why Inflation?



This degree of homogeneity and flatness suggests all parts of the universe were causally connected in early universe, requiring early accelerated expansion

Slow-roll Inflation

$$H^2 = \frac{1}{3M_{\mathrm{Pl}}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

Scalar potential $V(\varphi)$ fully specifies the dynamics

Slow-Roll Condition

Accelerated expansion requires

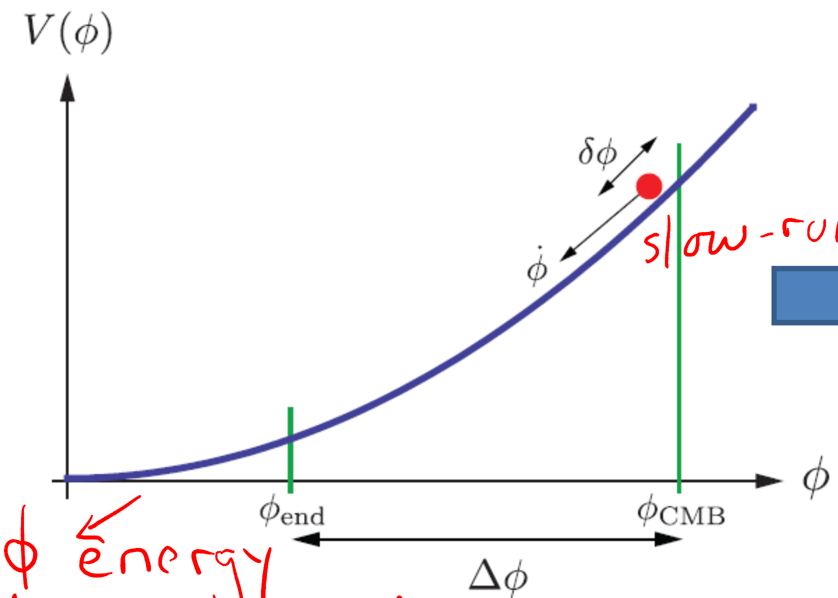
$$\epsilon = -\frac{\dot{H}}{H^2} \approx \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 < 1$$

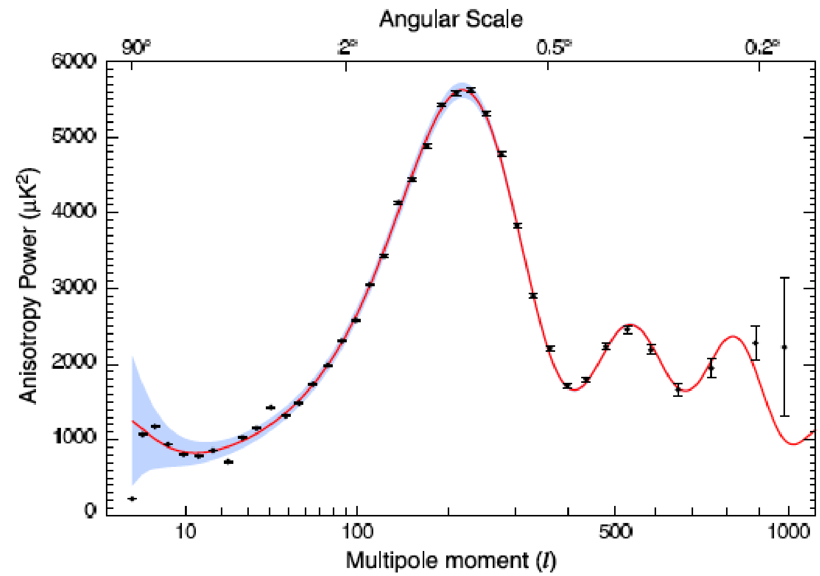
Consider a “generic” scalar field potential

$$V = m^2\phi^2 + g\phi^3 + \ldots \Rightarrow V'/V \approx 1/\phi$$

Inflation then occurs only when $\boxed{\phi \gtrsim M_{\rm pl}}$

Slow-roll Inflation





ϕ energy decays to SM

$\equiv$ REHEATING

Quantum fluctuations of the inflation generate density perturbations, CMB anisotropies

→ We can observe the scalar field dynamics with data!

Lyth Bound and Gravitational Waves

$$\mathcal{N}_{\text{e-folds}} = \int dt\, H = \int d\phi \frac{1}{M_{\text{pl}}^2} \frac{V}{V'}$$

$$= \int \frac{d\phi}{M_{\text{pl}}} \frac{1}{\sqrt{\epsilon}}$$

Flatness of Universe requires $\mathcal{N}_{\text{e-folds}} \gtrsim 60$

Observable CMB tensor modes requires $\epsilon \gtrsim 10^{-3}$

⇒ Observing tensor modes implies $\boxed{\Delta\phi > M_{\text{pl}}}$

$M_{\rm Pl}$ is the scale at which both GR and EFT break down!
Generically expect higher-dimension operators:

$$V \supset \frac{\phi^5}{\Lambda} + \frac{\phi^6}{\Lambda^2} + \ldots \qquad \Lambda \lesssim M_{\rm pl}$$

For $\phi \gtrsim M_{\rm pl}$, potential is completely out of control!
Do we need to work in e.g. a full string theory? Does even string theory allow this?

The EFT way: Assume symmetries of the UV theory, but not detailed dynamics. Can this work for inflation?

Global Symmetries?

Inflaton as a PNGB

Freese, Frieman, Olinto '90

Consider a Nambu-Goldstone boson ϕ :

$$\psi = \rho e^{i\phi/f} \quad U(1) : \phi \to \phi + c$$

$$\langle \psi \rangle = f \Longrightarrow V(\phi) = 0$$

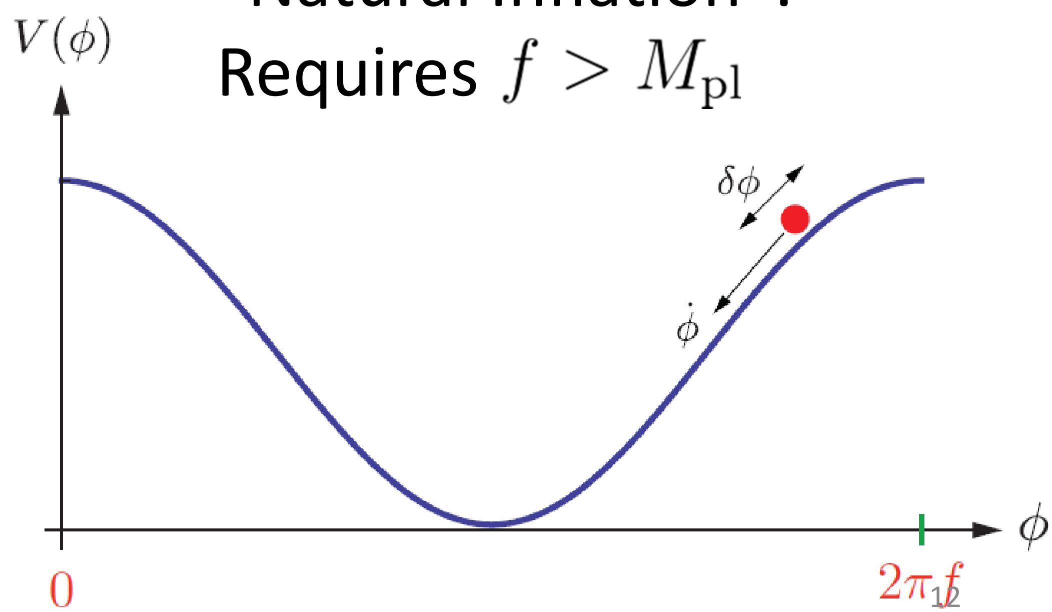
If U(1) symmetry is broken by small term

$$\delta \mathcal{L} = \epsilon \mu^3 \psi$$

Then ϕ gets a potential

$$V(\phi) = \epsilon \mu^3 f \cos\left(\frac{\phi}{f}\right)$$

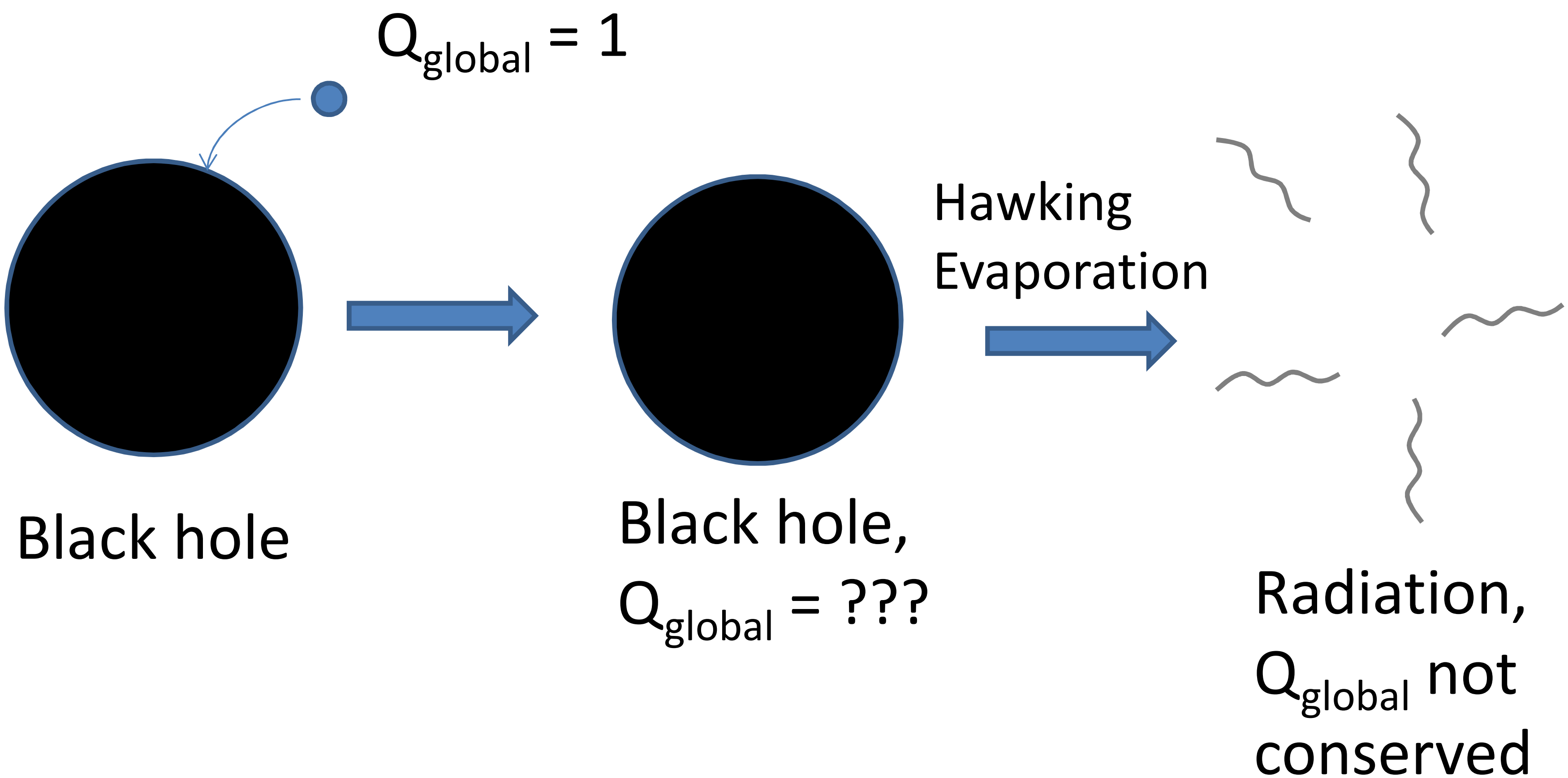
"Natural Inflation": Requires $f > M_{\rm pl}$



We assumed that the U(1) global symmetry was broken in a controlled way, but…

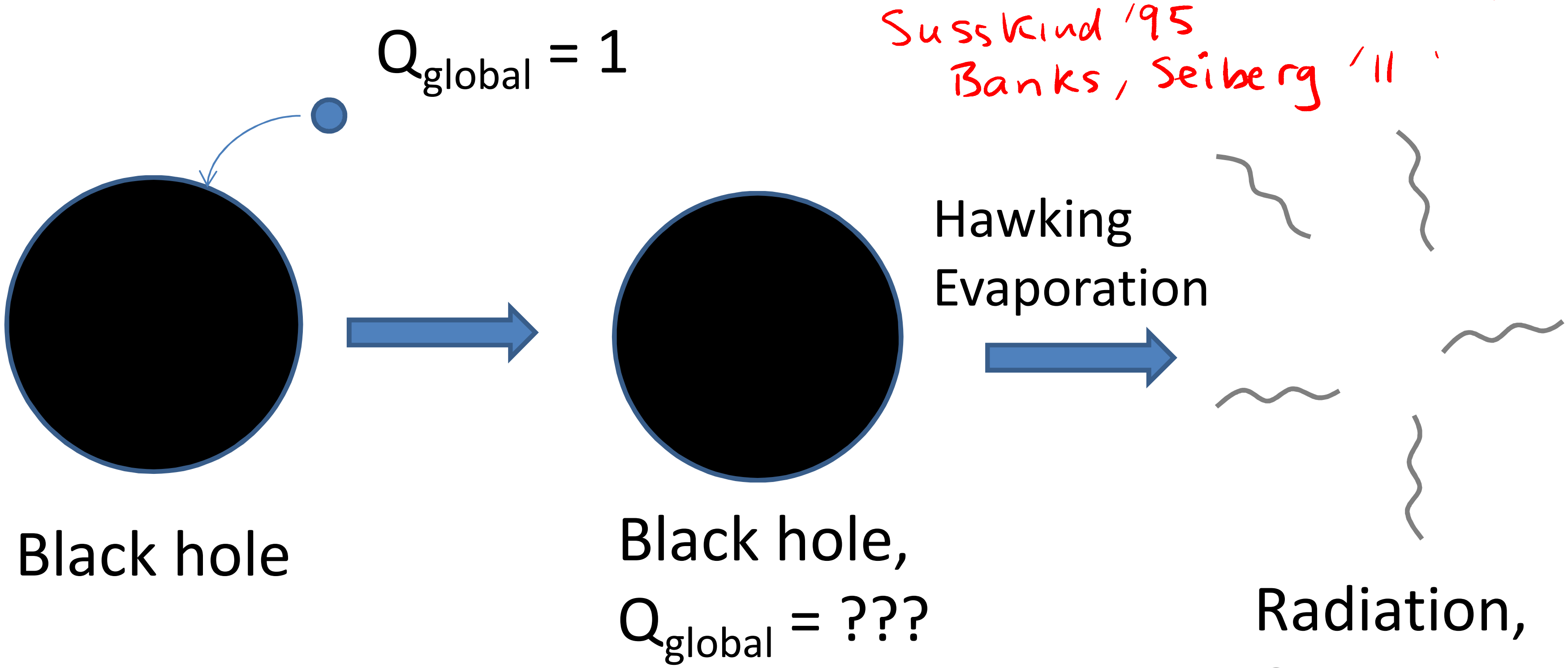
Black holes seem to violate all continuous global symmetries

Black holes violate global symmetries



Black holes violate global symmetries

Kallosh, A. Linde, D. Linde, Susskind '95
Banks, Seiberg '11



Radiation, Q_{global} not conserved

Can a black hole remember it's global charge?

If so, infinitely many microstates for each black hole → thermodynamic problems, violation of entropy bounds

No global symmetries in UV

If QG ultimately respects no global symmetries, no reason not to write down terms like $\dfrac{\psi|\psi|^4}{M_{\rm pl}}$

Inflaton potential gets corrections

$$V(\phi) \supset \frac{f^5}{M_{\rm pl}} \cos\frac{\phi}{f} + \ldots$$

Which are uncontrolled for $f > M_{\rm pl}$!

Related: in string theory, axions with $f > M_{\rm pl}$ tend to have unsuppressed higher harmonics

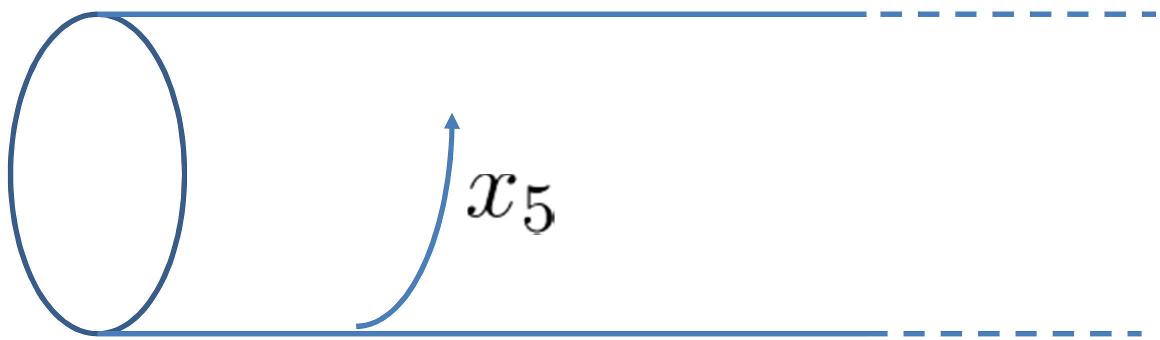
Banks, Dine, Fox, Gorbatov hep-th/0303252

Gauge Symmetries?

Extra-Natural Inflation

Arkani-Hamed, Cheng, Creminelli, Randall
hep-th/0301218

$U(1)$ gauge field in the bulk of an extra dimension S^1



A_5 component gives a light scalar field in 4D; gauge-invariant observable is the Wilson loop $e^{ig \oint_{S^1} dx^5 A_5}$

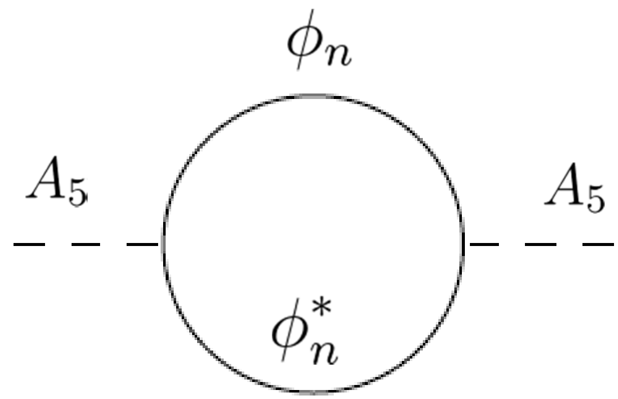
→ Periodic potential for A_5 field

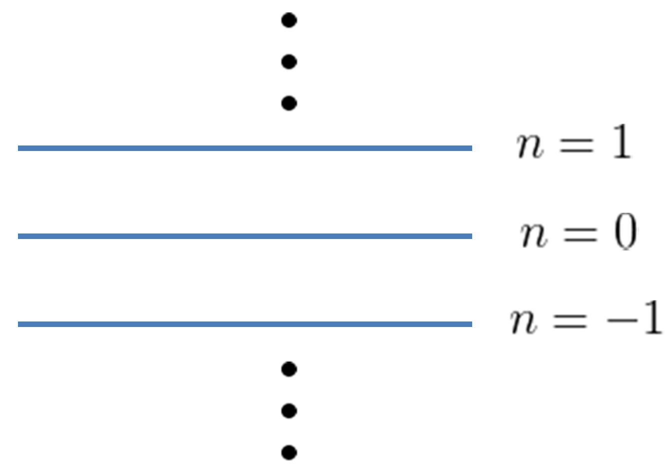
$$V \sim \frac{1}{R^4} \cos\left(2\pi R g A_5\right)$$

PNGB of accidental global symmetry, protected by UV gauge symmetry

Potential from charged KK tower

$$V = \sum_n$$





Coleman-Weinberg potential from a KK mode is a function of the field-dependent mass:

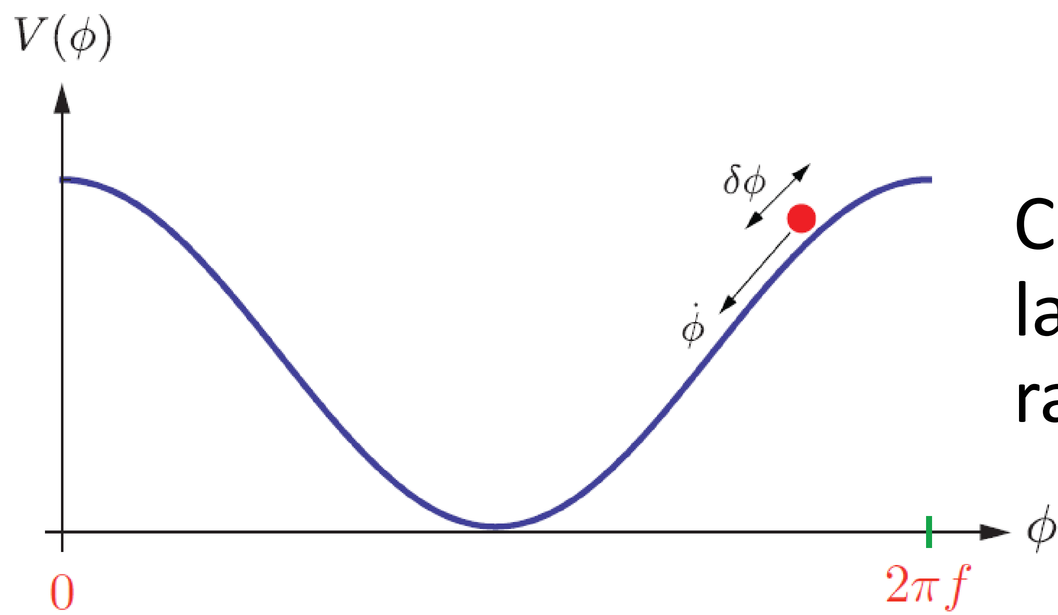
$$m_{\mathrm{KK,n}}^2 = m_5^2 + \left(\frac{n}{R} - gA_5\right)^2$$

$A_5 \to A_5 + \dfrac{1}{gR}$ simply shifts the whole KK tower

1-loop potential from a charged field is:

$$V(\phi) = \frac{3(-1)^S}{4\pi^2} \frac{1}{(2\pi R)^4} \sum_n c_n e^{-2\pi n R m_5} e^{in\phi/f} \quad + \text{h.c.}$$

$$c_n(2\pi R m_5) = \frac{(2\pi R m_5)^2}{3n^3} + \frac{2\pi R m_5}{n^4} + \frac{1}{n^5} \qquad f = \frac{1}{2\pi R g}$$



Can get arbitrarily large inflaton field range by taking $g \ll 1$

Extra-Natural Inflation: Success?

5D gauge symmetry and locality guarantee that physics above the compactification scale gives small corrections to the A_5 potential

Long inflation in limit of small coupling g, with fixed R: perturbativity guaranteed

…but doesn't the limit $g \to 0$ bring us to a global symmetry, which was problematic?

Weak Gravity Conjecture

Arkani-Hamed, Motl, Nicolis, Vafa hep-th/0601001

Claim: In any theory with gravity and a gauge field with coupling strength *g*, effective field theory must break down at a scale Λ, where

$$\Lambda < g M_{\rm pl}$$

So small *g* limits the validity of EFT!

Familiar in string theory: string states are BELOW Planck scale at weak coupling

“Lemma”: Gravity implies charge quantization (compact gauge groups)

Suppose there exist incommensurate electric charges, e.g. $q_A = 1$ and $q_B = \pi$

Then in addition to electric charge there exists an exactly conserved global symmetry, $A - B$ number

Once again, issues with entropy bounds etc.

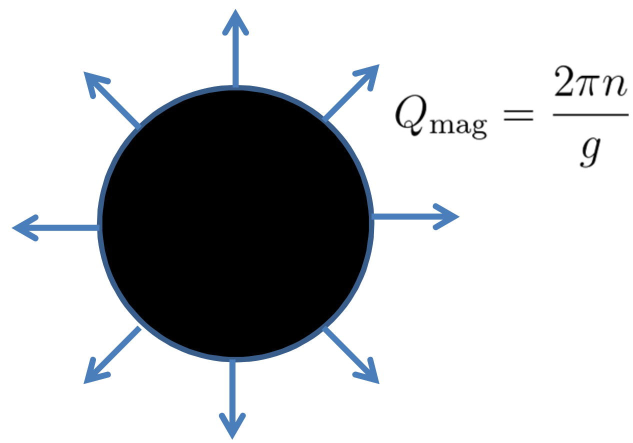
Entropy of magnetic black holes

Compactness of gauge groups implies existence of magnetically charged black hole solutions

Minimal (extremal) magnetic BH has finite entropy:

$$M \sim Q_{\mathrm{mag}} M_{\mathrm{pl}} = \frac{2\pi}{g} M_{\mathrm{pl}}$$

$$S \sim 1/g^2$$



Conjecture: There must be a fundamental monopole that is not a black hole to explain this entropy in terms of microstates

Magnetic monopole cannot be pointlike; its size defines a cutoff length scale 1/Λ

Mass of monopole (magnetic self-energy) is

$$M_{\text{monopole}} \sim \int d^3x B^2 \sim \frac{\Lambda}{g^2}$$

Can actually relax this assumption in a slightly longer proof…

Require Schwarzschild radius to be less than 1/Λ:

$$\frac{\Lambda}{g^2 M_{\text{pl}}^2} \lesssim \frac{1}{\Lambda} \quad \to \quad \boxed{\Lambda \lesssim g M_{\text{pl}}}$$

Downfall of Extra-Natural Inflation

Recall that extra-natural inflation required

$$f \sim \frac{1}{gR} > M_{\text{pl}}$$

But the WGC tells us EFT is only valid up to $\Lambda < gM_{\text{pl}}$

This implies

$$\Lambda < 1/R$$

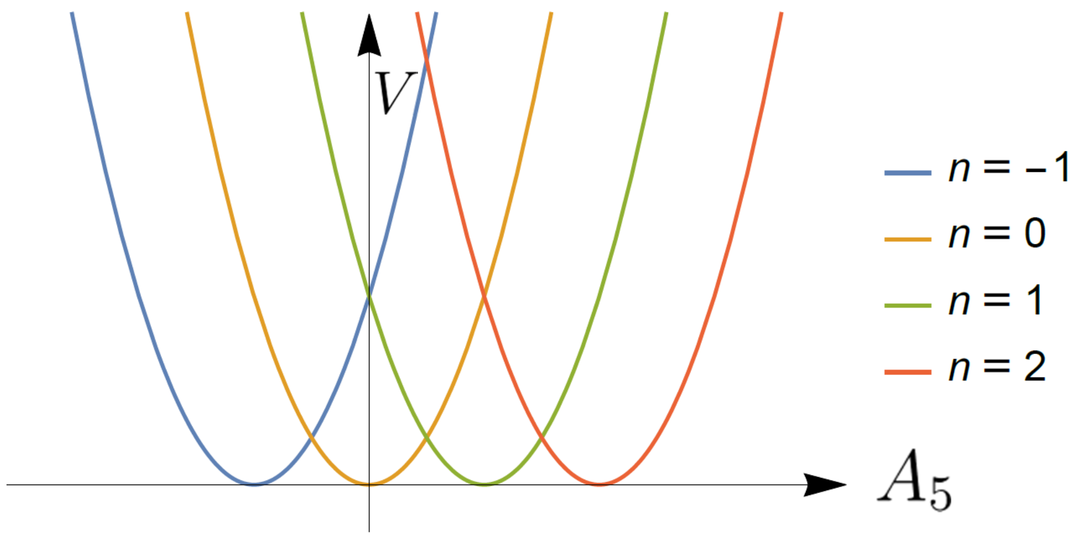
→ 5D theory is not within EFT control!

Attempt #2: Toy "Axion Monodromy"

Consider Higgsing the $U(1)$:

$$S \supset \int d^5x |(\partial_5 - igA_5)\phi|^2 + \frac{\lambda}{4}|\phi^2 - v^2|^2$$

$$\langle\phi\rangle = ve^{iny/R} \to V(A_5) = g^2v^2\left(A_5 - \frac{n}{R}\right)^2$$



Branches are related by gauge transformation:

$$A_5 \to A_5 + 1/R$$

$$\langle\phi\rangle \to \langle\phi\rangle e^{iy/R}$$

The Higgs mass-squared in this model is

$$m_H^2 \sim \frac{1}{2}\lambda v^2 - g^2\left(A_5^2 - \frac{n}{R}\right)^2$$

Negative mass-squared → Classical instability to roll to another branch (different value of *n*).

Maximum displacement on a given branch is

$$\Delta A_{5,\max} \sim \sqrt{\frac{\lambda}{2}}\frac{v}{g}$$

Infinite in Stückelberg limit (g → 0)

But there is a WGC cutoff at $\Lambda < g M_{\mathrm{pl}}$. We need $v < \Lambda$ for EFT control. But then

$$\Delta A_{5,\max} \lesssim \sqrt{\frac{\lambda}{2}} M_{\mathrm{pl}}$$

Cannot get parametrically many *e*-folds of inflation!

Axion Monodromy within String Theory

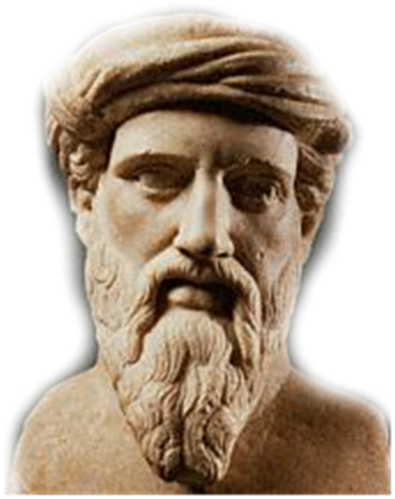
Silverstein, Westphal '08

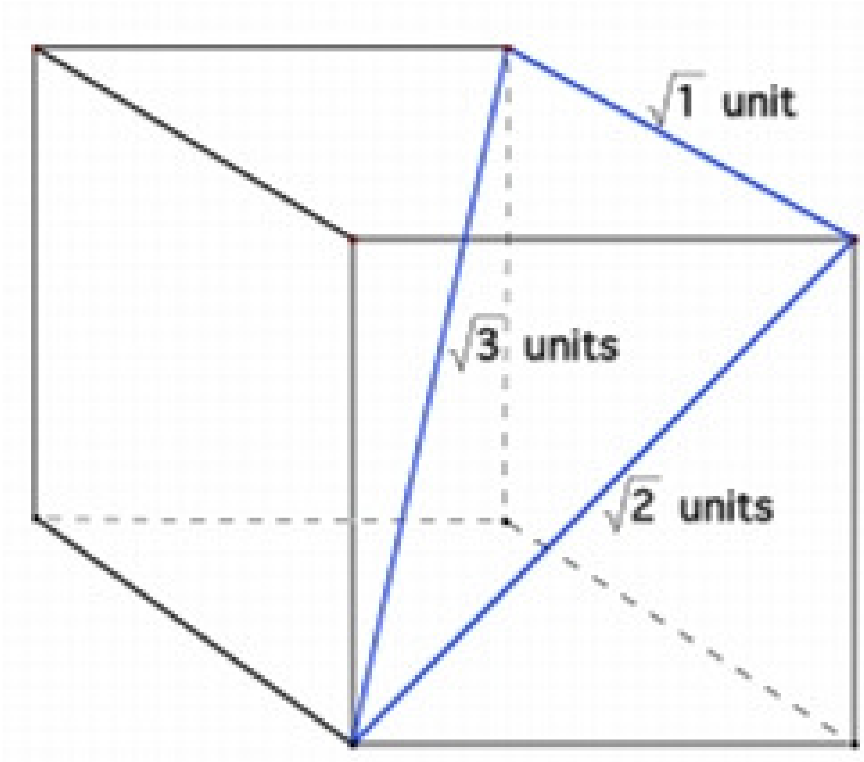
Role of "uneaten" physical Higgs scalar played by extra-dimensional radii (moduli). Success of inflation tied to complex dynamics of moduli stabilization, many d.o.f.
Attractive for automatic consistency with QG but full realism from first principles still to be demonstrated.

Multiple Fields?

N-flation: With *N* axion fields, radius of field space is increased by factor $\sim \sqrt{N}$; can achieve transplanckian range with $f \sim M_{pl}/\sqrt{N}$

Dimopoulos, Kachru, McGreevy, Wacker'05





Attempt #3: Extra-Natural *N*-flation

But for *N* U(1) gauge fields, there is a stronger WGC!
Imagine breaking U(1)'s to the diagonal:

$$U(1) \times U(1) \times \ldots \to U(1)_D$$

Coupling of $U(1)_D$ is $g/\sqrt{N}$. But then WGC requires

$$\Lambda \lesssim \frac{g}{\sqrt{N}} M_{\rm pl}$$

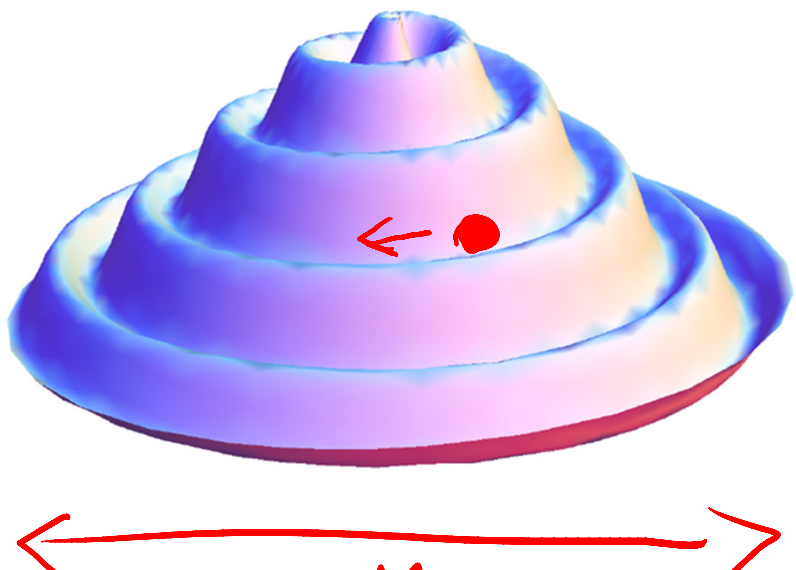
See e.g. Cheung, Remmen 1402.2287

Then for each axion has $f < M_{pl}/\sqrt{N}$, so even with *N*-flation we must have $f_{\rm eff} < M_{pl}$!

A Theorem???: The effective field space of axions from higher dimension gauge fields must be smaller than M_{Pl}.

Incapable of supporting inflation?

Viable Models: Winding Inflation

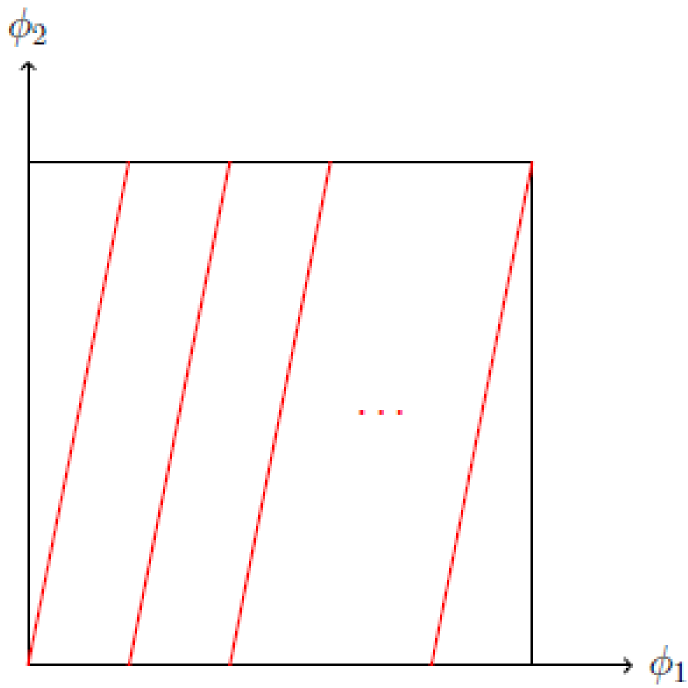


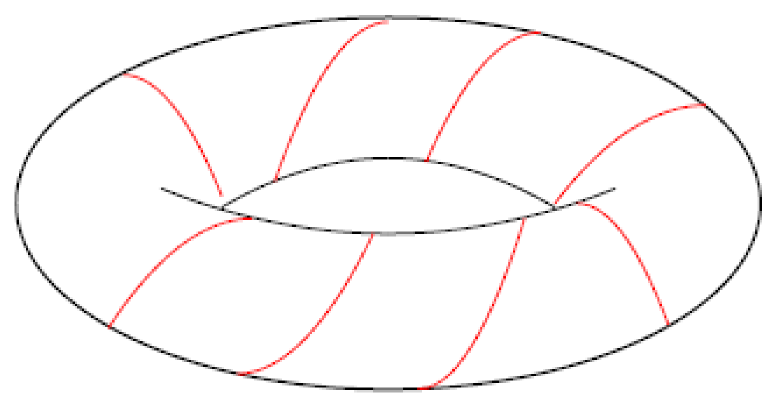
$< M_{Pl}$

Bi-Axion Models

early forms: Kim, Nilles, Peloso '05

Even if the radius of scalar field space subplanckian, there are paths with long distance which one can traverse





Extra-Natural Bi-Axion Models

Bai, Stefanek '14 (but WGC-violating)

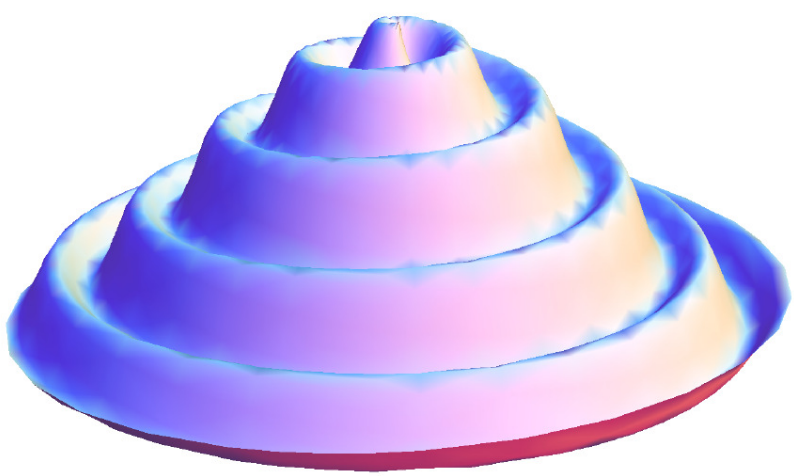
Consider two *U(1)* gauge fields *A* and *B* and two light particles with charges (N, 1) and (1, 0) under (A, B)

$$V = V_0 \left[\cos\left(\frac{NA}{f_A} + \frac{B}{f_B} \right) + \cos \frac{A}{f_A} \right]$$

$$V_0 = \frac{3}{4\pi^2} \frac{1}{(2\pi R)^4}$$

"Groove" potential

"Hill" potential



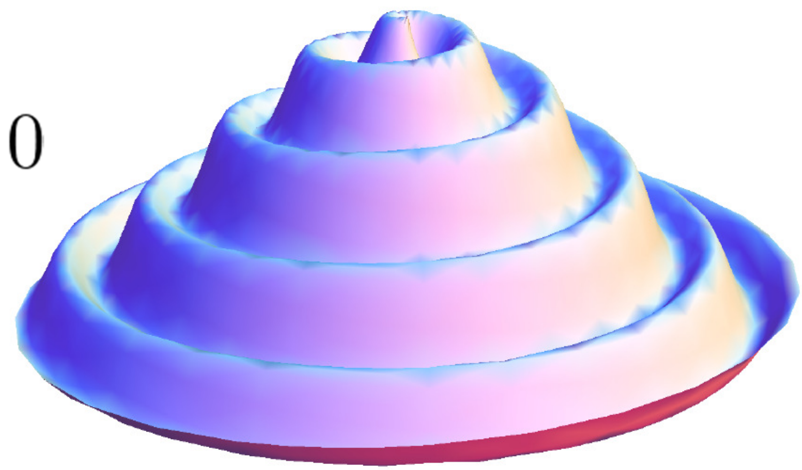
Radial direction: *A*
Angular direction: *B*

Bi-Axion Models

Heavy mode (orthogonal to groove):

$$\phi_H \sim \frac{NA}{f_A} + \frac{B}{f_B}$$

Integrate out: frozen at $\phi_H \approx 0$



Light mode (inflaton):

$$\phi_L \sim B - \frac{f_A A}{N f_B}$$

$$V(\phi_L) \sim \cos \frac{\phi_L}{N f_B} \Rightarrow f_{\text{eff}} = N f_B$$

Potentially transplanckian

Constraints for EFT Control

5D gauge theory is non-renormalizable, with strong coupling scale

$$\Lambda_g = \frac{8\pi}{N^2 g^2} \frac{1}{R}$$

WGC implies an EFT cutoff

$$\Lambda_{\mathrm{WGC}} = g M_{\mathrm{pl}}$$

Requiring both of these to be above the compactification scale 1/R implies the bound

$$\frac{f_{\mathrm{eff}}}{M_{\mathrm{pl}}} \equiv N \frac{1}{2\pi R M_{\mathrm{pl}}} \lesssim M_{\mathrm{pl}} R$$

Constraints on Inflationary Phenomenology

$$\mathcal{N}_{\text{e-folds}} \sim \frac{f_{\text{eff}}}{M_{\text{pl}}} \lesssim M_{\text{pl}} R$$

But $1/R$ also controls the Hubble scale:

$$H \sim \frac{\sqrt{V}}{M_{\text{pl}}} \sim M_{\text{pl}} \frac{1}{(M_{\text{pl}} R)^2} \lesssim \frac{M_{\text{pl}}}{\mathcal{N}_{\text{e-folds}}^2}$$

To fit the real world data we need

$$\frac{H}{M_{\text{pl}}} \sim 10^{-4} \qquad \mathcal{N}_{\text{e-folds}} \gtrsim 60$$

On the edge of the controlled parameter space…

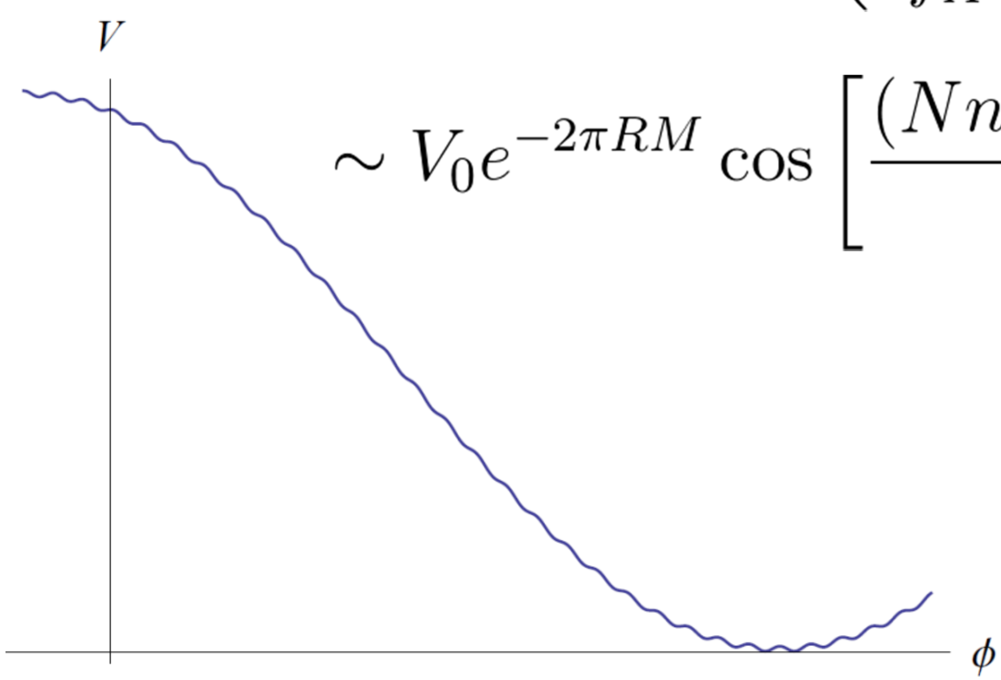
Effects of UV Physics

At the cutoff of EFT, new states with unknown quantum numbers may exist (possibly mandated by the quantum gravity theory), affecting the potential. For a particle of mass *M* with charges (n_A, n_B):

$$\delta V \sim V_0 e^{-2\pi RM} \cos\left(\frac{n_A A}{f_A} + \frac{n_B B}{f_B}\right)$$

$$\sim V_0 e^{-2\pi RM} \cos\left[\frac{(N n_B - n_A)\phi}{f_{\text{eff}}}\right]$$

Expect
$M \sim \Lambda$
typically



Potential generically receives small-amplitude but high-frequency perturbations

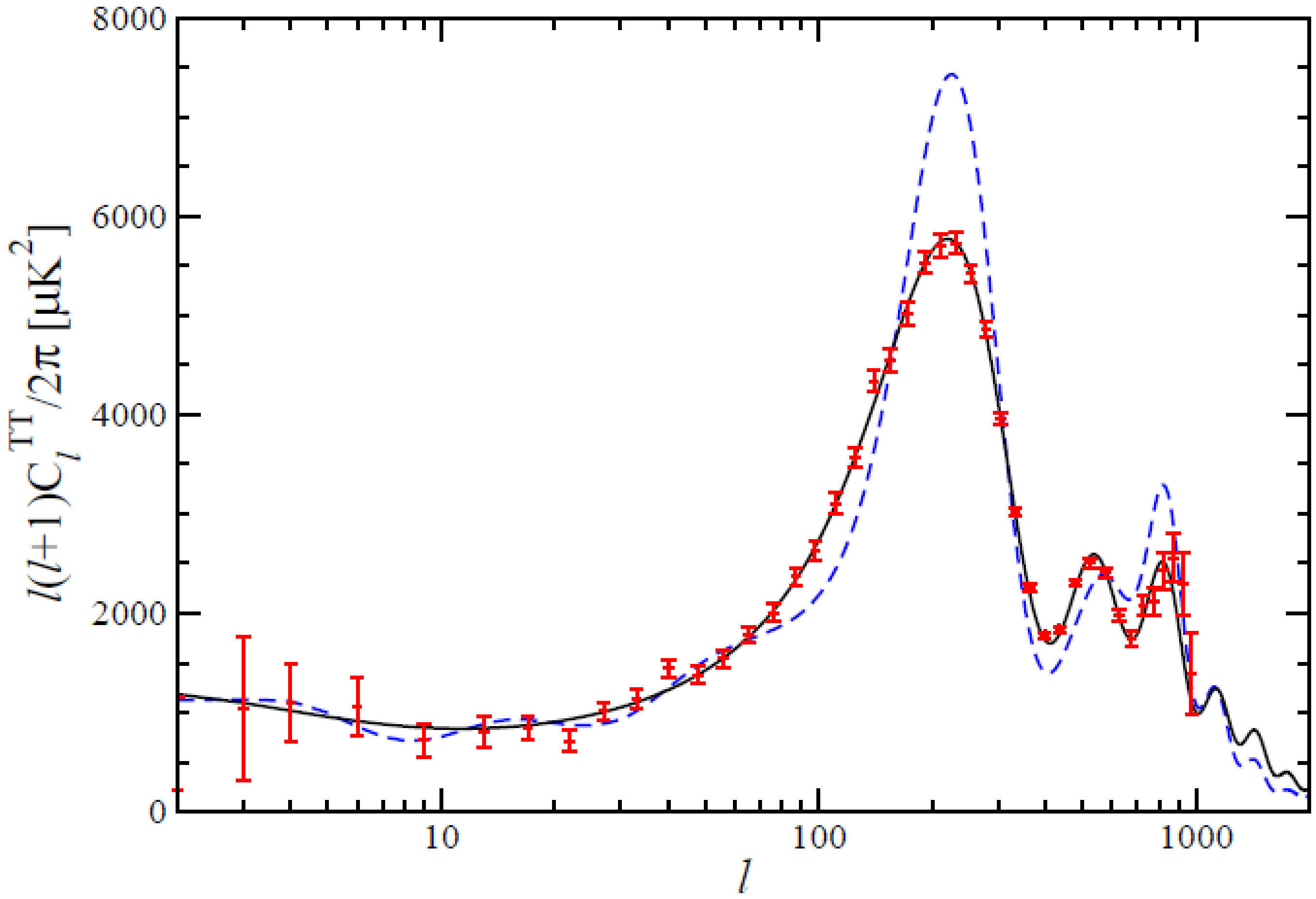


FIG. 2: The CMB power spectrum corresponding to the models shown in Fig. 1. The WMAP5 data are superimposed [16]. The error bars include both the cosmic variance and instrumental noise.

Pahud, Kamionkowski, Liddle '08

Corrections to CMB Power Spectrum

Flauger, McAllister, Pajer, Westphal, Xu '10
Flauger, McAllister, Silverstein, Westphal '14

$$\epsilon = \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2$$

Scalar power spectrum goes as 1/ε

$$\frac{\delta\epsilon}{\epsilon} \sim 2(Nn_B - n_A)e^{-2\pi RM}\sin\left[(Nn_B - n_A)\frac{\phi}{f_{\rm eff}}\right]$$

Searches for oscillations in the CMB power spectrum at the relevant frequencies require

$$\frac{\delta\epsilon}{\epsilon} \lesssim 1 - 5\%$$

→ Additional charged particles must have mass > few times 1/R

Fitting the data

Cosmological data can be fit by inflation with

$$V = V_0 \cos \frac{\phi}{f_{\rm eff}} \qquad\qquad \begin{array}{c} V_0 \sim 10^{-2} M_{\rm pl} \\ f_{\rm eff} \sim 10 M_{\rm pl} \end{array}$$

This can achieved in this model by choosing e.g.

$$M_{\rm pl} R = 8 \qquad\qquad N = 40 \qquad\qquad g = .08$$

Then additional charges with mass at EFT cutoff Λ give

$$\frac{\delta\epsilon}{\epsilon} \sim 2\%$$

So if there are extra charges near the cutoff, we may observe a “smoking gun” signal with further data…

Just for fun . . .

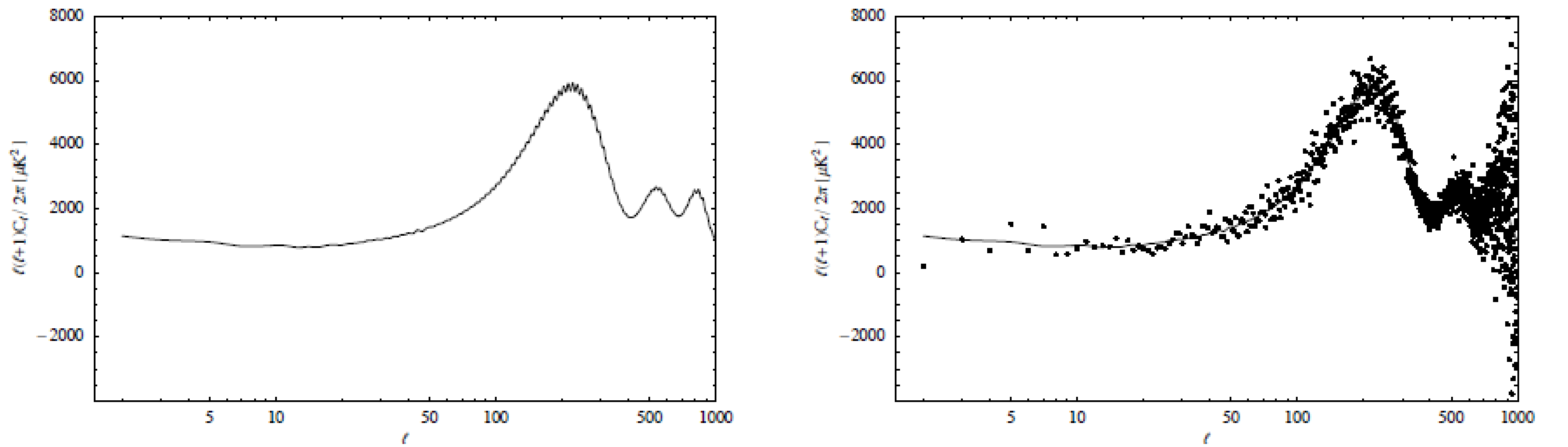
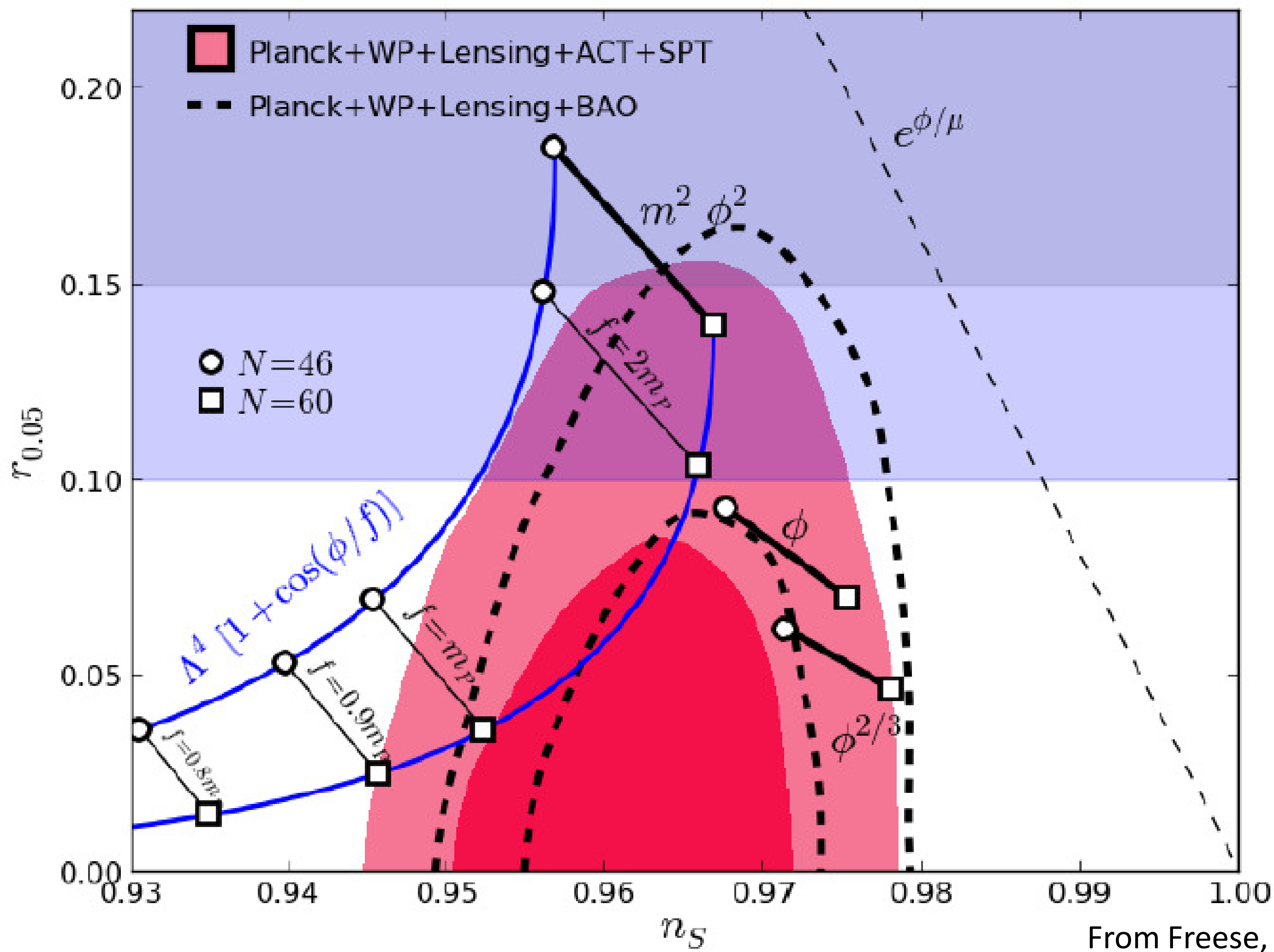


Figure 3: The left plot shows the angular power spectrum for the best fit point $f = 6.67\times10^{-4}$, and $\delta n_s = 0.17$. The right plot shows the angular power spectrum for the best fit point together with the unbinned WMAP five-year data.

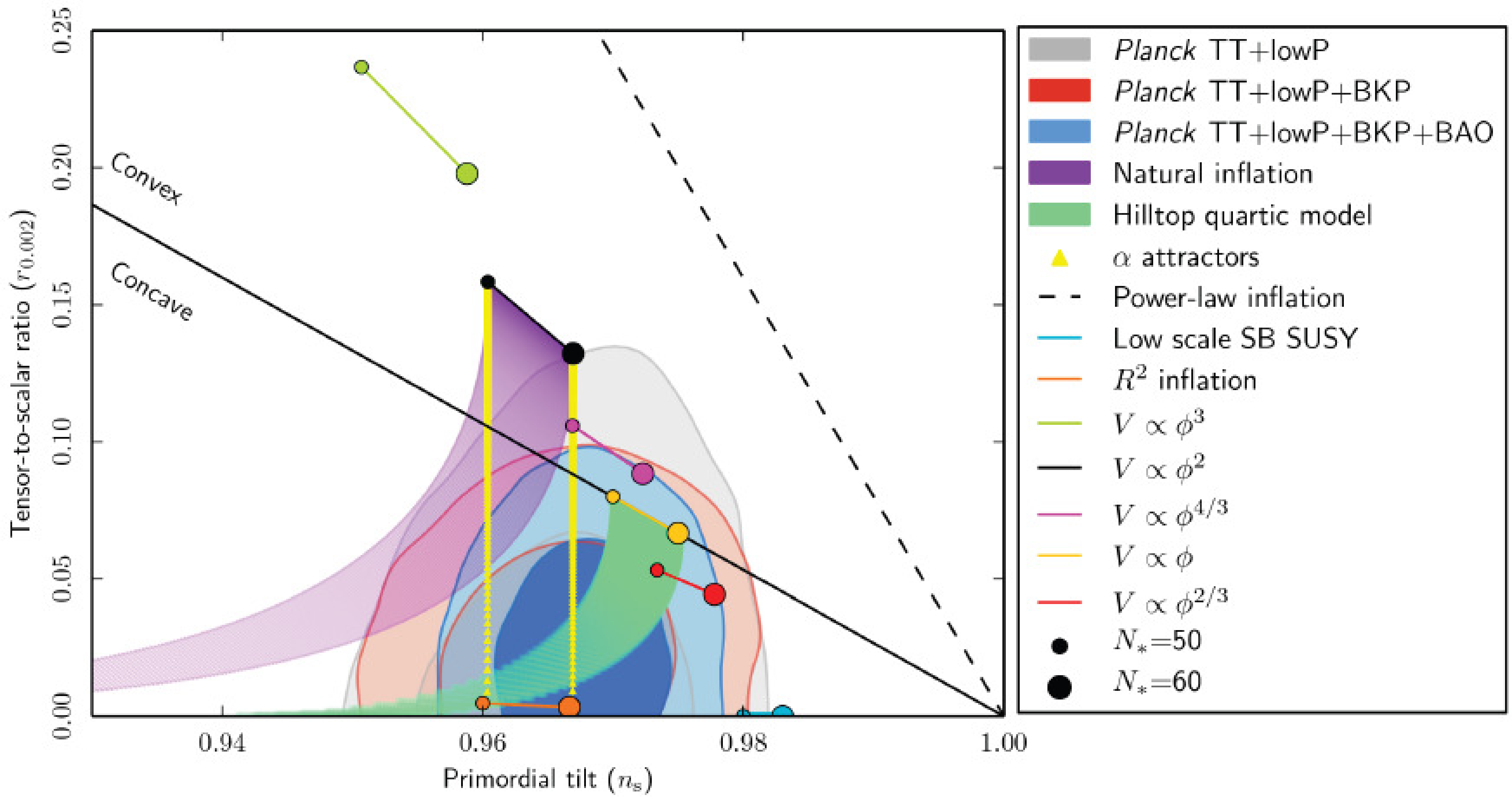
Flauger, McAllister, Pajer, Westphal, Xu '10

If true: Gravitational Waves in the next few years



From Freese, Kinney 1403.5277

If true: Gravitational Waves in the next few years



Planck 2015

Does the Theory Need to Have a Large *N*?

We required that the theory give us a light field with a parametrically large charge *N* in this model– looks strange. Perhaps the UV theory can’t actually realize this low-energy EFT?

With a slightly different model, we can avoid assuming that the dynamical theory has parametrically large integers “built-in.”

Chern-Simons model

Consider coupling the 5D gauge field to a non-Abelian sector:

$$S \supset \int d^5x \frac{N}{64\pi^2} \epsilon^{LMNPQ} G^a_{LM} G^a_{NP} A_Q.$$

In 4D:

$$S \supset \int d^4x \frac{N}{64\pi^2} \frac{A}{f} \epsilon^{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}. \qquad f = \frac{1}{2\pi R g}$$

If the non-Abelian group confines in the IR, one obtains an axion-like potential:

$$V(A) \sim V_0 \cos\left(\frac{NA}{f}\right)$$

Can recover the bi-axion model without charged particles; the large *N* is in a coupling

Large Integer *N* from Flux

We can UV complete the 5D Chern-Simons model without introducing *N* in the action by considering a 7D model (in $\mathbb{R}^4 \times S^1 \times S^2$):

$$S \supset \int d^7x \frac{1}{32\pi^2} \mathrm{d}A \wedge A \wedge G \wedge G$$

A flux of $F = \mathrm{d}A$ can wrap the two-sphere:

$$\oint_{S^2} \mathrm{d}A = \frac{N}{2\pi}.$$

Integrating out the S^2 then gives the previous 5D coupling with a large *N*.

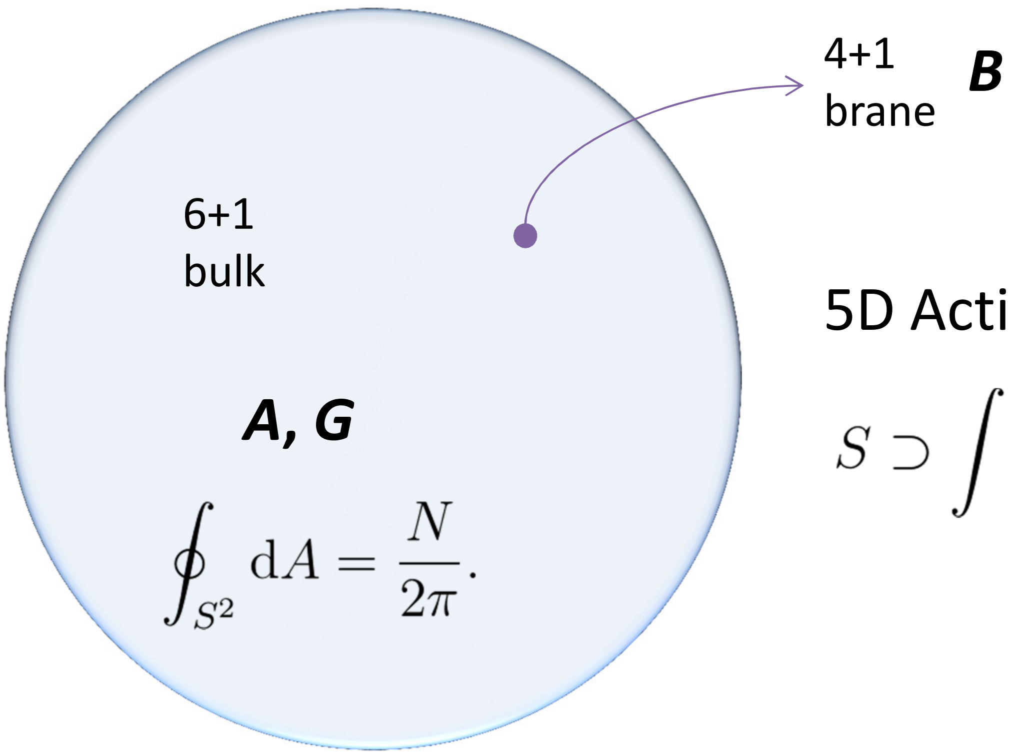
N is no longer in the action of the theory

Instead, there is a *landscape* of solutions with different values of *N*.

NOTE: “Anthropic selection” not necessary, since *N* does not need to be tuned!

Price for large *N*: large flux can destabilize the S^2

To obtain a large *N* for only one axion, one could imagine having one live in the 6+1 bulk while the other is localized to a 4+1 brane:



$$\oint_{S^2} \mathrm{d}A = \frac{N}{2\pi}.$$

5D Action:

$$S \supset \int d^5x \frac{1}{32\pi^2} (NA + B) \wedge G \wedge G$$

→ "Charges" of the form $(n_A, n_B) = (N, 1)$ do not require tuning

Conclusions

- In a theory with gravity, there are limits to how effectively global or even gauge symmetries can protect a scalar potential
- “Winding” models with axions from gauge fields can be theoretically controlled and suggest high-frequency oscillations of the power spectrum + gravitational waves
- This is a framework for thinking about inflation in EFT: can start to explore other possibilities