The Orbifold Higgs

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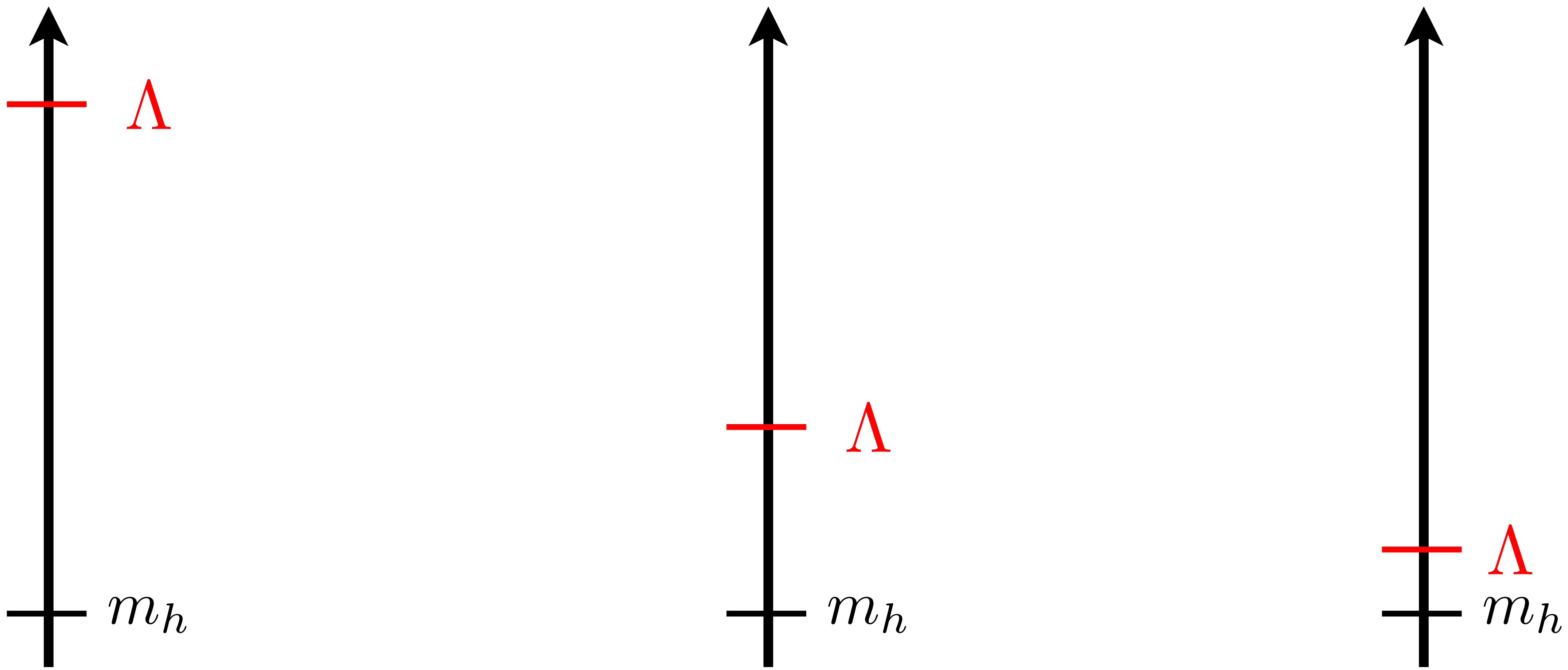
1411.7393: N. Craig, SK, P. Longhi
1410.6808: N. Craig, SK, P. Longhi

Hierarchy problem

Elementary scalars are quadratically sensitive to new physics at high scales



Finetuning
anthropic selection

Requires symmetry/symmetries
to protect the Higgs mass
(or some finetuning)

Experimentally
disfavored

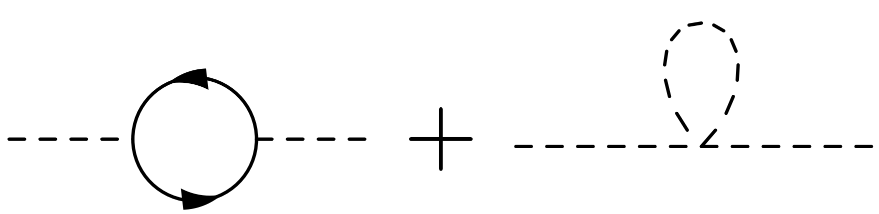
- EW precision
- lack of new particles

Top partner 'theorem' *

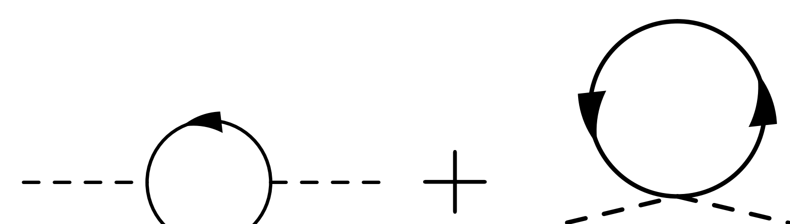
* only for symmetry-based solutions to hierarchy problem

- Symmetry must act non-trivially on H
- Top quark must be in a rep of this symmetry because y_t H Q U
  - ➡ Top quark has a partner particle
- Top partner must be 'light' or the symmetry is badly broken
- The usual suspects: SUSY or global symmetry
  - ➡ Commute with QCD → Can we relax this assumption?
  - ➡ Top partner must be colored

supersymmetry

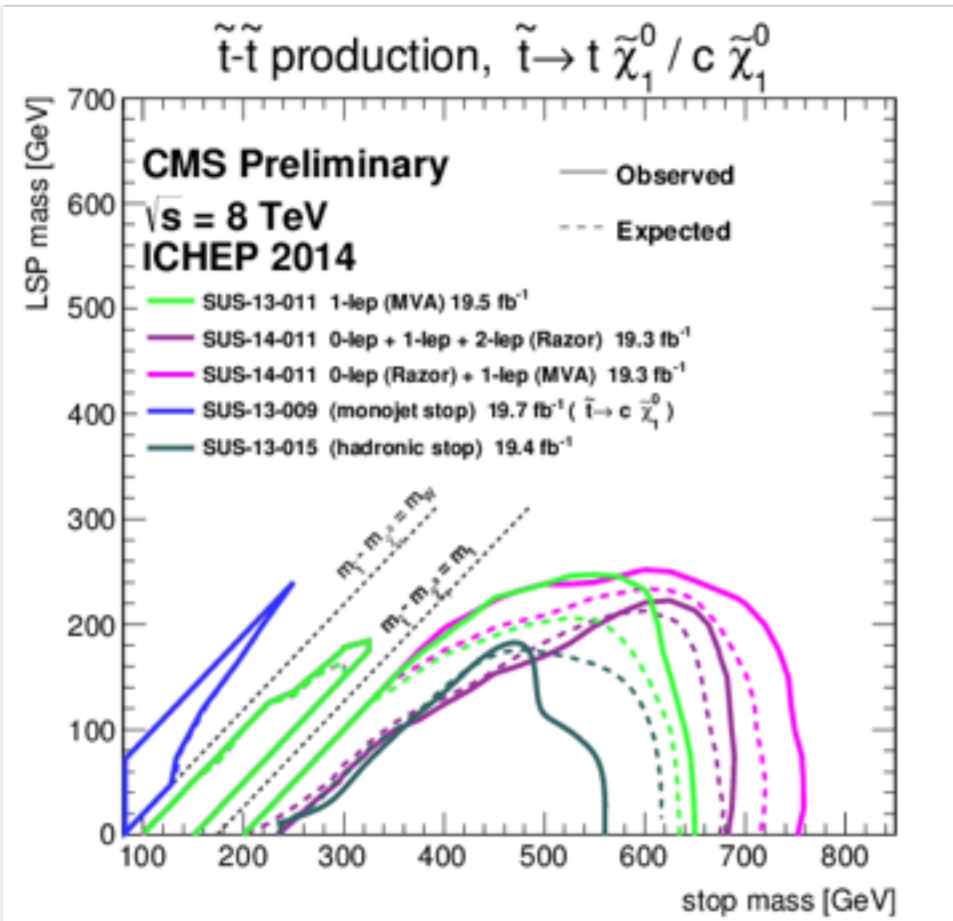


global symmetry

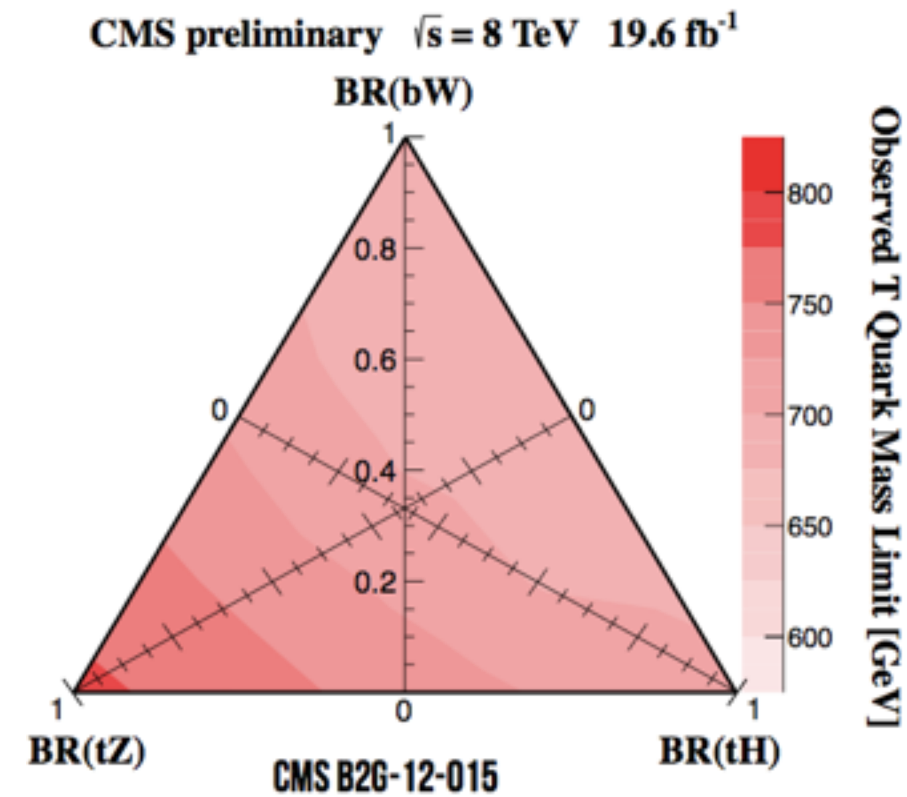


Colored top partners

Supersymmetry



Global symmetry



Plenty of possible caveats

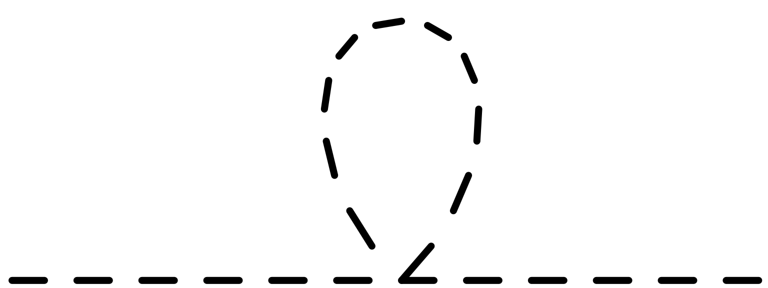
- RPV
- squeezed spectra
- stealth
- ...

Can the top partner(s) be neutral under QCD?

Canceling the divergence

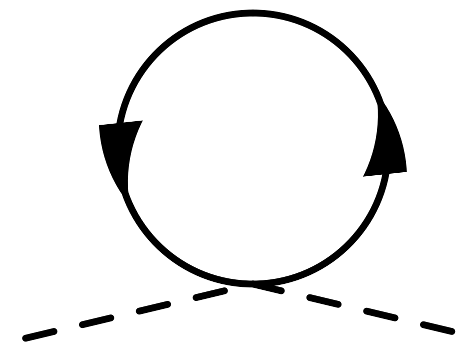
Bottom-up

supersymmetry



$$\sim \frac{N_c}{16\pi^2} y_t^2 \Lambda^2$$

global symmetry



$$\sim \frac{N_c}{16\pi^2} y_t^2 \Lambda^2$$

At 1 loop, only the number of colors enters

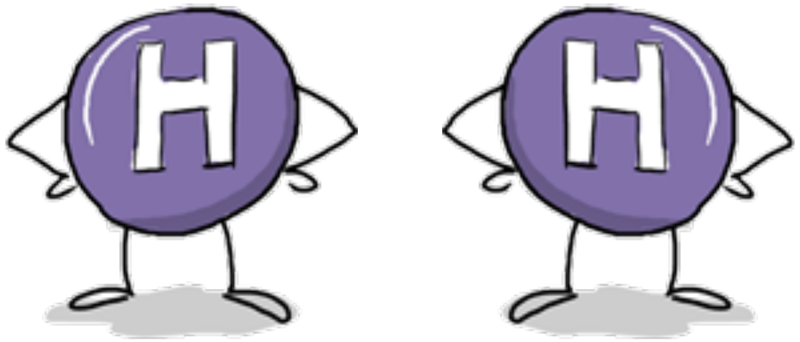
Top-down

Charge the top under a symmetry that does not commute with QCD

'Accidental' symmetry

Z. Chacko, H, Goh, R. Harnik, '05
...

The Twin Higgs



Take H in the fundamental of a global SU(4)

$$V(H) = -m^2|H|^2 + \lambda|H|^4$$

Spontaneously breaks SU(4) → SU(3): 7 goldstones

Now gauge $SU(2)_A \times SU(2)_B \subset SU(4)$ (eat 6 goldstones)

$$H = \begin{pmatrix} h_A \\ h_B \end{pmatrix}$$

$$V(H) \supset \frac{1}{16\pi^2}\frac{9}{4}\Lambda^2\left(g_A^2 h_A^2 + g_B^2 h_B^2\right)$$

Spoils the SU(4) symmetry

Extra ingredient: Z_2 symmetry A ↔ B such that $g = g_A = g_B$

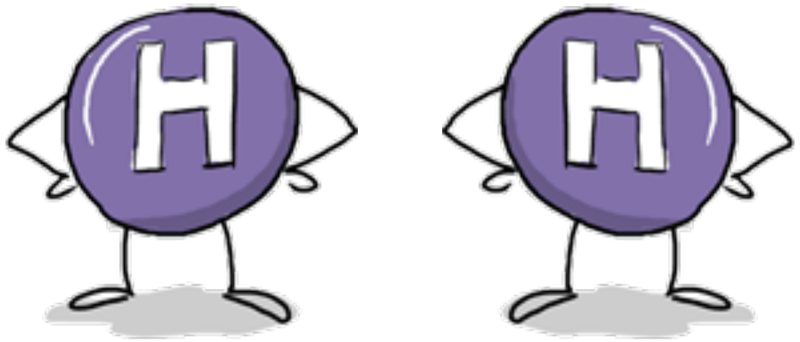
$$V(H) \supset \frac{1}{16\pi^2}\frac{9}{4}g^2\Lambda^2\left(h_A^2 + h_B^2\right) = \frac{1}{16\pi^2}\frac{9}{4}g^2\Lambda^2|H|^2$$

Accidental SU(4) symmetry preserved in the 1 loop effective potential

(quadratic piece only)

Z. Chacko, H, Goh, R. Harnik, '05
...

The general idea



The Higgs is a pseudo goldstone boson of an accidental global symmetry

The global symmetry is explicitly broken by the gauge interactions, but nevertheless preserved in the 1 loop effective potential due to a Z_2 symmetry

The Twin Standard Model

$$[SU(3)_c \times SU(2)_w \times U(1)_Y]^2 \times Z_2$$

SM top — Twin top

$$V(H) \supset -m^2|H|^2 + \lambda|H|^4 + y_t h_A q_A u_A + y_t h_B q_B u_B$$

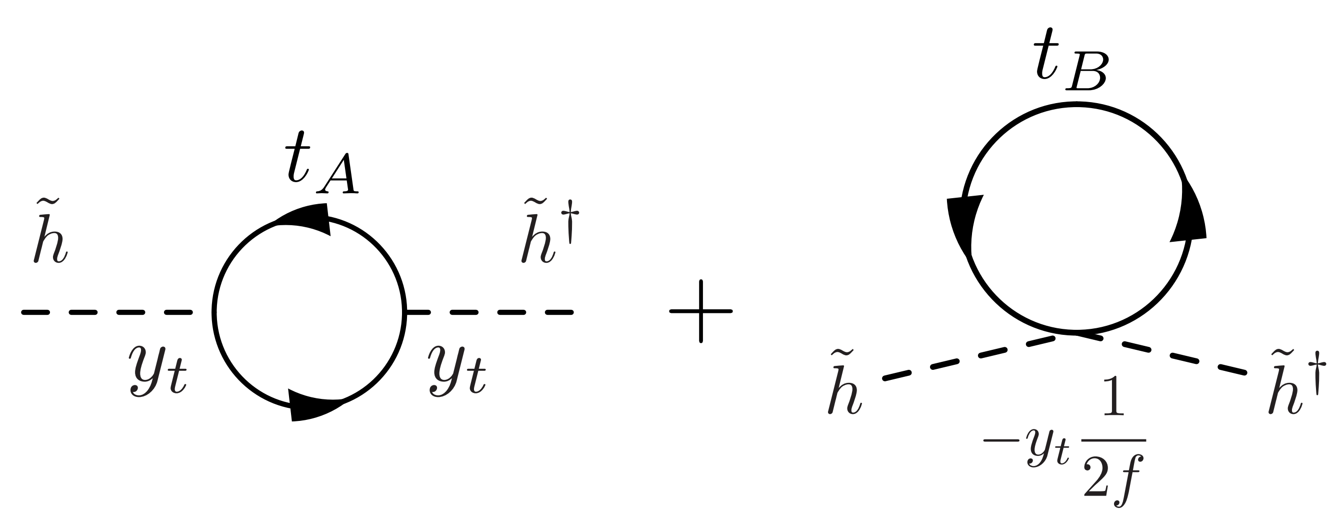
Spontaneously breaks SU(4) → SU(3): 7 goldstones

$$H = \begin{pmatrix} h_A \\ h_B \end{pmatrix} = \exp(\frac{i}{f}\Pi)\begin{pmatrix} 0 \\ 0 \\ 0 \\ f \end{pmatrix} \qquad \Pi = \left(\begin{array}{ccc|c} 0 & 0 & 0 & \tilde{h}_1 \\ 0 & 0 & 0 & \tilde{h}_2 \\ 0 & 0 & 0 & 0 \\ \hline \tilde{h}_1^* & \tilde{h}_2^* & 0 & 0 \end{array}\right) + \cdots$$

$$h_A = i\tilde{h} + \cdots$$

$$h_B = f - \frac{1}{2f}\tilde{h}^\dagger\tilde{h} + \cdots$$

QCD singlet!



However....

- What with the light fermions? Twin neutrino's ?
- Where do the Z_2 and the accidental SU(4) come from? (Accidental symmetries are not radiatively stable)
- Is the twin Higgs just a pathological case or is there a more general story?

Outline

1. Introduction
2. Orbifolding to the Twin Higgs
3. Orbifolding more general field theories
4. A recipe for generalized Orbifold Higgs models

A 'Twin Unified Theory' ?

$$y_t = y_t' \quad \text{at} \quad 1\% \text{ level}$$
$$g_2 = g_2' \quad \text{at} \quad 10\% \text{ level}$$
$$g_3 = g_3' \quad \text{at} \quad 15\% \text{ level}$$

at $\Lambda \sim 5 \text{ TeV}$

1501.05310: N. Craig, A. Katz, M. Strassler, R. Sundrum

$$SU(6) \times SU(4)$$
$$\downarrow$$
$$\Big[SU(3) \times SU(2)\Big] \times \Big[SU(3) \times SU(2)\Big]$$

Use the tools from GUT model building

example:

$$SU(5)/\mathbb{Z}_2 \to SU(3) \times SU(2) \times U(1)$$

Orbifolds are a clean way of reducing symmetries

Orbifold Correspondence

…
Kachru, Silverstein '98
Bershadsky, Johansen '98
Schmaltz '99
…

mother theory $\xrightarrow{\text{orbifold}}$ daughter theory

In the large N limit, the correlation functions of the daughter are identical* to those of the mother

Intuition:

Exact symmetry in mother $\longleftrightarrow$ Accidental symmetry in daughter

(UV complete in higher dimension or by deconstruction)

* up to a rescaling of the couplings

Orbifolds

Orbifolds in field theory

Map between two field theories: “Mother” → “Daughter”
(Mother does not necessarily flow to the daughter)

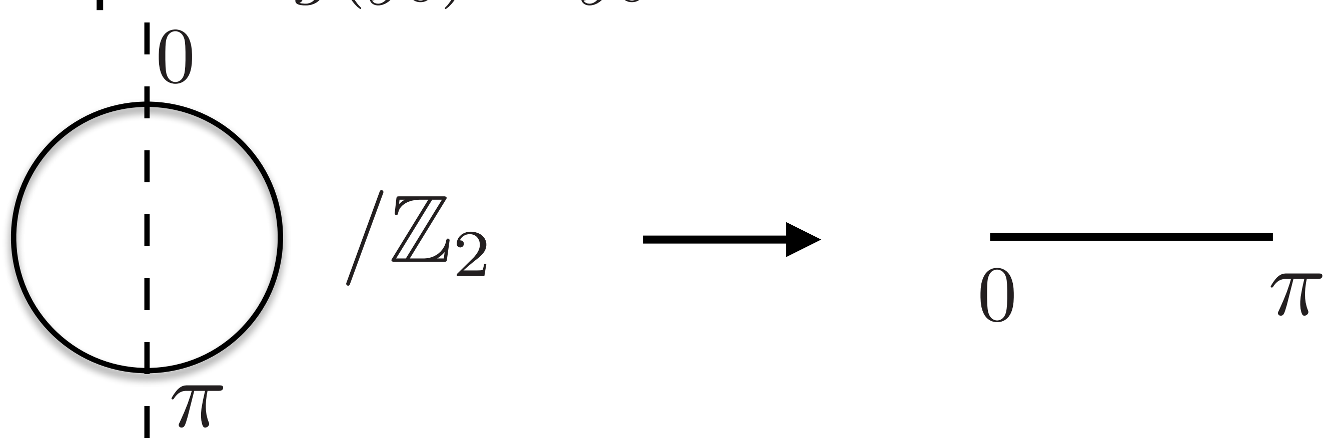
Geometric interpretation

Quotient space of manifold modded out by a discrete group $\mathcal{G}$

$$\mathcal{G}: \quad \phi^i[y] \quad \to \quad R(g)_{ij}\, \phi^j[g(y)]$$

Need a space time fixed point: $g(y_0) = y_0$

Example: $S^1/\mathbb{Z}_2$



Field Theory Orbifold

Recipe:

1. Identify discrete symmetry in the mother theory

2. Eliminate DOF that are not invariant to obtain the daughter theory

<u>example</u>

$$SU(4)/Z_2 \to SU(2) \times SU(2) \times U(1)$$

Z_2 action: $\gamma = \begin{pmatrix} \mathbb{1}_2 & \\ & -\mathbb{1}_2 \end{pmatrix}$

Invariant if $A_\mu^a = \gamma A_\mu^a \gamma$

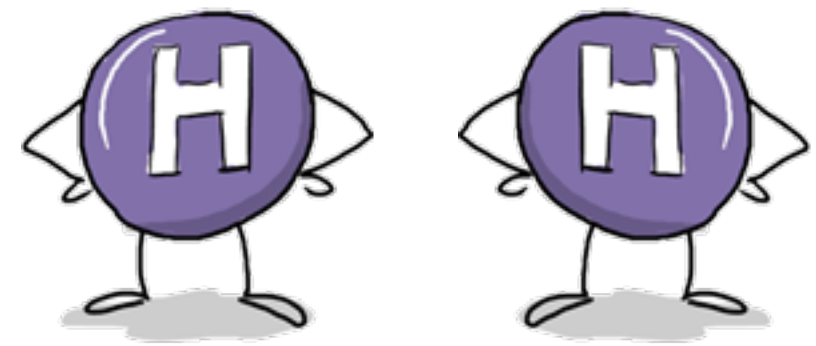
$$\begin{pmatrix} A_\mu^a & 0 \\ 0 & 0 \end{pmatrix} \longrightarrow SU(2)$$

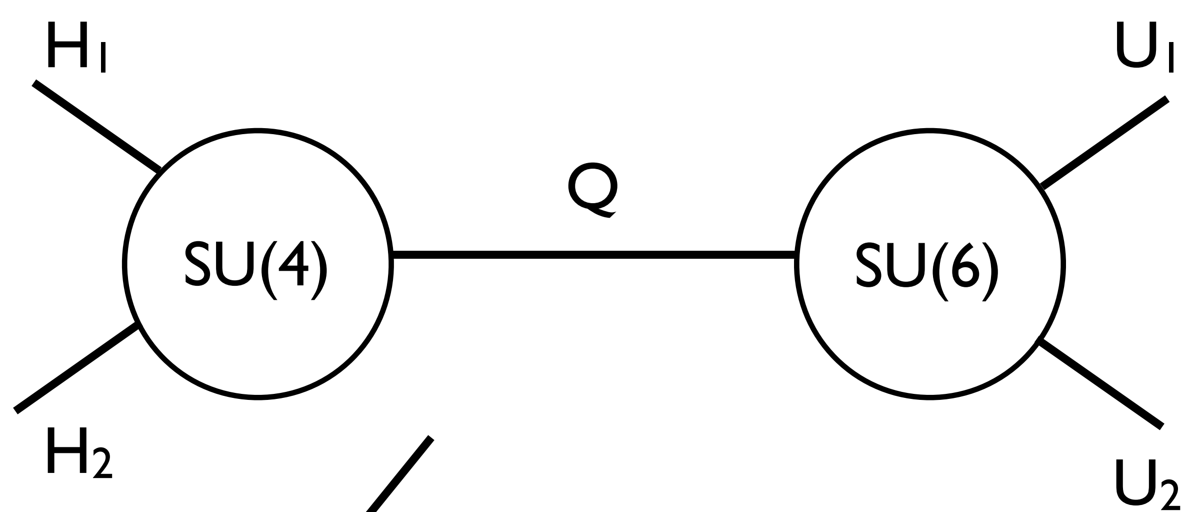
$$\begin{pmatrix} 0 & 0 \\ 0 & A_\mu^a \end{pmatrix} \longrightarrow SU(2)$$

$$\begin{pmatrix} A_\mu \times \mathbb{1}_2 & 0 \\ 0 & -A_\mu \times \mathbb{1}_2 \end{pmatrix} \longrightarrow U(1)$$

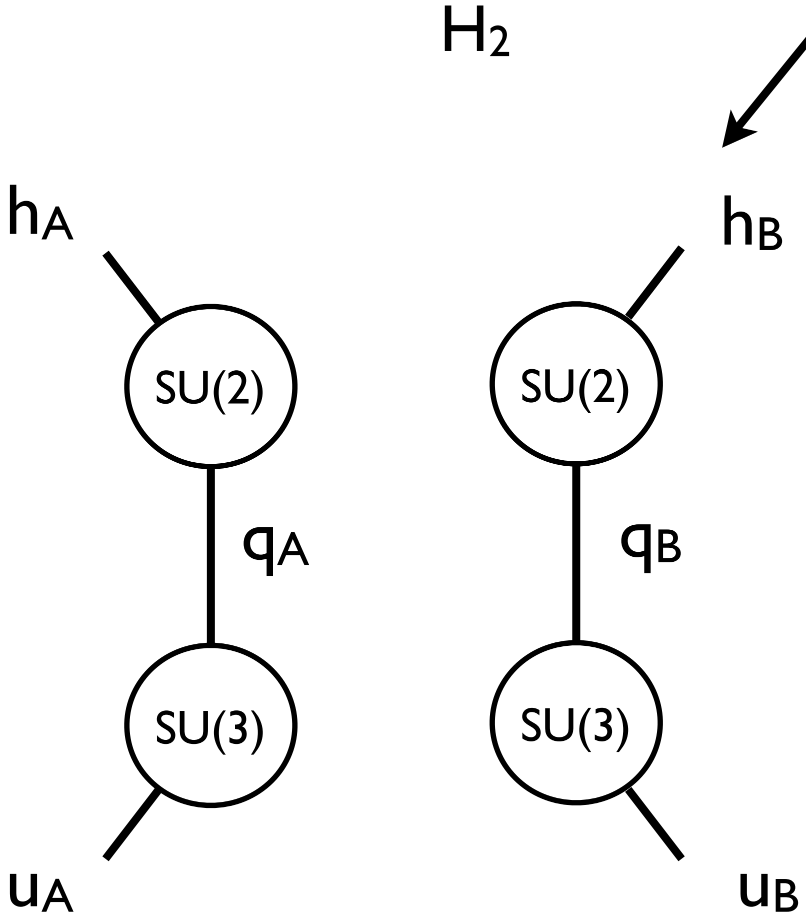
Twin Higgs IS an orbifold

$$[SU(6)\times SU(4)]/Z_2 \to [SU(3)\times SU(2)]^2 \times U(1)\times U(1)\times Z_2$$





with $\quad y_t H_1 Q U_1 + y_t H_2 Q U_2$



<u>Twin Higgs?</u>

- Tree-level potential SU(4) symmetric ✔

$$-m^2|H|^2+\lambda|H|^4 \to -m^2(|h_A|^2+|h_B|^2)+\lambda(|h_A|^2+|h_B|^2)^2$$

- 1 loop quadratic potential SU(4) symmetric ✔

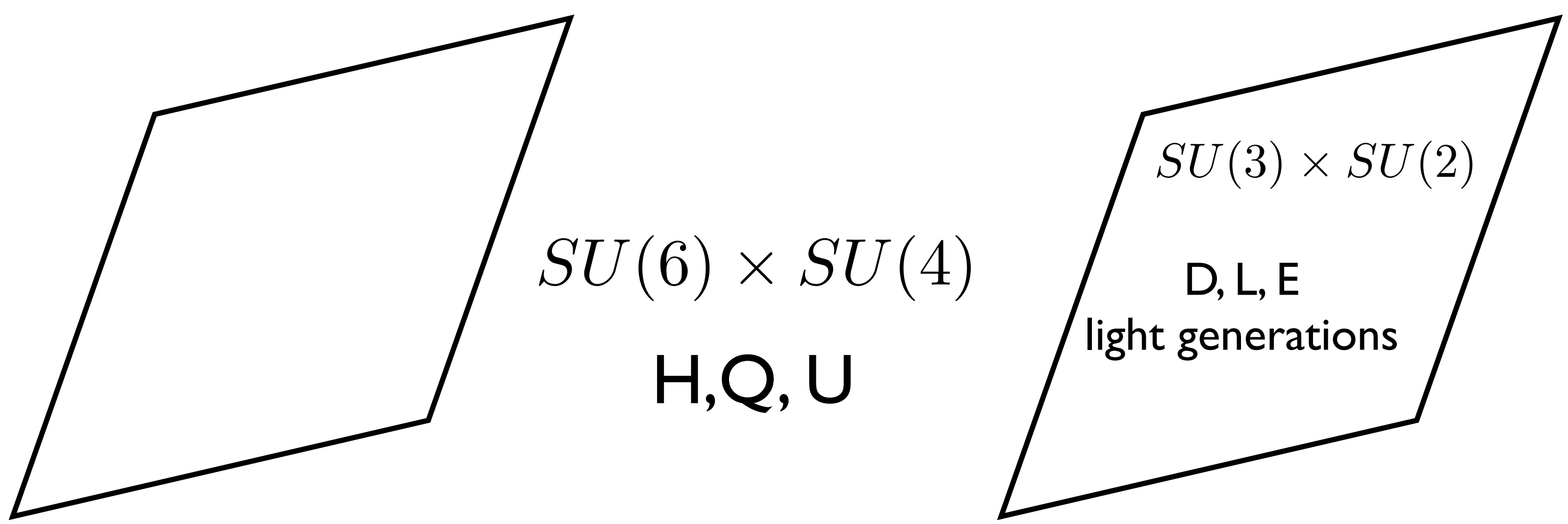
$$g_2 = g_2^{(A)} = g_2^{(B)}, \quad g_3 = g_3^{(A)} = g_3^{(B)}, \quad y_t = y_t^{(A)} = y_t^{(B)}$$

$$\longrightarrow \quad V(H) \supset \frac{1}{16\pi^2}\frac{9}{4}g^2\Lambda^2\left(h_A^2+h_B^2\right) = \frac{1}{16\pi^2}\frac{9}{4}g^2\Lambda^2|H|^2$$

Twin Higgs IS an orbifold

Geometrical interpretation in 5D

UV completion for twin Higgs



(Deconstruction is also possible)

General Strategy

Mother theory with
exact symmetry

Provides
inspiration for

Classify new natural
effective theories

Realistic daughter theory
with accidental symmetry

examples:

- Twin Higgs
- Folded Supersymmetry
- …

Geometric UV
completion

Integrate out KK-towers

Outline

1. Introduction

2. Orbifolding the Twin Higgs

3. Orbifolding more general field theories

4. A cookbook for generalized orbifold Higgs models

Can we generalize the orbifold twin Higgs?

$$SU(3\Gamma) \times SU(2\Gamma)/\mathcal{G} \qquad\qquad \Gamma = |\mathcal{G}|$$

A bit of group theory

Consider $\mathcal{G} = \{g_1, g_2, \cdots, g_\Gamma\}$

The regular representation is given by $g_a \circ g_i = \gamma^a_{ij} g_j$

The regular representation is reducible

$$\gamma^a = \begin{pmatrix} (r_1^a) & & & & & & \\ & (r_2^a) & & & & & \\ & & \ddots & & & & \\ & & & \overbrace{(r_l^a) \quad \ddots \quad (r_l^a)}^{d_l \text{ times}} & & & \\ & & & & & & \ddots \end{pmatrix}.$$

Every irrep shows up d_l times, with d_l the dimension of the irrep

In more compact notation: $\gamma^a = \bigoplus_l r_l^a \otimes \mathbb{1}_{d_l}$

Schmaltz '99

How to orbifold a field theory?

1. First, find embedding of $\mathcal{G}$ in SU(N Γ)

$$\gamma_N^a = \begin{pmatrix} \overbrace{\begin{pmatrix} (r_1^a) & & \\ & \ddots & \\ & & (r_1^a) \end{pmatrix}}^{d_1 N \text{ times}} & & \\ & \overbrace{\begin{pmatrix} (r_2^a) & & \\ & \ddots & \\ & & (r_2^a) \end{pmatrix}}^{d_2 N \text{ times}} & \\ & & \ddots \end{pmatrix} .$$

or

$$\gamma_N^a = \bigoplus_l r_l^a \otimes \mathbb{1}_{d_l} \otimes \mathbb{1}_N = \bigoplus_l r_l^a \otimes \mathbb{1}_{N d_l}$$

Fundamental $\quad Q \to \gamma_N Q$

Adjoint $\quad A \to \gamma_N A \gamma_N^\dagger$

2. Drop fields that not invariant

Schmaltz '99

How to orbifold an adjoint?

$$A \to \gamma_N A \gamma_N^\dagger$$

$$\gamma_N^a = \begin{pmatrix} \overbrace{\begin{pmatrix} (r_1^a) & & \\ & \ddots & \\ & & (r_1^a) \end{pmatrix}}^{d_1 N \text{ times}} & & \\ & \overbrace{\begin{pmatrix} (r_2^a) & & \\ & \ddots & \\ & & (r_2^a) \end{pmatrix}}^{d_2 N \text{ times}} & \\ & & \ddots \end{pmatrix} .$$

Use Shur's lemma $(r_l^a \otimes \mathbb{1}_{Nd_l}) A (r_l^a \otimes \mathbb{1}_{Nd_l})^\dagger = A \quad \Rightarrow \quad \mathbb{1}_{d_l} \otimes A_l$

$$\downarrow$$

$$N d_l \times N d_l$$

$$A = \begin{pmatrix} \mathbb{1}_{d_1} \otimes A_1 & & & \\ & \mathbb{1}_{d_2} \otimes A_2 & & \\ & & \ddots & \\ & & & \mathbb{1}_{d_n} \otimes A_n \end{pmatrix}$$

fixed from group theory

$$SU(N\Gamma) \to SU(d_1 N) \times SU(d_2 N) \times \cdots \times SU(d_n N) \times U(1)^{n-1}$$

Non-trivial breaking pattern!

$$\frac{1}{g^2} Tr F_{\mu\nu} F^{\mu\nu} \to \sum_l \frac{d_l}{g^2} Tr (F_l)_{\mu\nu} (F_l)^{\mu\nu} \longrightarrow \qquad g_l = \frac{g}{\sqrt{d_l}}$$

gauge couplings are rescaled

How to orbifold a fundamental?

$$Q \to \gamma_N Q$$

$$\gamma_N^a = \begin{pmatrix} \overbrace{\begin{pmatrix} (r_1^a) & & \\ & \ddots & \\ & & (r_1^a) \end{pmatrix}}^{d_1 N \text{ times}} & & \\ & \overbrace{\begin{pmatrix} (r_2^a) & & \\ & \ddots & \\ & & (r_2^a) \end{pmatrix}}^{d_2 N \text{ times}} & \\ & & \ddots \end{pmatrix}.$$

Construct a projector operator $P_{r_l} \equiv \frac{1}{\Gamma} \sum_{a=1}^{\Gamma} r_l^a$ for which $r_l^a P_{r_l} = P_{r_l}$

By construction, $P_{r_l} Q$ is then an invariant for the irrep r_l

Moreover $P_{r_l} = \begin{cases} 1 & \text{if} \quad r_l \quad \text{is trivial rep} \\ 0 & \text{otherwise} \end{cases}$

Full projector is thus

$$PQ = \bigoplus_l P_{r_l} Q = \begin{pmatrix} \mathbb{1}_N & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 \end{pmatrix} Q = \begin{pmatrix} Q_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

only a survivor in the first sector

How to orbifold a bifundamental?

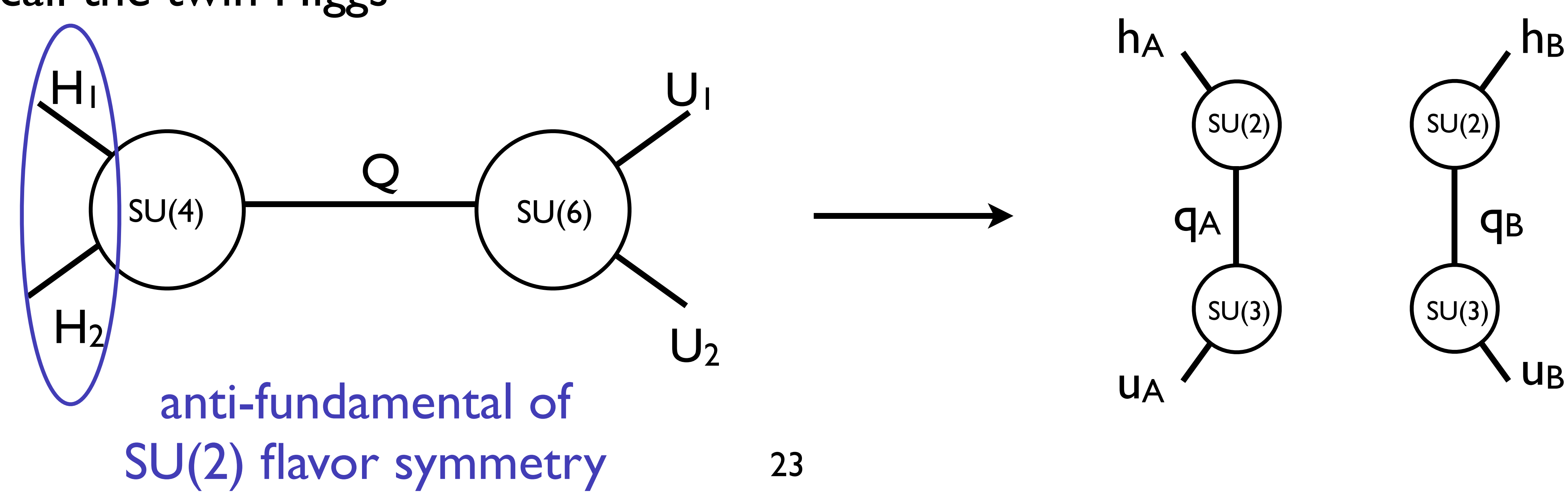
$$Q \to \gamma_N \otimes \gamma_{N'}^\dagger Q$$

$$\gamma_N^a = \begin{pmatrix} \overbrace{\begin{pmatrix} (r_1^a) & & \\ & \ddots & \\ & & (r_1^a)\end{pmatrix}}^{d_1 N \text{ times}} & & \\ & \overbrace{\begin{pmatrix} (r_2^a) & & \\ & \ddots & \\ & & (r_2^a)\end{pmatrix}}^{d_2 N \text{ times}} & \\ & & \ddots \end{pmatrix}.$$

Construct a projector operator $$P_{r_l \otimes r_m^\dagger} = \frac{1}{\Gamma}\sum_{a=1}^{\Gamma} r_l^a \otimes (r_m^\dagger)^a = \frac{1}{d_l}\delta_{lm}\mathbb{1}_{d_l}$$

Precisely one survivor for every sector

recall the twin Higgs



Outline

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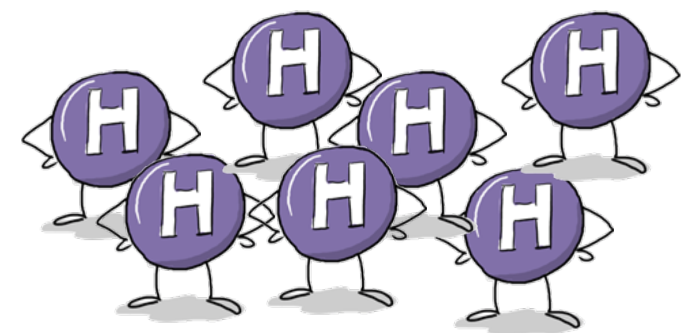
Example 1: The Z_Γ orbifold Higgs

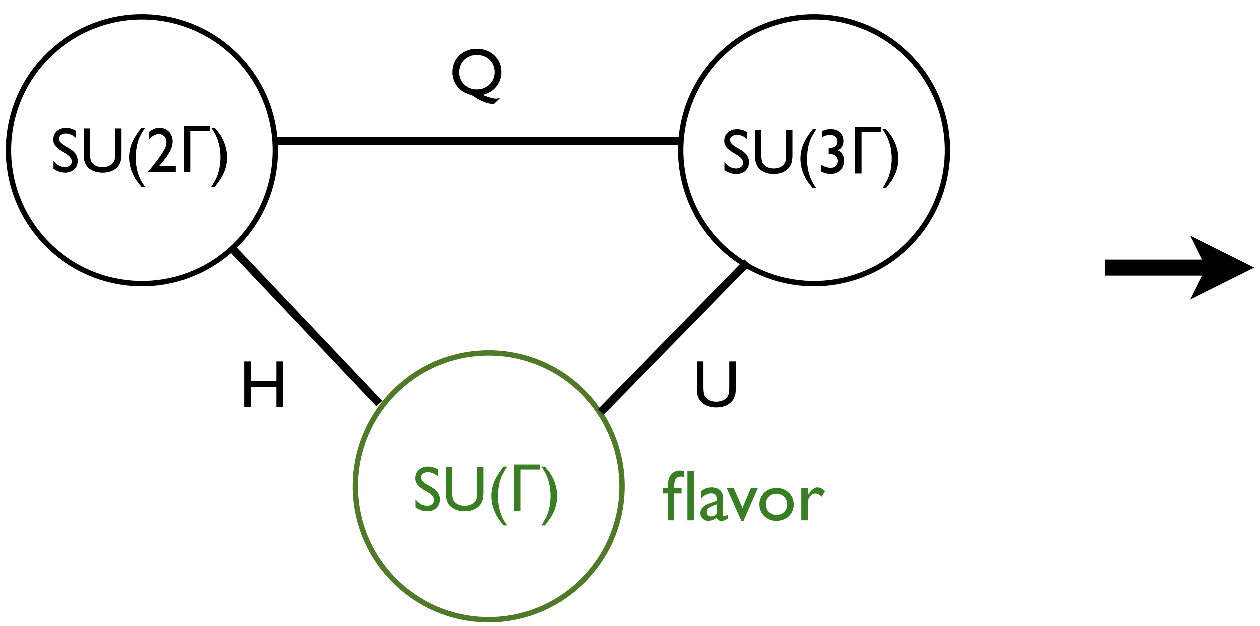
For abelian groups, all irreps are dimension one $\quad d_l = 1 \quad \forall l$

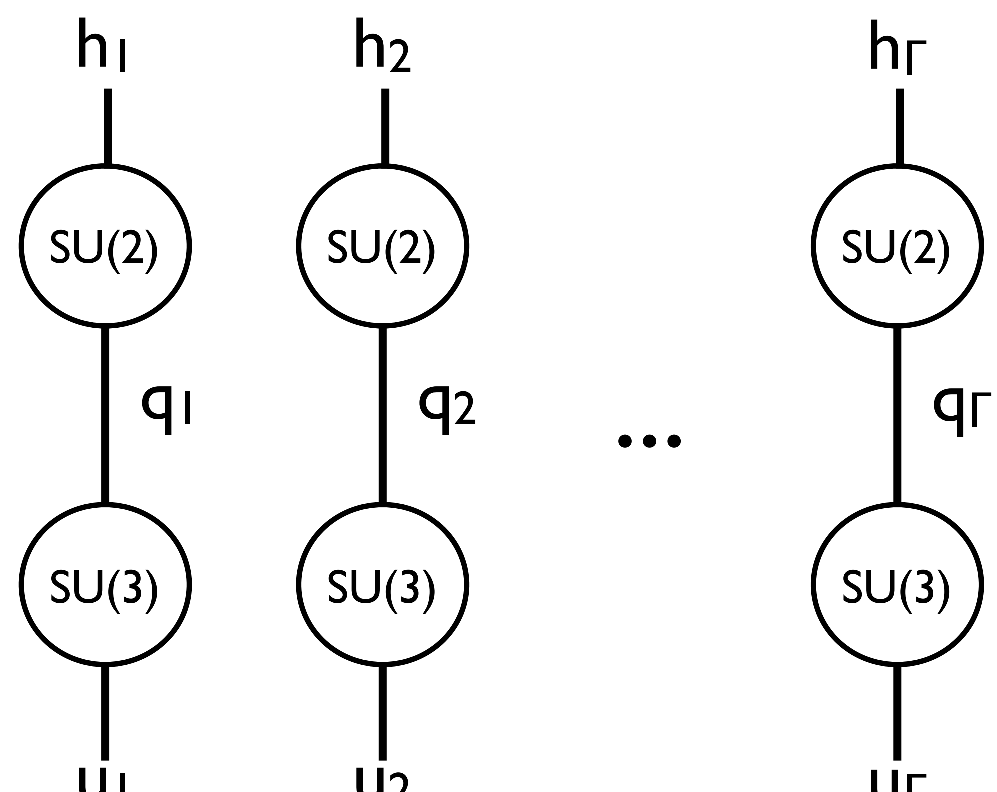


$$[SU(3\Gamma) \times SU(2\Gamma)]/Z_\Gamma \to [SU(3) \times SU(2)]^\Gamma \times U(1)^{\Gamma-1} \times S_\Gamma$$

$$g^{(1)} = g^{(2)} = \cdots g^{(\Gamma)}$$

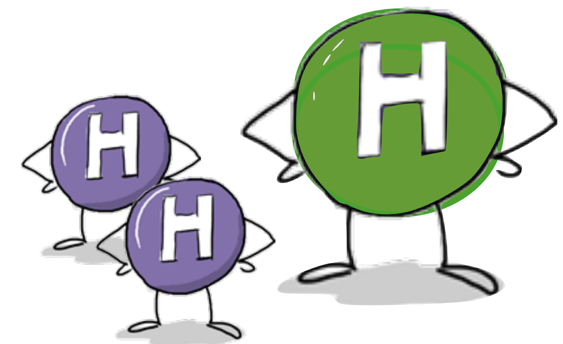
'Twin' Higgs mechanism goes through as before





Example 2: The S_3 orbifold Higgs

- S_3 is a 6-dimensional, non-abelian group
- S_3 is has 2 dim one irreps and one dim 2 irrep

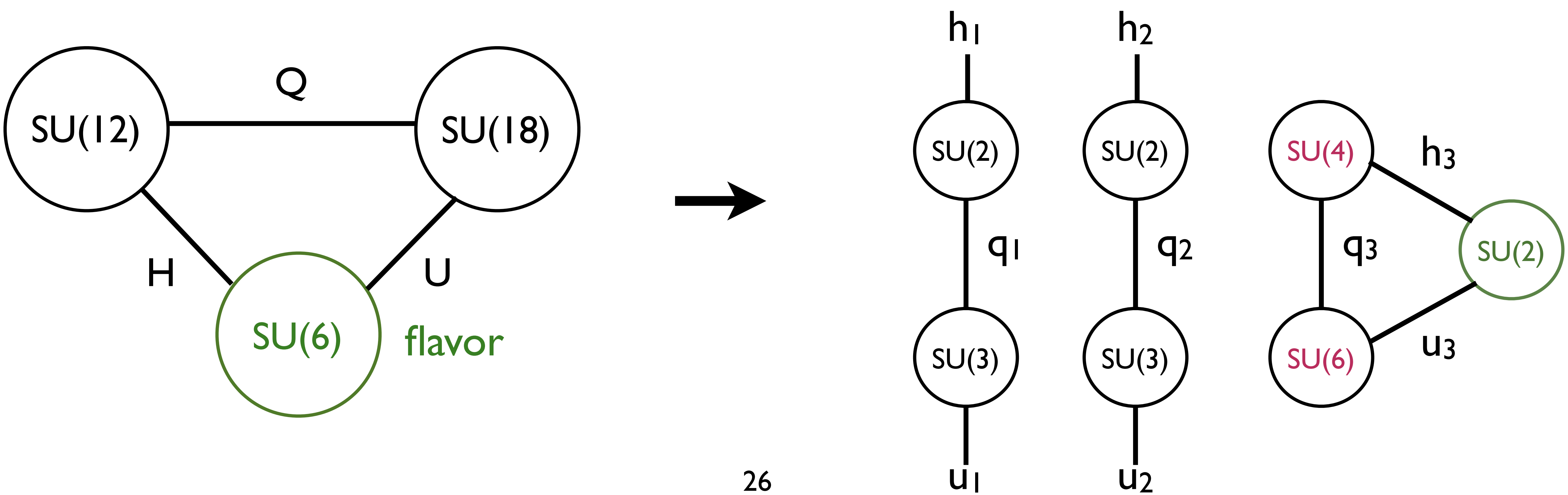


$$d_1 = d_2 = 1 \quad d_3 = 2 \qquad\qquad \Gamma = 6$$

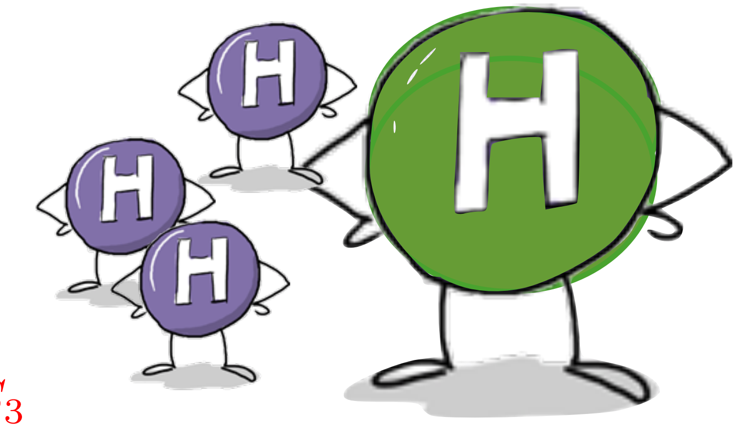
$$[SU(18) \times SU(12)]/S_3 \to [SU(3) \times SU(2)]^2 \times SU(6) \times SU(4) \times U(1)^2 \times {\color{red}Z_2}$$

$$g^{(1)} = g^{(2)} = \frac{g^{(3)}}{\sqrt{2}}$$

'Twin' Higgs mechanism goes through as before, but non-trivially



Example 3: The A_4 orbifold Higgs



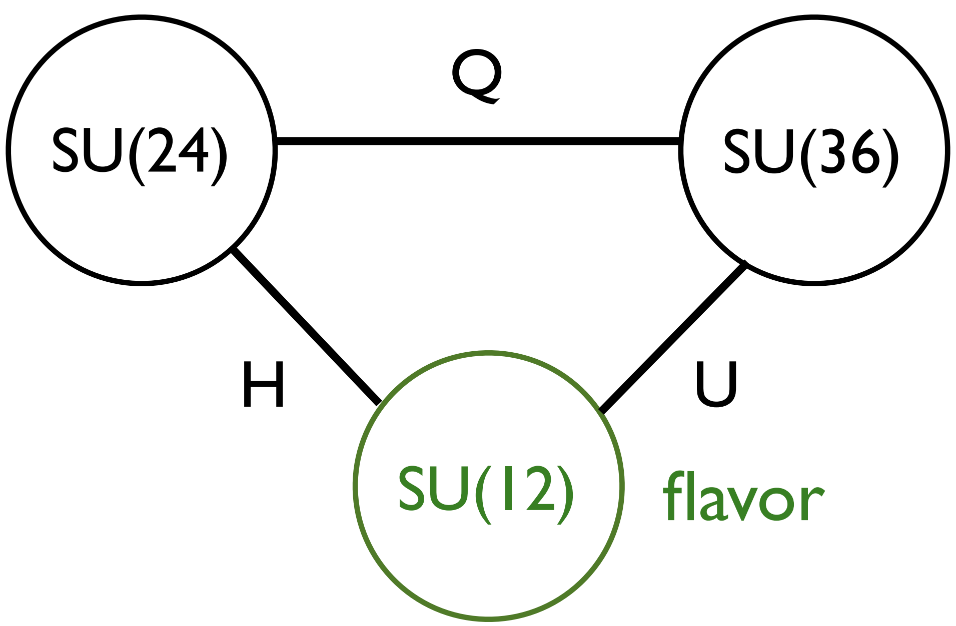
- A_4 is a 12-dimensional, non-abelian group
- A_4 is has 3 dim one irreps and one dim 3 irrep

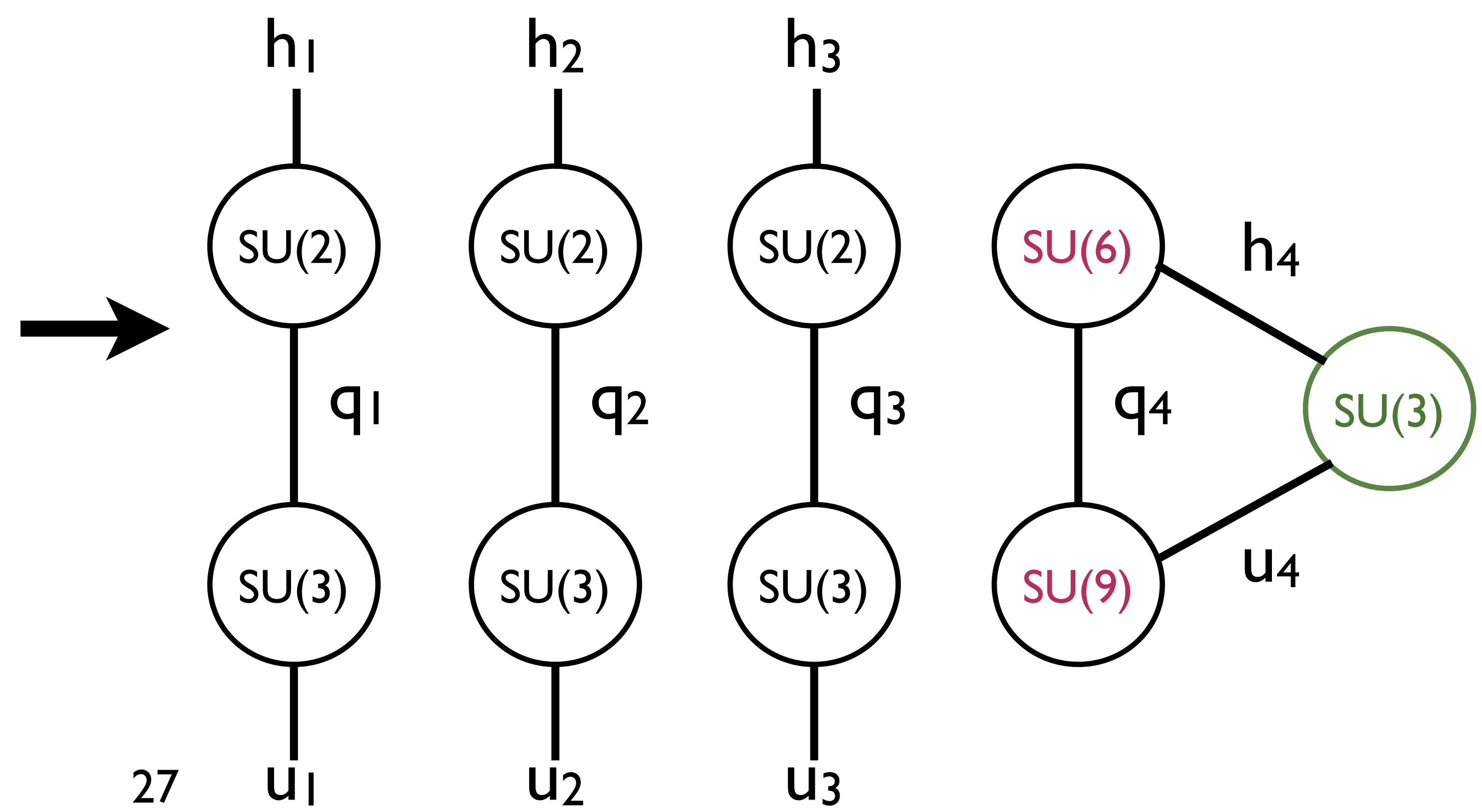
$$d_1 = d_2 = d_3 = 1 \quad d_4 = 3 \quad \Gamma = 12$$

$$[SU(36) \times SU(24)]/A_4 \to [SU(3) \times SU(2)]^3 \times SU(9) \times SU(6) \times U(1)^3 \times S_3$$

$$g^{(1)} = g^{(2)} = g^{(3)} = \frac{g^{(4)}}{\sqrt{3}}$$

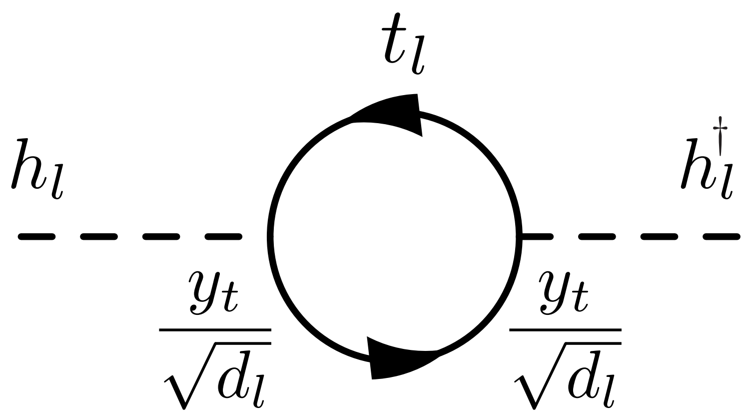
'Twin' Higgs mechanism goes through as before, but non-trivially

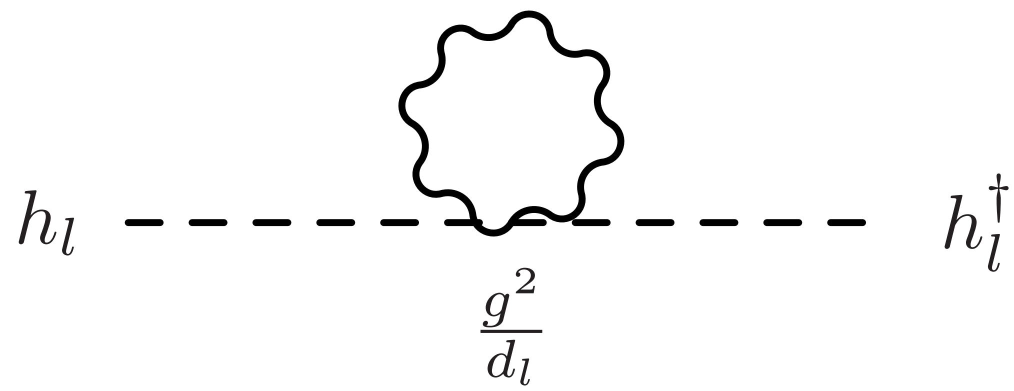




The accidental symmetry

Yukawa and gauge couplings get rescaled





$$\delta m_{h_l}^2 = -\frac{N d_l}{8\pi^2}\frac{y_t^2}{d_l}\Lambda^2 = -\frac{N}{8\pi^2}y_t^2\Lambda^2$$

Independent on sector

$$\delta m_{h_l}^2 = \frac{3}{16\pi^2}\frac{g^2}{d_l}\frac{(d_l N)^2-1}{2d_l N}\Lambda^2 = \frac{3}{32\pi^2}g^2\left(N-\frac{1}{d_l^2 N}\right)\Lambda^2$$

Independent on sector up to large N effects

$$V_{eff} = \frac{1}{16\pi^2}(-6y_t^2+3g^2)\Lambda^2\underbrace{(|h_1|^2+\cdots+|h_n|^2)}_{\text{SU}(2\Gamma)\text{ symmetric}} \underbrace{-\frac{3g^2}{16\pi^2}\Lambda^2\sum_{l=1}^{n}\frac{1}{4d_l^2}|h_l|^2}_{\text{not SU}(2\Gamma)\text{ symmetric}}$$

Orbifold Higgs

<u>Ingredients</u>

1 Discrete group G

1 SU(3Γ) x SU(2Γ) gauge theory with Γ=|G|

1 SU(Γ) Flavor symmetry

3 Bifundamentals (H, Q, U)

<u>Preparation</u>

* *Find all irreps of G*
* *Write a SU(3d_I) x SU(2d_I) sector for each irrep*
* *Decorate with other SM particles and interactions (boundary or bulk)*

Where are the bodies buried?

- What about all these extra U(1)'s?
- What about hypercharge?
- What about anomaly cancellation?
- What do higher dimensional UV completions look like?

Hypercharge

- A shared hypercharge is trivial, but difficult experimentally

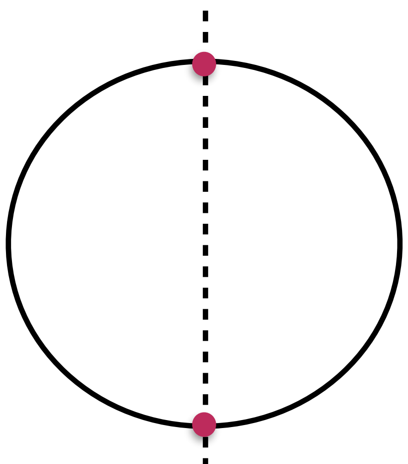
- A private hypercharge for the Standard Model is possible by extending the model
    - $U(3\Gamma) \times U(2\Gamma) \times SU(\Gamma) / G$
    - $SU(4\Gamma) \times SU(2\Gamma) \times SU(2\Gamma) / G$ (Pati-Salam unification)
    - $SU(3\Gamma) \times SU(3\Gamma) \times SU(3\Gamma) / G$ (Trinification)

- A private hypercharge is also possible by going beyond regular representation

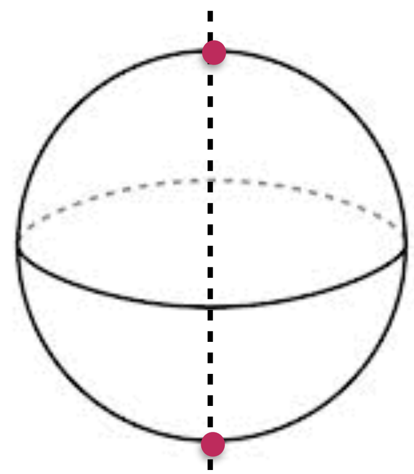
Where needed, anomalies can be cancelled by adding spectator boundary fields

Geometric UV completions

Z_2 in 5D



Z_Γ in 6D



orbifold fixed points

Sn in (n-1+4)D

on (n-1)-sphere

$$\sum_i^n x_i^2 = 1$$

$$\pm\frac{1}{\sqrt{n}}(1, 1, \cdots, 1)$$

Exact implementation depends on how you handle the hypercharge

Both 1 Higgs doublet and 2 Higgs doublet models are possible

(detailed study necessary)

Qualitative Phenomenology

Usual Twin Higgs + Hidden Valleys

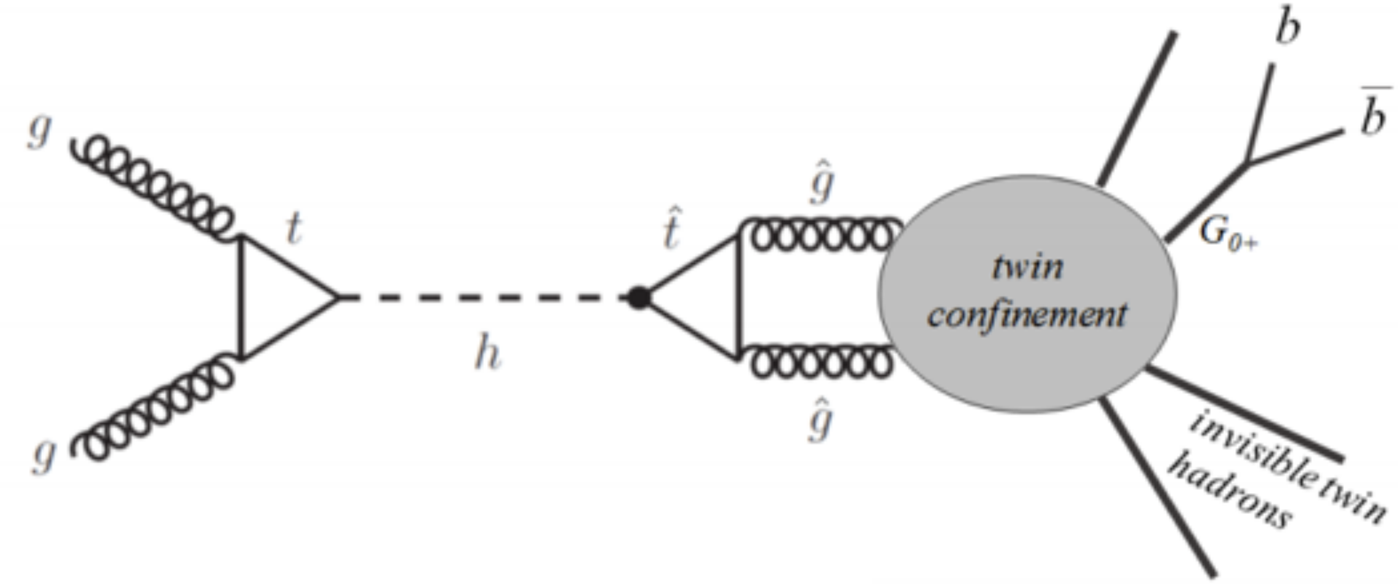
Higgs mixes with dark Higgs

Deviations in Higgs couplings

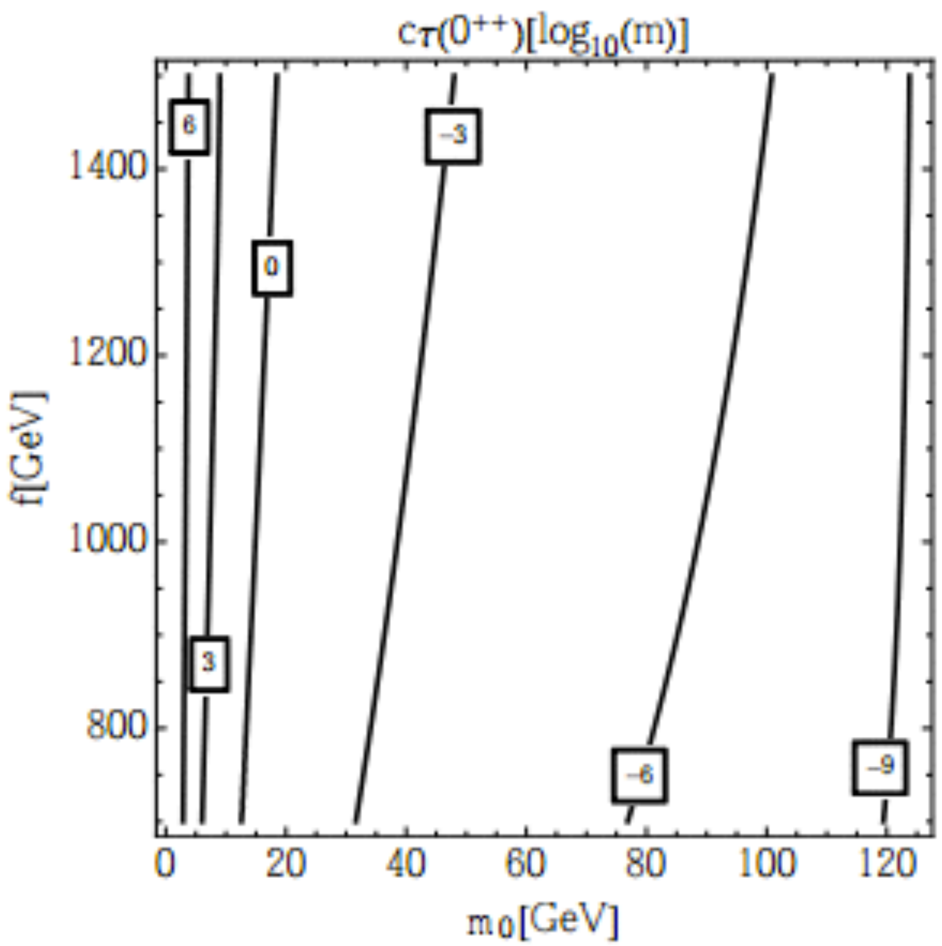
Higher KK modes charged under both SM and dark sector(s)

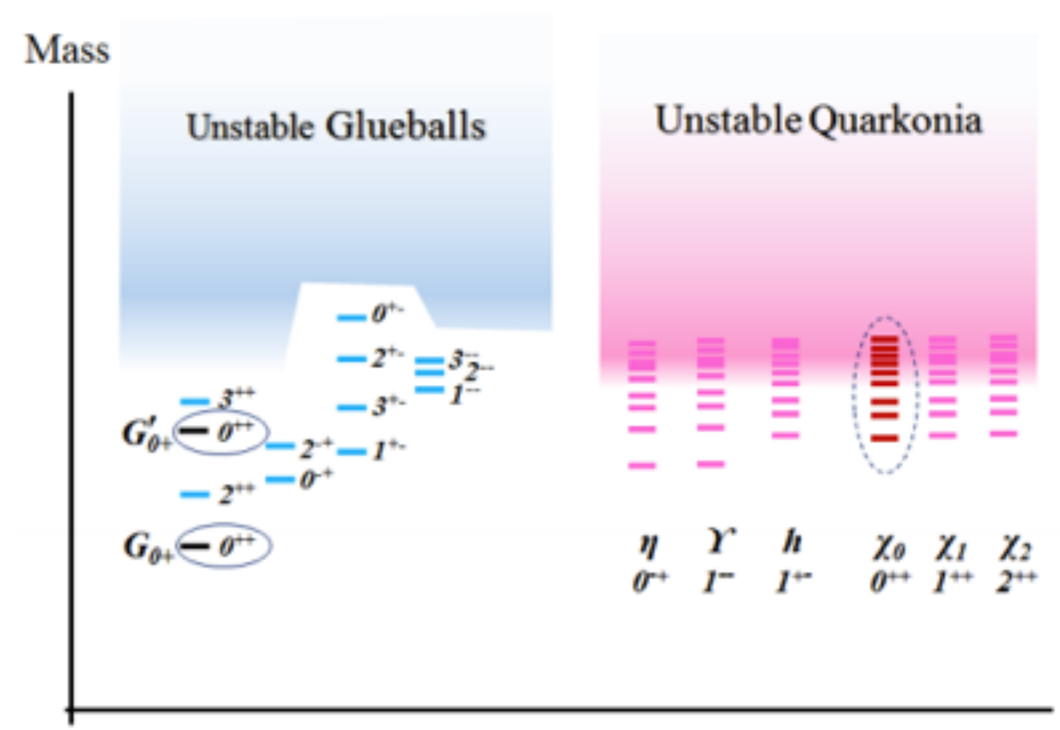
Exotic collider signatures may be possible

Should we be depressed?



$$\mathcal{L} \supset -\frac{\hat{\alpha}_3}{6\pi}\frac{v}{f}\frac{h}{f}\hat{G}^a_{\mu\nu}\hat{G}^{\mu\nu}_a \,.$$





1501.05310: N. Craig, A. Katz, M. Strassler, R. Sundrum

Summary

“Twin Unified Theories”

Systematic exploration of Neutral Naturalness

Orbifolds are a powerful tool

- The Twin Higgs is the simplest example of an Orbifold Higgs
- The hidden symmetries are uniquely fixed by the dimensions of the irreps of the orbifold group
- The number of top partners does not have to be 3

Some open questions

- Adding in SUSY: can we classify folded SUSY models?
- How far can we further generalize this: Beyond regular representation? Orientifolds?
- How is the phenomenology of NN affected by its UV completion?
- A full-fledged model