Questioning Λ: where can we go?

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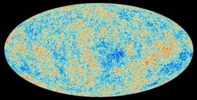
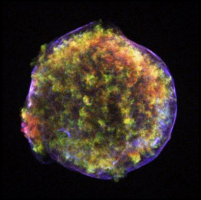
New Physics from Cosmology





- Cosmic microwave Background $\delta T/T \sim 10^{-6}$
  $\Rightarrow$ Cosmic Inflation
- Large scale structure $\Rightarrow$ Dark Matter $\rho_{DM} \approx 5 \times \rho_{SM}$
- Acceleration $\Rightarrow$ small Λ $\sim 10^{-120} G^{-1}$

Probes complementary and consistent

3 exotic ingredients $\Rightarrow$ revise basic assumptions

The Standard Cosmological Model

Theory of Gravity

$$\sqrt{-g}\frac{R}{16\pi G}$$

The Standard Cosmological Model

Metric Ansatz

$$-dt^2 + a(t)^2 d\vec{x}^2$$

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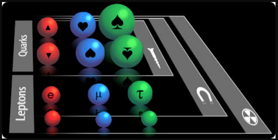

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Inflation

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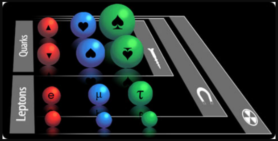

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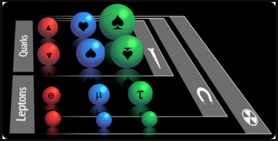

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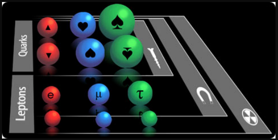

Theory of Gravity

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Why inhomogeneous cosmologies?

Precision Cosmology

Precission Goals:

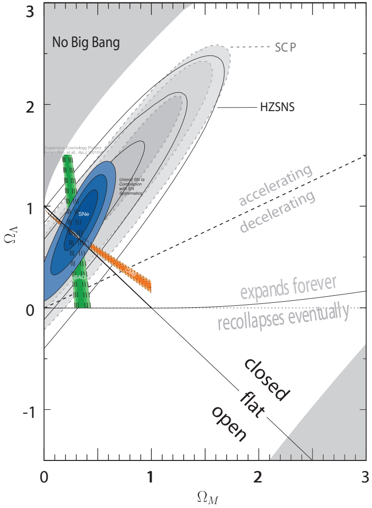
- ⋆ Search for surprises
- ⋆ Gain confidence on the results
  - $\Rightarrow$ Model independence

The case for Λ:

- $\Omega_\Lambda \neq 0$ in flat-FRW
- $w = -1$ in flat-FRW
- $w = -1$ in curved-FRW

$\dots$

- $\Omega_\Lambda \neq 0$ for non-FRW?



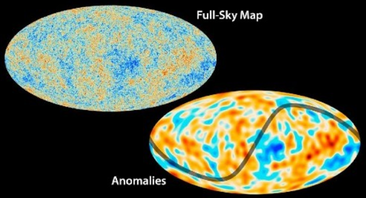
Goobar & Leibundgut 1102.1431

Post Planck Cosmology

Precission Goals:

- ⋆ Search for surprises
- ⋆ Gain confidence on the results

  ⇒ Model independence



ESA and the Planck Collaboration

Great support for ΛCDM model, but

- Minor discrepancies: $H_0 \rightarrow$ local void (Marra et al. PRL '13)
- Anomalies (i.e. statistical anisotropy)

  fluctuations? ⇒ include in the model

Spherical Symmetry $\Rightarrow$ LTB

The Lemaitre-Tolman-Bondi metric ($p = 0$)

$$ds^2 = -dt^2 + \frac{A'^2(r,t)}{1-k(r)}\,dr^2 + A^2(r,t)\,d\Omega^2$$

- Two expansion rates: $H_R = \dot{A}'/A' \neq \dot{A}/A = H_T$
- Matter/curvature profile $k(r) \Rightarrow \Omega_M(r)$

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- Expansion rate $H_0(r) \Rightarrow$ time to big-bang $t_{\rm BB}(r)$
  $t_{BB}(r) = t_0 \Rightarrow$ ◀◀ homogeneous state at early times

Spherical Symmetry ⇒ LTB

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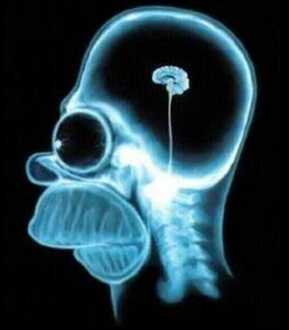
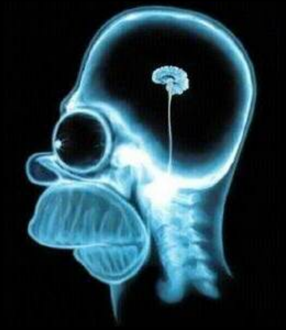
  $t_{BB}(r) = t_0 \Rightarrow$ ◀◀ homogeneous state at early times
- Observations on $ds^2 = 0 \Rightarrow$ mix time & space variations
  time acceleration $\rightarrow$ spatial variation
- CMB isotropy $\Rightarrow$ Central location $r \approx 0 \pm 10\text{Mpc}$



A non-copernican void model



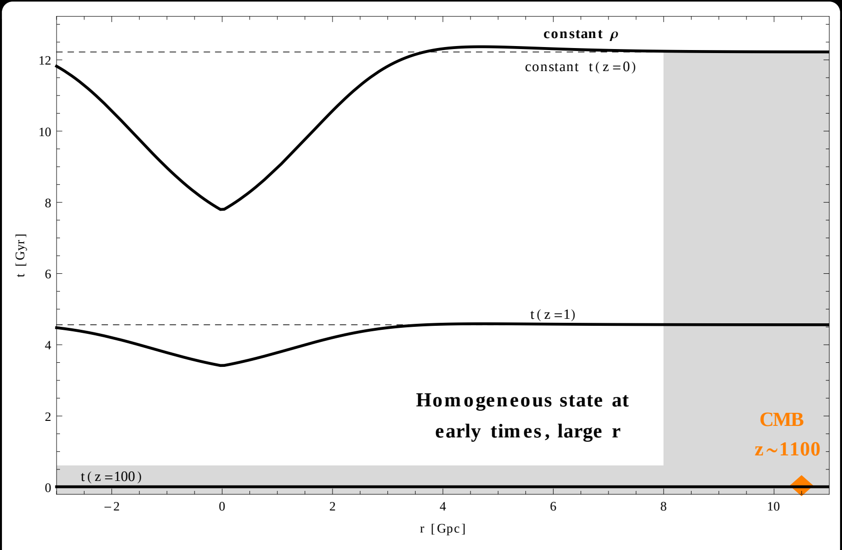
A non-copernican void model

Can't prove models right $\rightarrow$ we have to disprove alternatives

and find observational answers

$\Rightarrow$ Cosmological Justice: judge models by the facts

Observations in LTB universes, constant t_0



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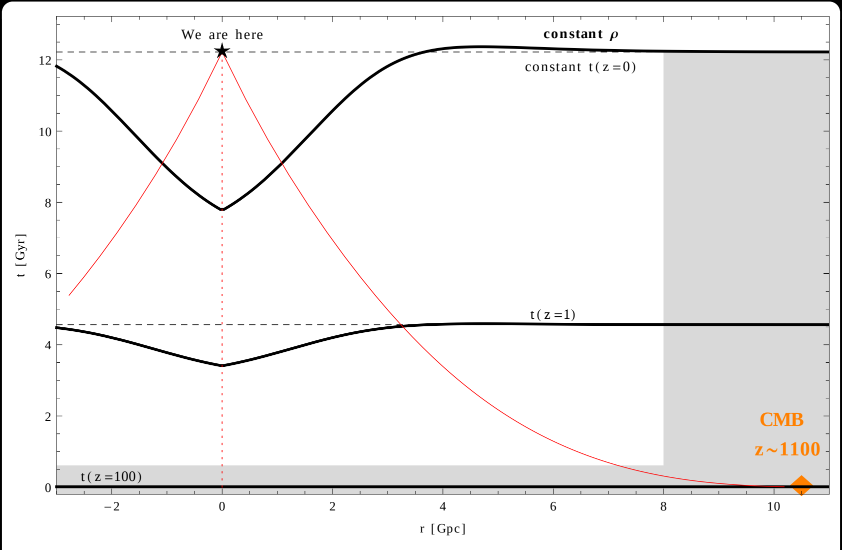

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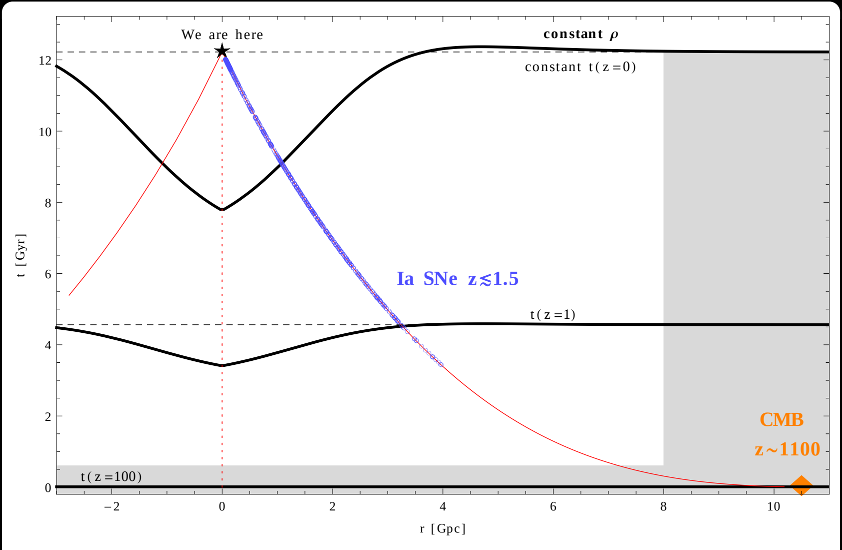

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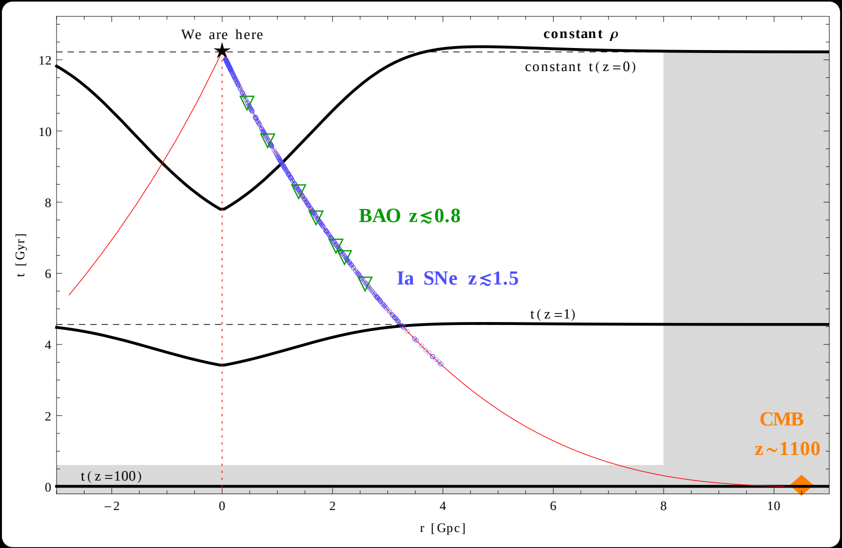

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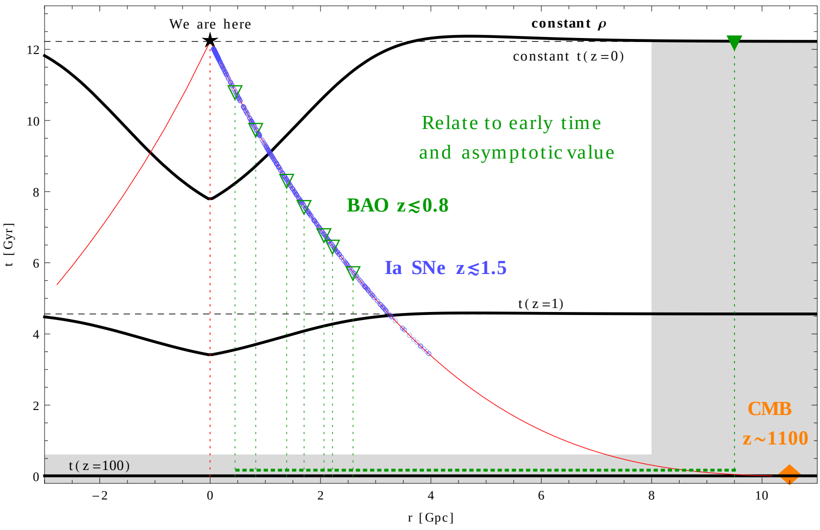
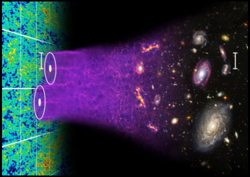


Baryon acoustic oscillations - Standard Rulers

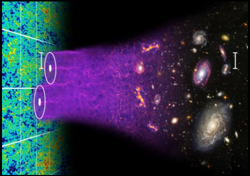


-Sound waves in the baryon-photon plasma travel a finite distance

-Initial baryon clumps $\rightarrow$ more galaxies

-Statistical standard ruler

Baryon acoustic oscillations - Standard Rulers



-Sound waves in the baryon-photon plasma travel a finite distance

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- LTB: Do these clumps/galaxies move? $\rightarrow$ Geodesics

$$\ddot{r} + \left[\frac{k'}{2(1-k)} + \frac{A''}{A'}\right]\dot{r}^2 + 2\frac{\dot{A}'}{A'}\dot{t}\dot{r} = 0$$

$\dot{t} \gg \dot{r} \Rightarrow$ Constant <u>coordinate</u> separation (zero order)

- Supported by N-body simulations (Alonso *et al.* 1204.3532) and Linear PT (February *et al.* 1206.1602)

Physical BAO scale

Early $\rightarrow$ Coordinate BAO scale ✓

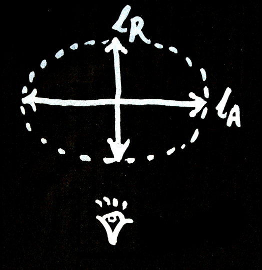
Coordinates $\rightarrow$ distances: $g_{rr} = \frac{A'^2(r,t)}{1-k(r)r^2}$, $g_{\theta\theta} = A^2(r,t)$

Physical BAO scales

$$l_R(r,t) = \frac{A'(r,t)}{A'(r,t_e)} l_{\text{early}}$$

$$l_A(r,t) = \frac{A(r,t)}{A(r,t_e)} l_{\text{early}}$$

l_{early} = early physical BAO scale at t_e



⋆ LTB: evolving (t), inhomogeneous (r) and anisotropic $l_A \neq l_R$

Observed BAO scale

Early → Coordinate → Physical BAO scale ✓

⋆ Geometric mean $\boxed{d \equiv \left(\delta\theta^2 \delta z\right)^{1/3}}$

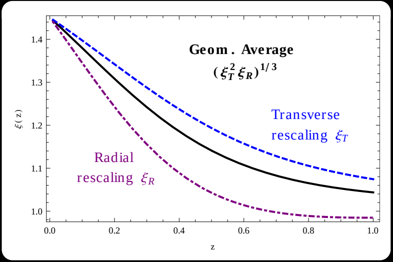
$$\delta\theta = \frac{l_A(z)}{D_A(z)}, \quad \delta z = (1+z) H_R(z)\, l_R(z)$$

Compare with FRW:

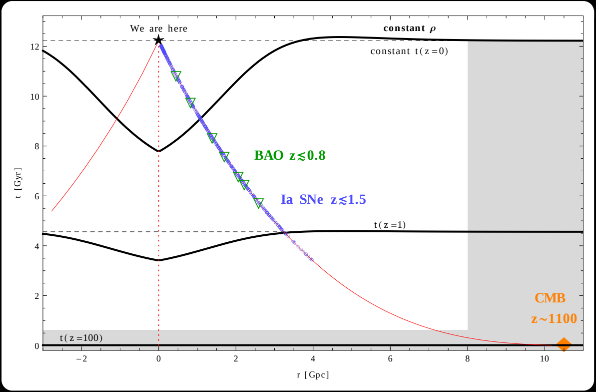
$$d_{\rm LTB}(z) \approx \xi(z)\, d_{\rm FRW}(z)$$

Rescaling: $\xi(z) = (\xi_A^2\, \xi_R)^{1/3}$

→ Inhomogeneous, anisotropic



Data and models



- GBH model (6 param)
- WiggleZ + SDSS + 6dF
- Union 2 Compilation
- $H_0 \to$ luminosity prior
- CMB peaks information (simplified analysis)

GBH + Homogeneous BB + asymptotically flat

Uncalibrated candles & rulers:

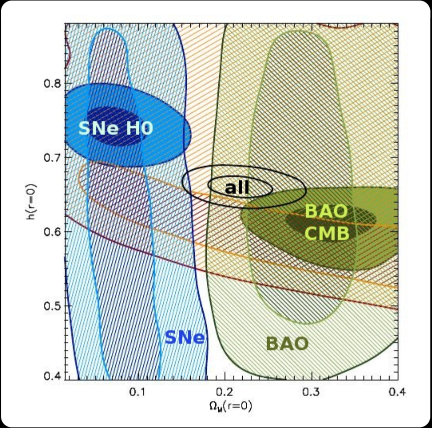
- SNe $\rightarrow \Omega_{\rm in} \approx 0.1$
- BAO $\rightarrow \Omega_{\rm in} \approx 0.3$

$\star$ $\Omega_M(0)$ 3σ discrepant!

Adding calibration:

- **SNe+H0** $\rightarrow h_{\rm in} \approx 0.74$
- **BAO+CMB** $\rightarrow h_{\rm in} \approx 0.62$

worse using full CMB!



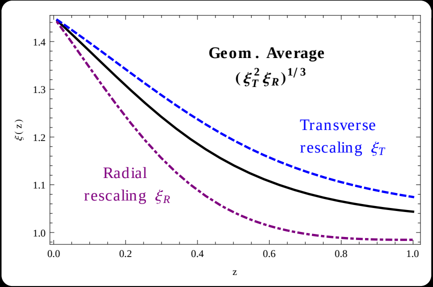
Asymptotically open $\rightarrow$ not any better

Tension in the Void

- Bad fit to SNe and BAO
- LTB Strongly ruled out!

⋆ SNe: measure distance

⋆ BAO: distance + rescaling



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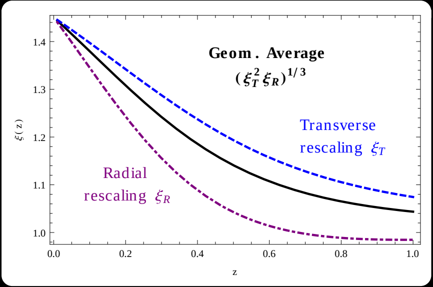

Other problems of LTB models with $\Lambda = 0$:

- $H0$: CMB vs Local - Moss et al '10
- kSZ bounds - Zhang & Stebbins '10
- Standard clocks - De Putter et al '12

Geometrical result, whatever Luminosity & BAO scale
$\Rightarrow$ Standard Rulers vs Standard Candles

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Initial Conditions

Inflation

Structure formation

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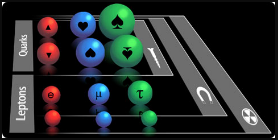
Λ

Metric Ansatz

$-dt^2 + a(t)^2 d\vec{x}^2 \quad \checkmark$

Standard Matter

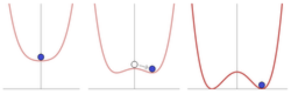


Theory of Gravity

$$\boxed{\sqrt{-g}\frac{R}{16\pi G}}$$

Why alternative gravities?

- Inflation again?
- Einstein's gravity not final: QG surprises at low energy?
- Classical Λ tuning: EW phase transition



- Test gravity on cosmological scales

Focus:

- Most general theory?
- Compatible with Local Gravity tests?

Other: Acceleration, make Λ small, cosmological signatures ...

What is the most general theory of gravity?

(that deserves our attention)

Ostrogradski's Theorem (1850)

Theories with $L \supset \frac{\partial^n q}{\partial t^n}$, $n \geq 2$ are unstable*

$$L(q(t), \dot{q}, \ddot{q}) \to \frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} + \boxed{\frac{d^2}{dt^2}\frac{\partial L}{\partial \ddot{q}}} = 0$$

$$q, \dot{q}, \ddot{q}, \dddot{q} \to Q_1, Q_2, P_1, P_2$$

$$H = \boxed{\mathbf{P_1 Q_2}} + \text{terms independent of } P_1$$

* <u>Assumes $\dddot{q}, \ddot{q} \leftrightarrow P_2, Q_2$</u>

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loophole for Degenerate Theories:

- 2^{nd} order equations
- Implicit constraints / reduced phase space

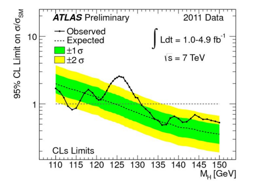
Lovelock's Theorem (1971)

$g_{\mu\nu}$ + Local + 4-D + Lorentz Theory with 2nd order Eqs.

$$\sqrt{-g}\frac{1}{16\pi G}(R-2\Lambda)+\sqrt{-g}\mathcal{L}_m(g_{\mu\nu},\psi_M)$$

Add a scalar field ϕ:

- simple, isotropic
- seem to exist ⟶
- arise in other theories (e.g. extra dimensions)



Jordan-Brans-Dicke

$$R \to \phi R$$

Modify Einstein-Hilbert action

Bekenstein

$$g_{\mu\nu} \to \phi g_{\mu\nu}$$

Modify the metric in $\mathcal{L}_m$

Horndeski's Theory (1974)

$g_{\mu\nu}$ + $\boxed{\phi}$ + Local + 4-D + Lorentz Theory with 2nd order Eqs.

$$\mathcal{L}_H = G_2(X,\phi) - G_3(X,\phi)\Box\phi$$
$$+G_4 R + G_{4,X}\left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right]$$
$$+G_5 G_{\mu\nu}\phi^{;\mu\nu} - \frac{G_{5,X}}{6}\left[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu}{}^{;\nu}\phi_{;\nu}{}^{;\lambda}\phi_{;\lambda}{}^{;\mu}\right]$$

4 × free functions of ϕ, $X \equiv -\frac{1}{2}\phi_{,\mu}\phi^{,\mu}$

- Jordan-Brans-Dicke: $G_4 = \frac{\phi}{16\pi G}$, $G_2 = \frac{X}{\omega(\phi)} - V(\phi)$

Horndeski's Theory (1974)

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- Kinetic Gravity Braiding - Deffayet *et al.* JCAP 2010

Horndeski's Theory (1974)

$g_{\mu\nu} + \boxed{\phi} + \text{Local} + \text{4-D} + \text{Lorentz}$ Theory with 2^{nd} order Eqs.

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- Deriv. couplings $G_4(X)$

Horndeski's Theory (1974)

$g_{\mu\nu}$ + $\boxed{\phi}$ + Local + 4-D + Lorentz Theory with 2^{nd} order Eqs.

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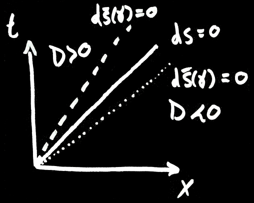
- Kinetic Gravity Braiding - Deffayet *et al.* JCAP 2010
- Deriv. couplings $G_4(X)$, $G_5 \neq 0$

Disformal Relation - Bekenstein (PRD 1992)

Matter sector $\sqrt{-\tilde{g}}\mathcal{L}_m(\tilde{g}_{\mu\nu},\ldots)$ with

$$\tilde{g}_{\mu\nu} = \underbrace{C(X,\phi)g_{\mu\nu}}_{\text{conformal}} + \underbrace{D(X,\phi)\phi_{,\mu}\phi_{,\nu}}_{\text{disformal}}$$

$\Rightarrow$ 2



$$\mathcal{L}_H \xrightarrow{C_{,X},D_{,X}=0} \tilde{\mathcal{L}}_H$$

$$\mathcal{L}_H \xrightarrow{C_{,X},D_{,X}\neq 0} \not{\mathcal{L}}_H$$

$C_{,X}, D_{,X} \neq 0 \rightarrow$ non-Horndeski!

Pure conformal: $\tilde{g}_{\mu\nu} = C(X,\phi) g_{\mu\nu} \quad \Longrightarrow \quad \Omega^2 R + \boxed{6\Omega_{,\alpha}\Omega^{,\alpha}}$

$$\nabla_\mu\Big((\Omega R - 6\Box\Omega)\Omega_{,X}\phi^{,\mu}\Big) + \Omega_{,\phi}(\Omega R - 6\Box\Omega) + \frac{1}{2}\frac{\delta\mathcal{L}_\phi}{\delta\phi} = 0$$

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$$2\Omega(g_{\mu\nu}\Box\Omega - \Omega_{;\mu\nu}) + \Omega^2 G_{\mu\nu}$$
$$- g_{\mu\nu}\Omega_{,\alpha}\Omega^{,\alpha} + 4\Omega_{,\mu}\Omega_{,\nu} - (\Omega R - 6\Box\Omega)\Omega_{,X}\phi_{,\mu}\phi_{,\nu} = 8\pi G T_{\mu\nu}$$

Pure conformal: $\tilde{g}_{\mu\nu} = C(X,\phi) g_{\mu\nu} \quad \Longrightarrow \quad \Omega^2 R + \boxed{6\Omega_{,\alpha}\Omega^{,\alpha}}$

$$\nabla_\mu\Big((\Omega R - 6\Box\Omega)\Omega_{,X}\phi^{,\mu}\Big) + \Omega_{,\phi}(\Omega R - 6\Box\Omega) + \frac{1}{2}\frac{\delta\mathcal{L}_\phi}{\delta\phi} = 0$$

Take trace with $g^{\mu\nu}$: $\quad 2\Omega(g_{\mu\nu}\Box\Omega - \Omega_{;\mu\nu}) + \Omega^2 G_{\mu\nu}$

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Take trace with $g^{\mu\nu}$: $2\Omega(3\Box\Omega) + \Omega^2(-R)$

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Take trace with $g^{\mu\nu}$: $\quad 2\Omega(3\Box\Omega) + \Omega^2(-R)$

$$- 0 - (\Omega R - 6\Box\Omega)\Omega_{,X}\phi_{,\mu}\phi_{,\nu} = 8\pi G T_{\mu\nu}$$

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Take trace with $g^{\mu\nu}$: $2\Omega(3\Box\Omega) + \Omega^2(-R)$

$$- 0 - (\Omega R - 6\Box\Omega)\Omega_{,X}(-2X) = 8\pi G T_{\mu\nu}$$

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Take trace with $g^{\mu\nu}$:
$$2\Omega(3\Box\Omega) + \Omega^2(-R) - 0 - (\Omega R - 6\Box\Omega)\Omega_{,X}(-2X) = 8\pi G T$$

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Implicit Constraint - MZ, García-Bellido 1308.4685

$$-(\Omega R - 6\Box\Omega) = \boxed{\frac{8\pi G\Omega_{,X} T}{\Omega - 2\Omega_{,X} X} \equiv \mathcal{T}_{\rm K}} \sim \partial\phi$$

Trace of metric eqs $\rightarrow$ solves high derivs!

Implicit Constraint - MZ, García-Bellido 1308.4685

$$\mathcal{T}_\mathrm{K} \equiv \frac{8\pi G \Omega_{,X} T}{\Omega - 2\Omega_{,X} X} = -(\Omega R - 6\Box\Omega)$$

Scalar Field eqs:

$$\nabla_\mu \left(\phi^{,\mu}\mathcal{T}_\mathrm{K}\right) + \frac{\Omega_{,\phi}}{\Omega_{,X}}\mathcal{T}_\mathrm{K} - \frac{1}{2}\frac{\delta\mathcal{L}_\phi}{\delta\phi} = 0$$

Metric eqs:

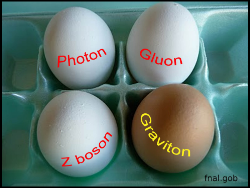
$$\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} \supset g_{\mu\nu}\Box\Omega - \Omega_{;\mu\nu} = \left(\begin{array}{c|c} g^{k\alpha}\Omega_{;k\alpha} & -\Omega_{;0i} \\ \hline -\Omega_{;0i} & g_{ij}\Box\Omega - \Omega_{;ij} \end{array}\right)$$

✓ No higher time derivatives in Jordan frame

How to satisfy local gravity tests?

Scalar-tensor theories

$\Rightarrow$ scalar mediated force



$\star$ Point Particle, coupled to $\tilde{g}_{\mu\nu}[\phi]$:

$$\ddot{x}^\alpha = -\big(\Gamma^\alpha_{\mu\nu} + \underbrace{\mathcal{K}^\alpha_{\mu\nu}}_{\gamma^{\alpha\lambda}\left(\nabla_{(\mu}\gamma_{\nu)\lambda} - \frac{1}{2}\nabla_\lambda \gamma_{\mu\nu}\right)}\big)\dot{x}^\mu \dot{x}^\nu$$

$$\Rightarrow \boxed{F^i_\phi \approx f[\phi]\nabla^i \phi} + \mathcal{O}(v^i/c)$$

Subtle the Force can be

$$F^i_\phi \approx f[\phi]\nabla^i\phi$$

"You must feel the Force around you;
here, between you, me, the tree, the rock,
everywhere, yes"

Master Yoda

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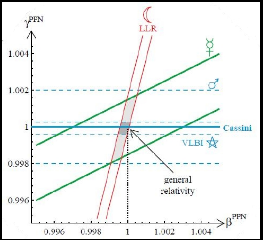

Screening Mechanisms

$$\left|\frac{F_\phi}{F_G}\right| \ll 1 \text{ when } \begin{cases} \rho \gg \rho_0 \\ r \ll H_0^{-1} \end{cases}$$

May the force *not* be with you

$\rho \gg \rho_0$ Chameleon Screening - Khoury & Weltman (PRL 2004)

Yukawa force: $\phi \propto \frac{1}{r} e^{-\phi/m_\phi}$ with $m_\phi(\rho)$ increases with ρ

(see also Symmetron - Pietroni PRD '05, Hinterbichler Khoury PRL '10)

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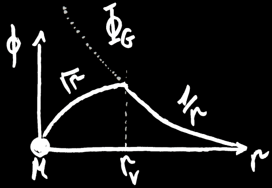
$r \ll H_0^{-1}$ Vainshtein Screening - Vainshtein (PLB 1972)

$\mathcal{L} \supset (\partial\phi) + \Box\phi X/m^2 + \alpha\phi T_m$ Non-linear derivative interactions

$$\Rightarrow \; \Box\phi + m^{-2}\left((\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu}\right) = \alpha M \delta(r)$$

$$\phi \propto \begin{cases} r^{-1} & \text{if } r \gg r_V \\ \sqrt{r} & \text{if } r \ll r_V \end{cases}$$

Vainshtein radius $r_V \propto (GM/m^2)^{1/3}$



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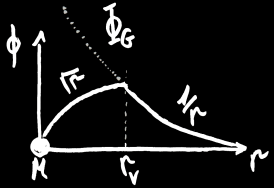

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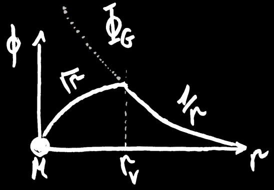

Vainshtein radius $r_V \propto (GM/m^2)^{1/3}$

Einstein gravity + disformal coupling

$$L_{EF} = \sqrt{-g}\left(\frac{R}{16\pi G} + \mathcal{L}_\phi\right) + \boxed{\sqrt{-\tilde{g}}\mathcal{L}_M(\tilde{g}_{\mu\nu}, \psi)}$$

$$\tilde{g}_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi)\phi_{,\mu}\phi_{,\nu}$$

- $G^{\mu\nu} = 8\pi G(T_m^{\mu\nu} + T_\phi^{\mu\nu})$
- Matter-field interaction: $\nabla_\mu T_m^{\mu\nu} = -Q\phi^{,\nu}$

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$$Q = \frac{D}{C}\nabla_\mu\left(T_m^{\mu\nu}\phi_{,\nu}\right) - \frac{C'}{2C}T_m + \left(\frac{D'}{2C} - \frac{DC'}{C^2}\right)\phi_{,\mu}\phi_{,\nu}T_m^{\mu\nu}$$

Kinetic mixing Conformal Disformal

Consequences of Kinetic Mixing - Koivisto, Mota, MZ (PRL 2012)

Static matter $\rho(\vec{x})$ + non-rel. $p = 0$

$$\left[1 + \frac{D\rho}{C - 2DX}\right]\ddot{\phi} + \mathcal{F}(\vec{\nabla}\phi_{,\mu}, \phi_{,\mu}, \rho) = 0$$

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Disformal Screening Mechanism

$$\ddot{\phi} \approx -\frac{D'}{2D}\dot{\phi}^2 + C'\left(\frac{\dot{\phi}^2}{C} - \frac{1}{2D}\right) \quad \text{(If } D\rho \to \infty\text{)}$$

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$\Rightarrow$ No $\vec{\nabla}\phi$ between massive bodies $\Rightarrow$ No fifth force!

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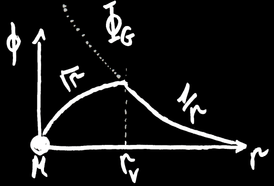
Signatures: fast matter, pressure, strong gravity, cosmic gradients

Kinetic Screening mechanisms:



⋆ Vainshtein screening:

within $r_V \propto (GM/m^2)^{1/3}$

⋆ Disformal coupling + canonical field

$$\Box\phi + \frac{-\mathcal{Q}_{\mu\nu}\delta T_m^{\mu\nu}}{C + D(\phi_{,r})^2} = 0 \quad \Rightarrow \quad \begin{cases} \text{asymptotic } \phi = \frac{S}{r} \\ \text{breaks down at } \tilde{r}_V = \left(\frac{DS^2}{C}\right)^{1/4} \end{cases}$$

Different screening mechanisms - MZ, Koivisto, Mota - PRD 2013

⋆ $D > 0 \Rightarrow$ stable disformal screening, complex r_V

⋆ $D < 0 \Rightarrow$ Positive r_V, but unstable (if canonical)

Scalar-tensor Generations

Jordan-Brans-Dicke $\rightarrow$ Horndeski $\rightarrow$ Implicit Constraints

The Standard Cosmological Model

Initial Conditions

Inflation

Structure formation

Dark Matter

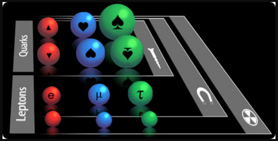
Acceleration

Λ

Metric Ansatz

$$-dt^2 + a(t)^2 d\vec{x}^2$$

Standard Matter



Theory of Gravity

$$\sqrt{-g}\frac{R}{16\pi G}$$

Conclusions

- Model independence & cosmological justice
- Spherically symmetric models <u>without</u> Λ <u>don't work</u> (perhaps if very involved)

Scalar-tensor gravity as alternative to Λ:

- can mimick Einstein's locally through screening

  Vainshten & disformal: related but different

- <u>and</u> have cosmological effects
- Ostrogradski's theorem $\rightarrow$ understand all the alternatives
- 3 theoretical generations

Backup Slides

Disformally Related Theories - MZ, Koivisto, Mota (PRD 2013)

$$\boxed{\gamma_{\mu\nu} = C(\phi) g_{\mu\nu} + D(\phi)\phi_{,\mu}\phi_{,\nu}}$$

Einstein Frame: $\mathcal{L}_{EF} = \sqrt{-g} R[g_{\mu\nu}] + \boxed{\sqrt{-\tilde{g}}\mathcal{L}_M(\tilde{g}_{\mu\nu}, \psi)}$

$$\boxed{\text{Einstein}}$$

$$g_{\mu\nu} \to \frac{1}{C} g_{\mu\nu} - \frac{D}{C}\phi_{,\mu}\phi_{,\nu}$$

$$\boxed{\text{Jordan}}$$

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$$\Big\downarrow \; g_{\mu\nu} \to \frac{1}{C} g_{\mu\nu} - \frac{D}{C}\phi_{,\mu}\phi_{,\nu}$$

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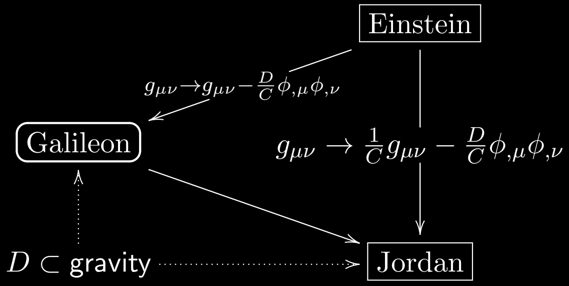

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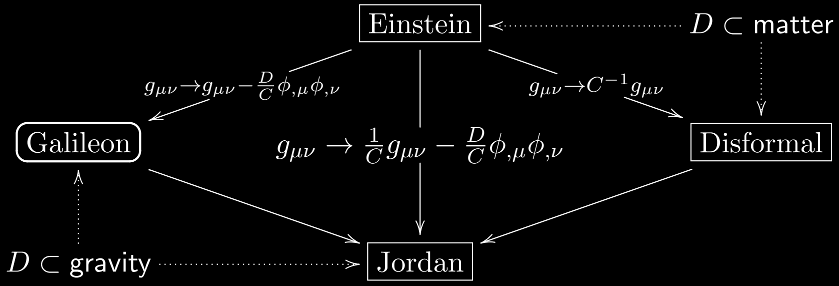

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Disformal screening - assumptions:

- Static $\partial_t\rho = 0$, Pressureless $Dp < \mathcal{X} \equiv C - 2DX$
- Neglect $\frac{p}{\rho}$, $\frac{p}{\rho}\left(\frac{\vec{\partial}\phi}{\partial_t\phi}\right)^2$, $\frac{\mathcal{X}}{D\rho}$, $\frac{\mathcal{X}}{D\rho}V'/\ddot{\phi}$, $\Gamma^{\mu}_{00}\phi_{,\mu}/\ddot{\phi}\ \sim 0$

Potential Signatures:

- **Matter velocity flows:** $T^{0i} \sim v/c \rightarrow$ Binary pulsars?
- **Pressure:** effects on radiation
- **Strong gravitational fields:** $\Gamma^{\mu}_{00}\phi_{,\mu} \rightarrow$ Compact objects?
- **Frozen cosmic gradients:** Evolution independent of $\partial_i\phi$

Properties of the Field Equation

Canonical scalar field $\mathcal{L}_\phi = X - V$, solve for $\nabla\nabla\phi$

$$\mathcal{M}^{\mu\nu}\nabla_\mu\nabla_\nu\phi + \frac{C}{C-2DX}\mathcal{Q}_{\mu\nu}T_m^{\mu\nu} - V = 0$$

$$\mathcal{M}^{\mu\nu} \equiv g^{\mu\nu} - \frac{D\,T_m^{\mu\nu}}{C-2DX}\,, \quad Q_{\mu\nu} \equiv \frac{C'}{2C}g_{\mu\nu} + \left(\frac{C'D}{C^2} - \frac{D'}{2C}\right)\phi_{,\mu}\phi_{,\nu}$$

Coupling to (Einstein F) perfect fluid $T^\mu{}_\nu = \mathrm{diag}(\rho, p, p, p)$

- $\mathcal{M}^0{}_0 = 1 + \frac{D\rho}{C-2DX}$, $D, \rho > 0 \Rightarrow$ no ghosts
- $\mathcal{M}^i{}_i = 1 - \frac{Dp}{C-2DX}$, $\Rightarrow$ potential instability if $p > C/D - X$

  - Does it occur dynamically?
  - Consider non-relativistic coupled species $\mathcal{M}^i{}_i > 0$