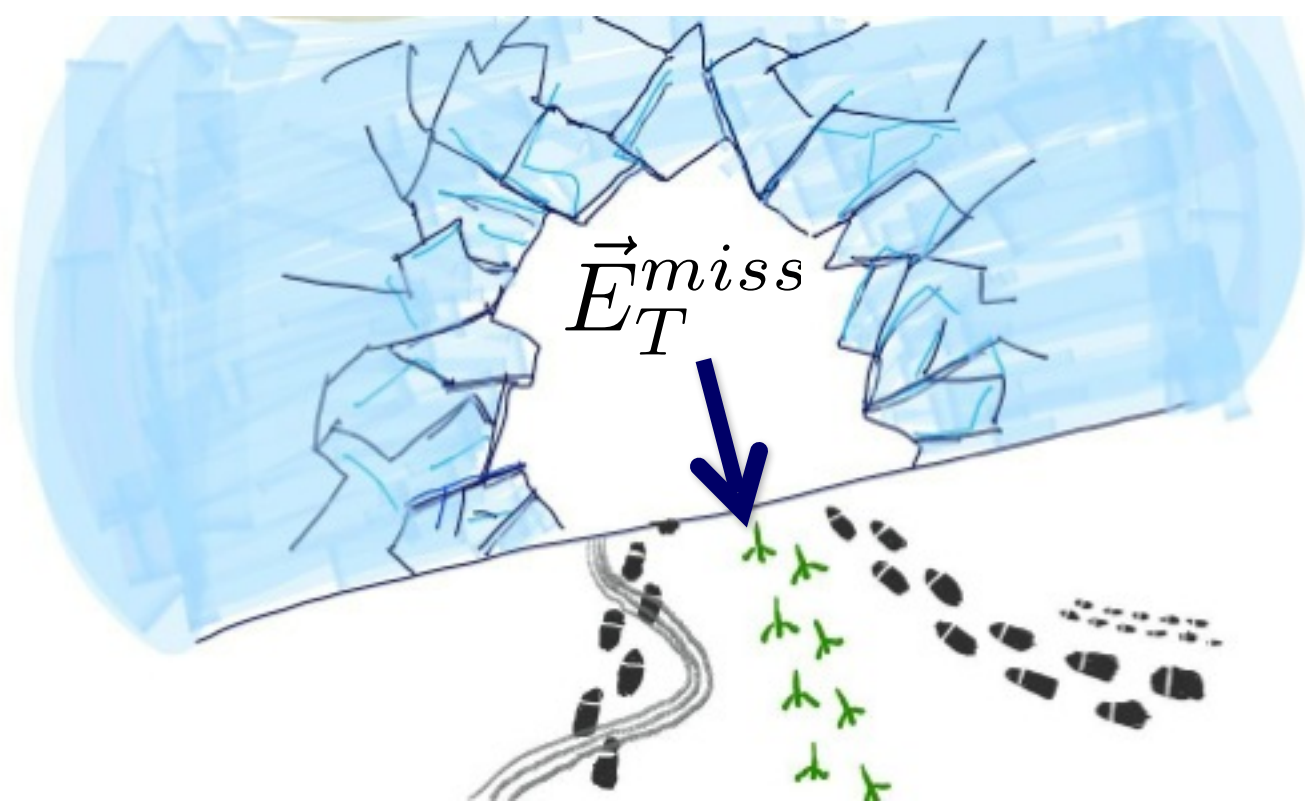




Kinematic variables for weakly-interacting particles at the LHC

Christopher Rogan



HARVARD
UNIVERSITY

UC Irvine Particle Physics Seminar – November 5, 2014



Talk Outline

- Weakly interacting particles at the LHC
 - Why? How?
- Kinematic handles for studying them
 - MET
 - singularity variables
- Recursive Jigsaw Reconstructions
 - ex. di-leptons (i.e. super-razor variables)
 - ex. di-leptonic tops/stops (something sexier)



Weakly interacting particles @ LHC

- **Why are they interesting?**

- Dark Matter

- It exists - but what is it? Would like to know if we're producing these particles at the LHC

- Electroweak bosons

- Decays of W and Z often produce neutrinos

- New symmetries

- Discrete symmetries (ex. R-parity) make lightest new 'charged' particles stable

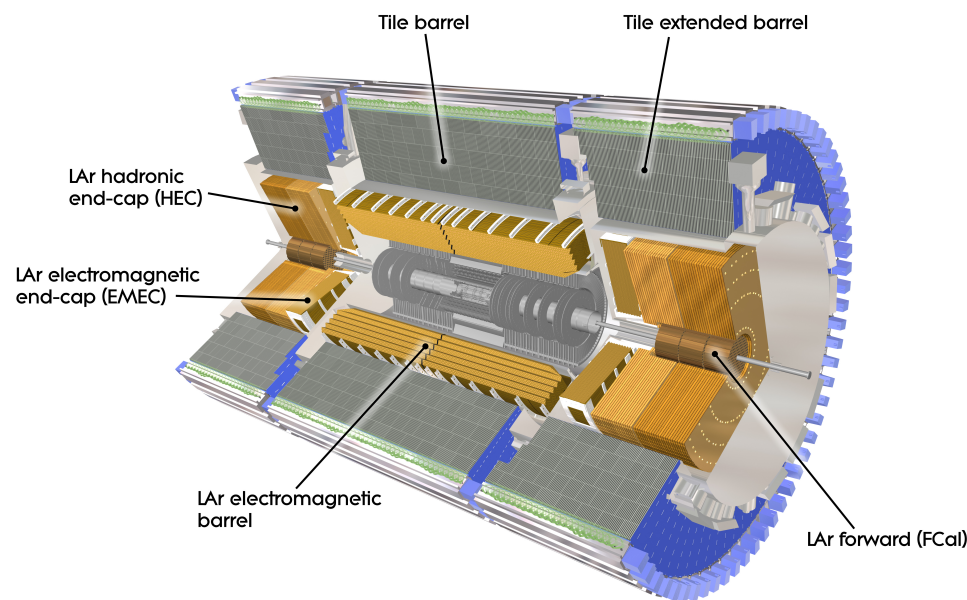


Weakly interacting particles @ LHC

■ How do we study them?

- Can infer their presence through *missing transverse energy*
- Hermetic design of LHC experiments allows us to infer *‘what’s missing’*

- full azimuthal coverage, up to $|\eta|$ of ~ 5
- stopping power of ~ 12 - 20 interaction lengths
- ECAL+HCAL components with segmentation comparable to lateral shower sizes



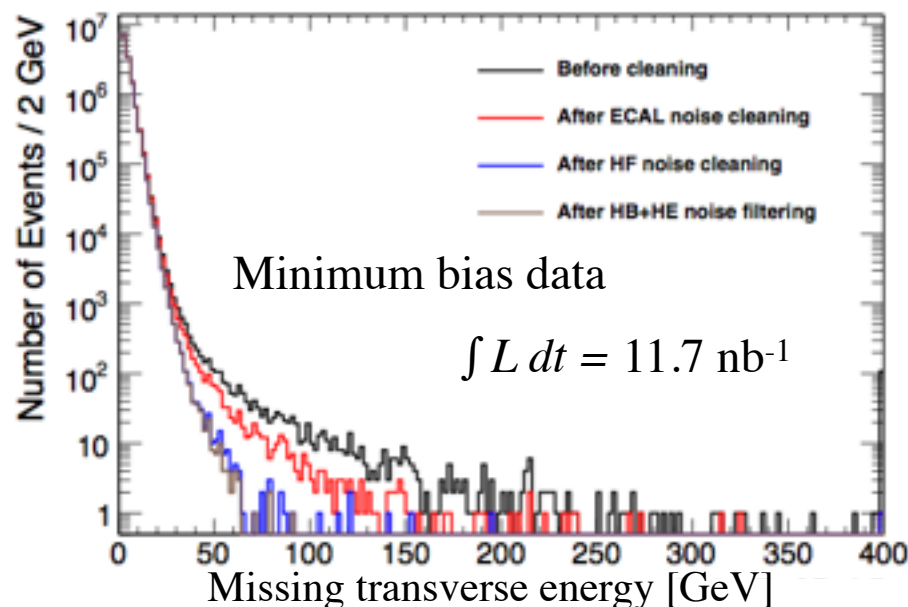
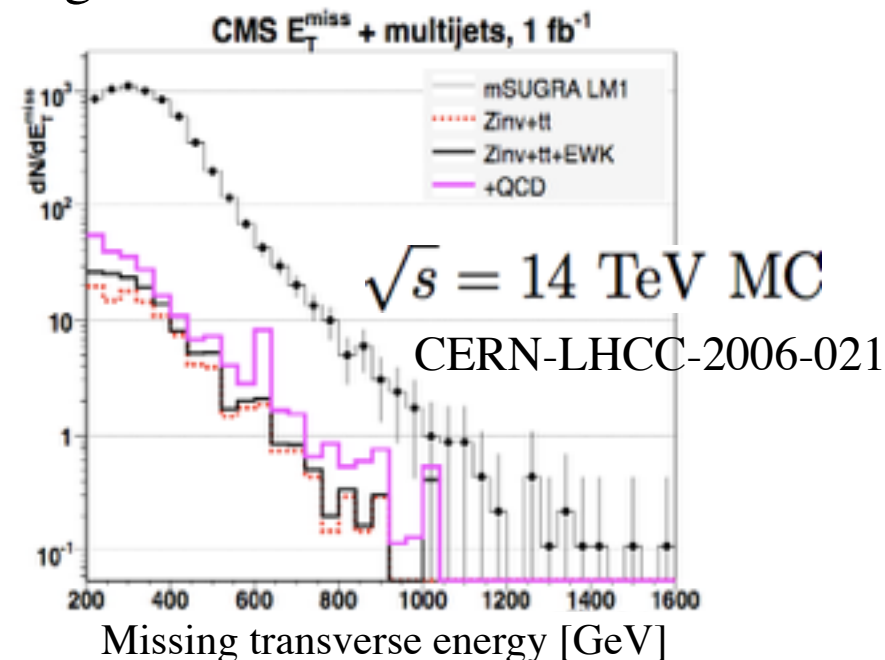
ATLAS Calorimeters

$$\vec{E}_T^{miss} = - \sum_i^{cells} \vec{E}_T$$



Missing transverse energy

Figures from SUSY10 conference talk:

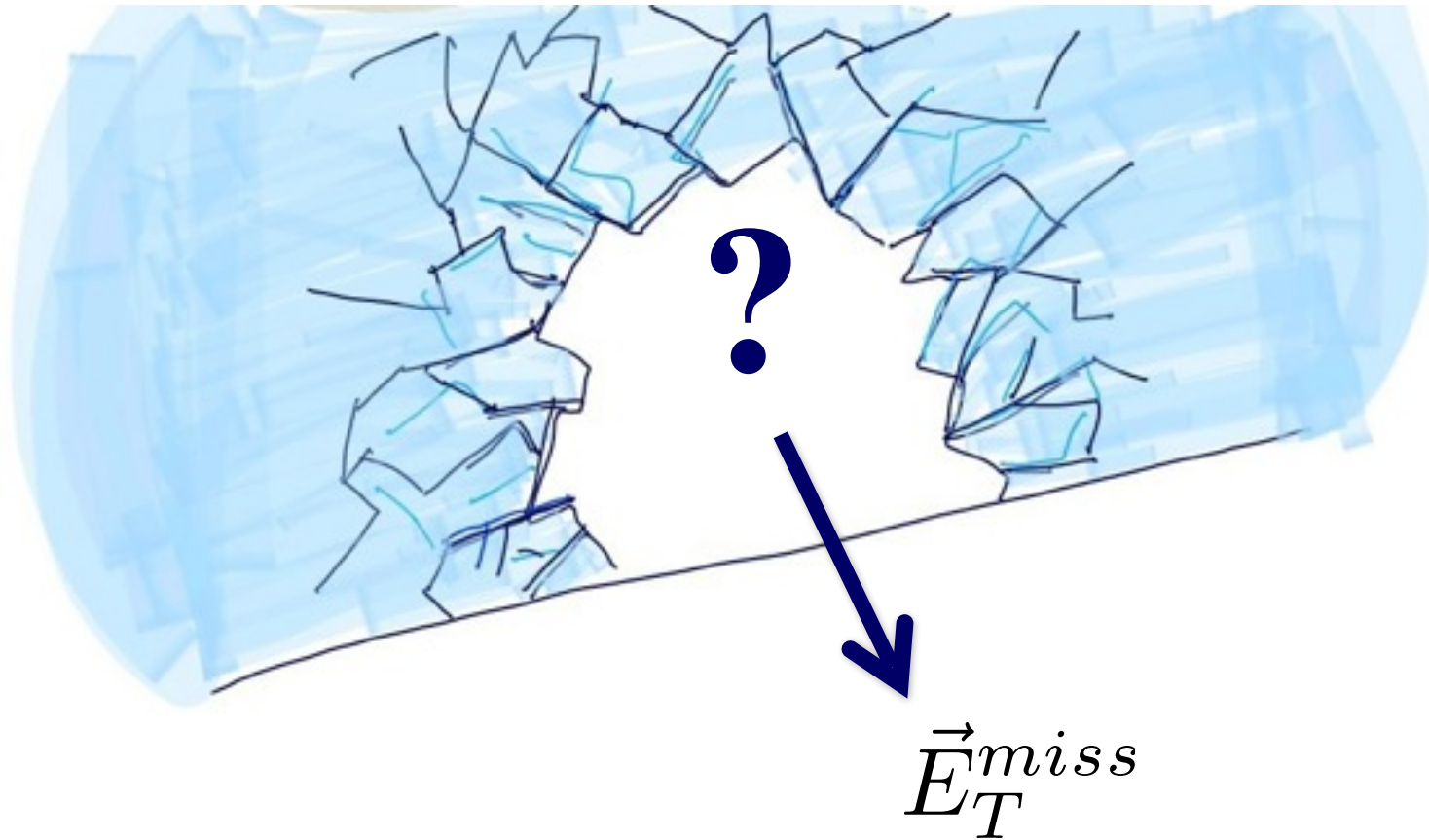


Missing transverse energy is a powerful observable for inferring the presence of weakly interacting particles

But, it only tells us about their transverse momenta – often we can better resolve quantities of interest by using additional information



Missing transverse energy



Missing transverse energy only tells us about the momentum of weakly interacting particles in an event...



Missing transverse energy



...not about the identity or mass of weakly interacting particles



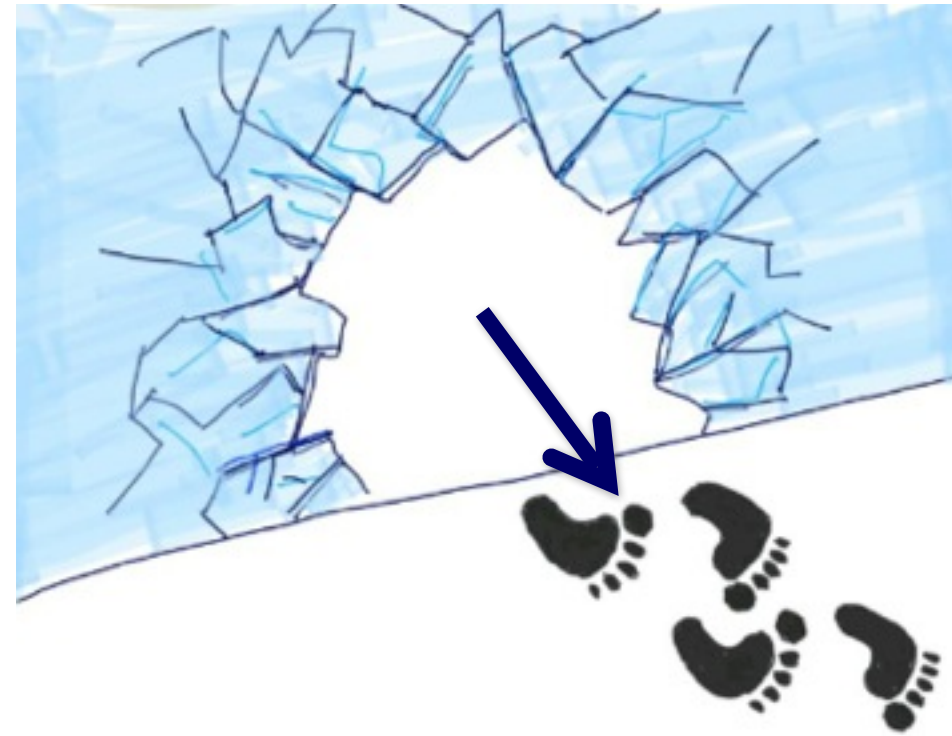
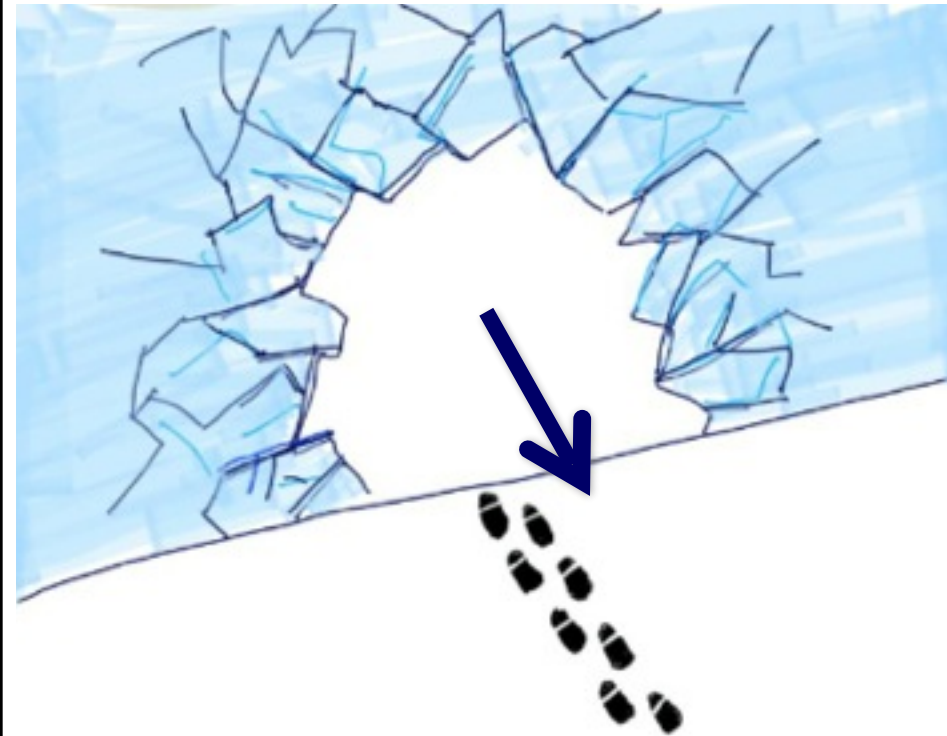
Missing transverse energy



...not about the identity or mass of weakly interacting particles



Missing transverse energy



We can learn more by using other information in an event to contextualize the missing transverse energy



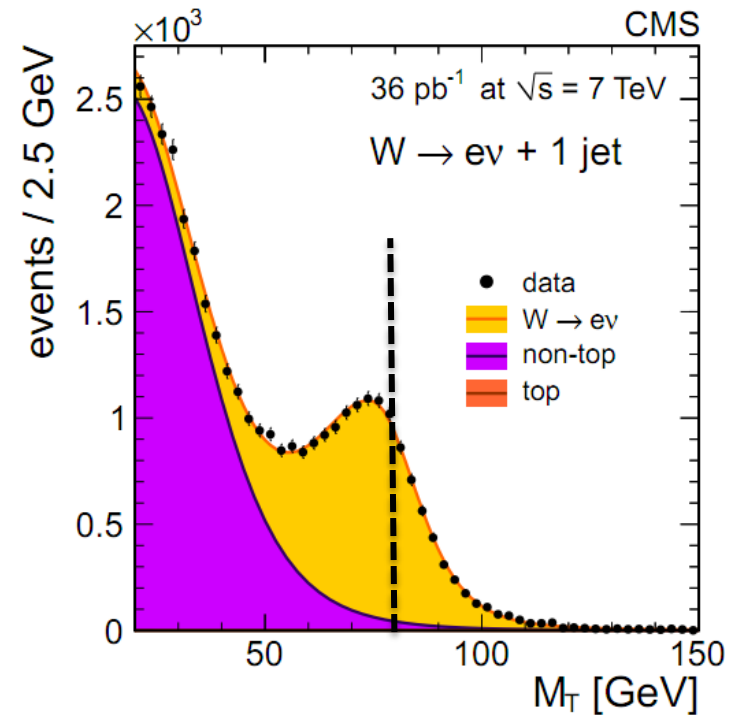
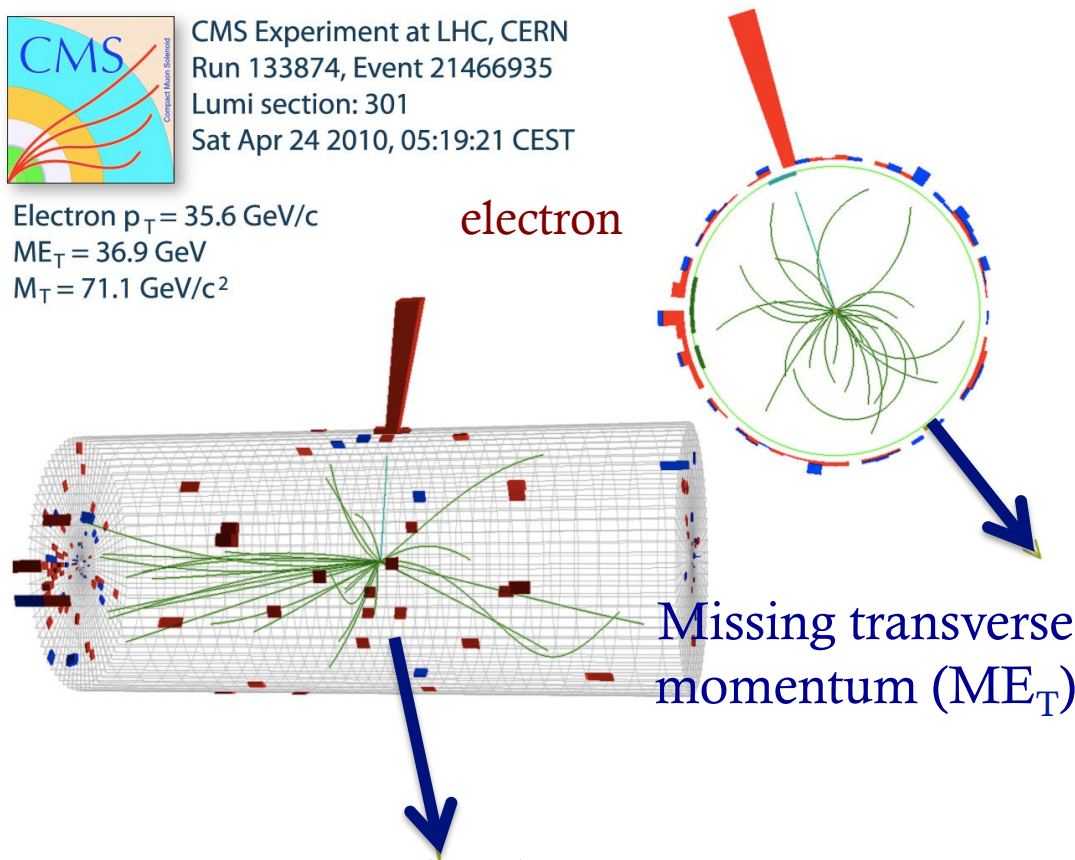
Resolving the invisible

$W(e\nu)$



CMS Experiment at LHC, CERN
Run 133874, Event 21466935
Lumi section: 301
Sat Apr 24 2010, 05:19:21 CEST

Electron $p_T = 35.6$ GeV/c
 $ME_T = 36.9$ GeV
 $M_T = 71.1$ GeV/ c^2



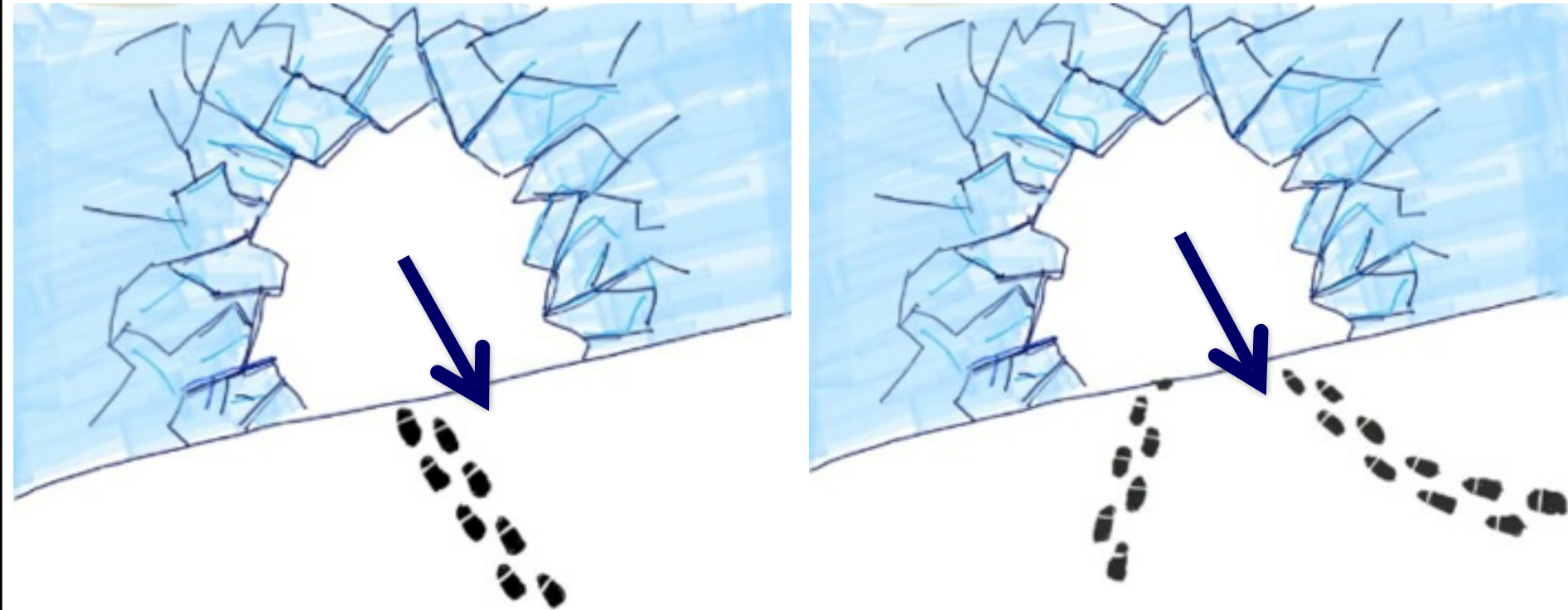
$$m_T = \sqrt{2p_T^e p_T^\nu (1 - \cos \phi)}$$

$m_T(\ell\nu)$ has kinematic *edge* at $m_W \sim 80$ GeV

Can use visible particles in events to contextualize missing transverse energy and better resolve mass scales



Missing transverse energy



We can learn more by using other information in an event to contextualize the missing transverse energy $\Rightarrow$ multiple weakly interacting particles?

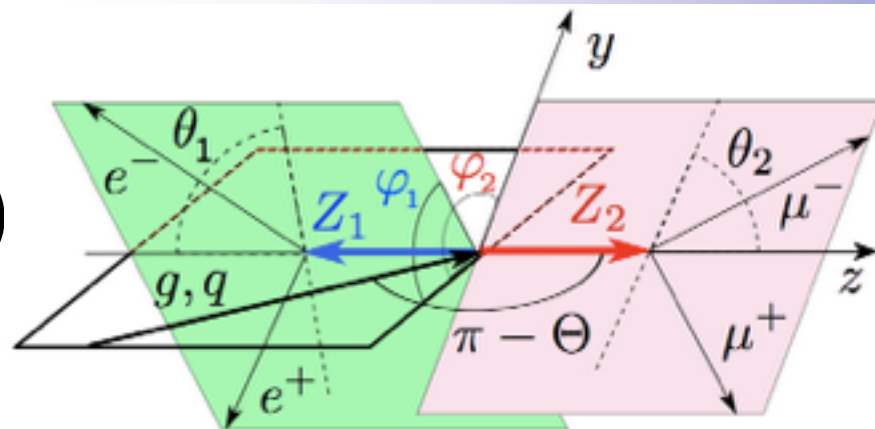


Open vs. closed final states

CLOSED $H \rightarrow Z(\ell\ell)Z(\ell\ell)$

Can calculate all masses,
momenta, angles

Can use masses for discovery, can use information
to measure spin, CP, etc.



OPEN $H \rightarrow W(\ell\nu)W(\ell\nu)$

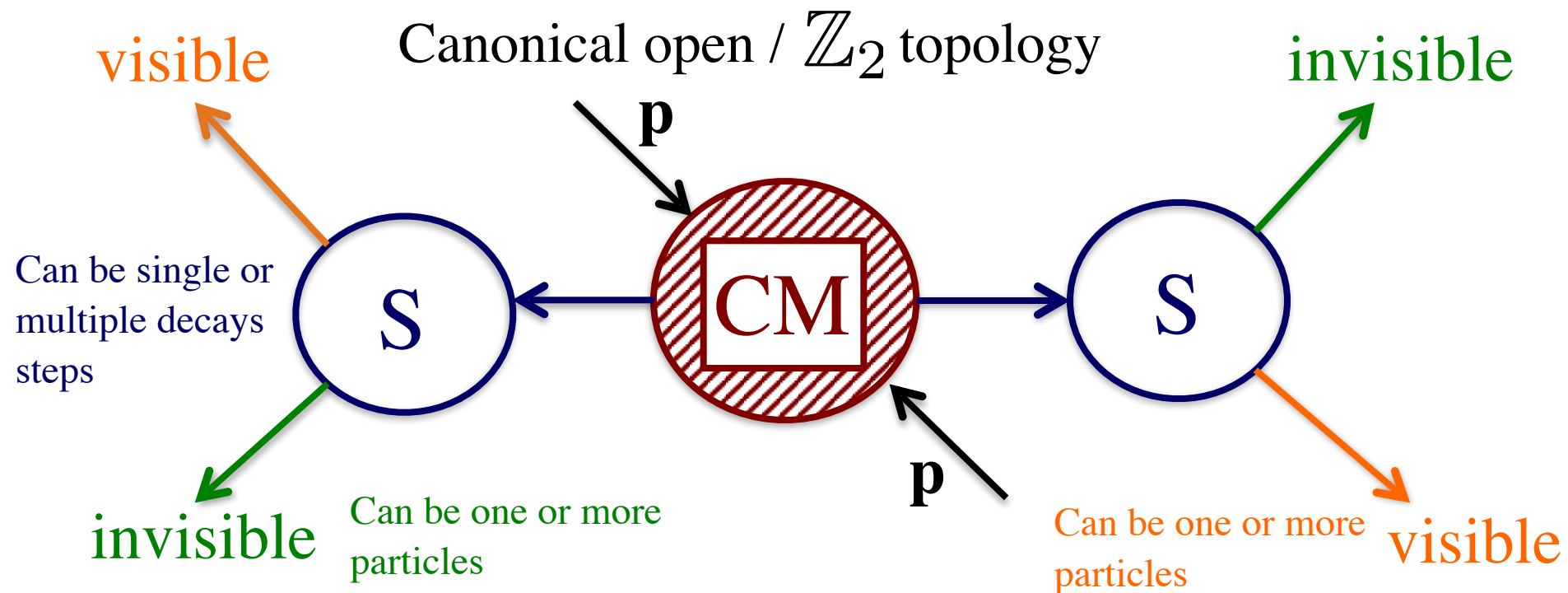
Under-constrained system with multiple weakly interacting
particles – can't calculate all the kinematic information

What useful information can we calculate?

What can we measure?



Multiple weakly interacting particles?



■ Dark Matter

■ Higgs quadratic divergences

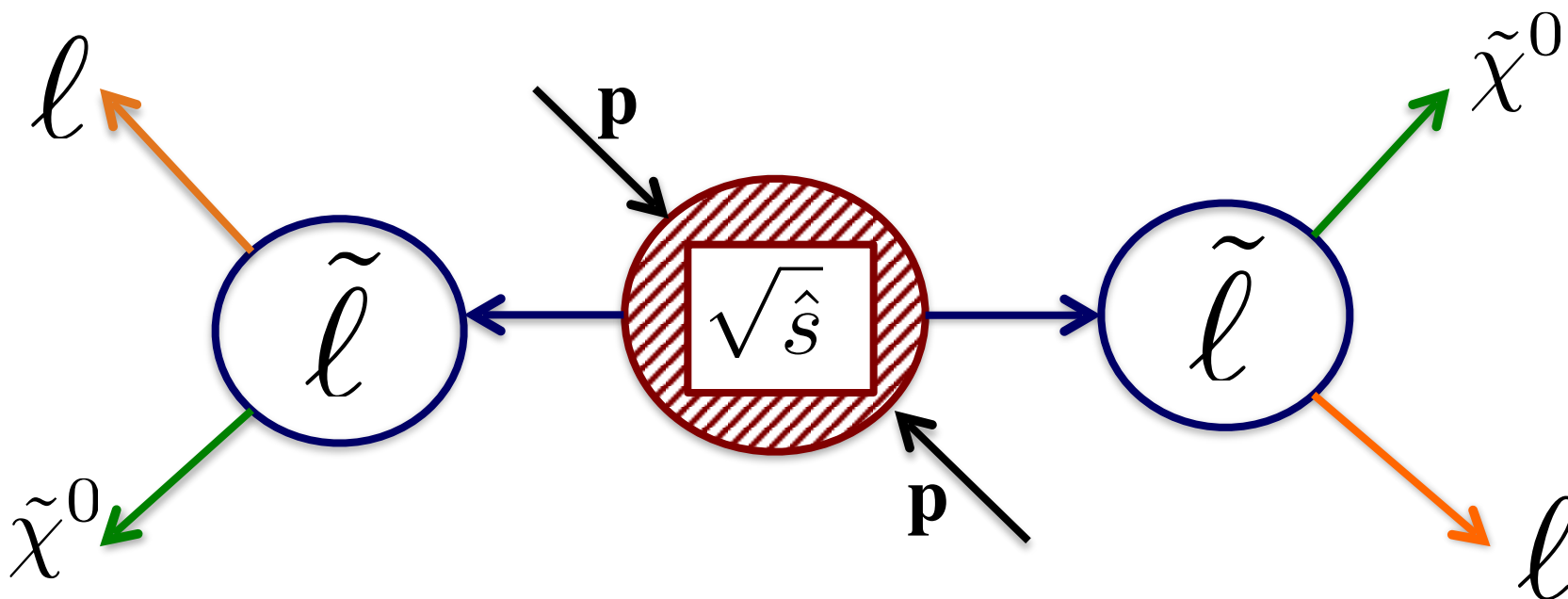
■

→ Theory $\mathbb{Z}_2$

SUSY	R-parity
Little Higgs	T-parity
UED	KK-parity



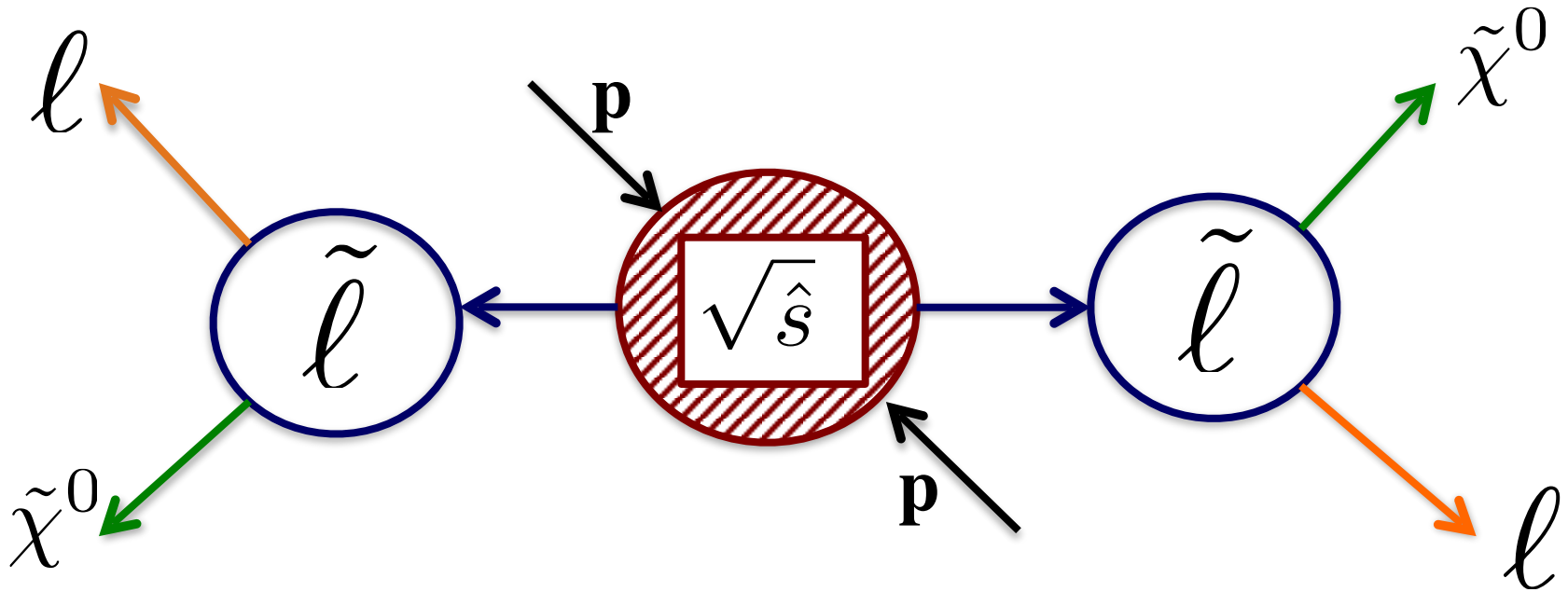
Example: slepton pair-production



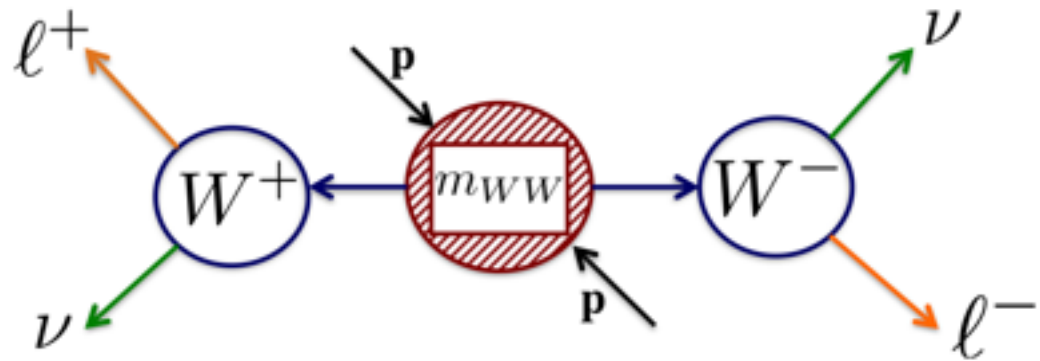
Experimental signature: di-leptons final states with
missing transverse momentum



Example: slepton pair-production

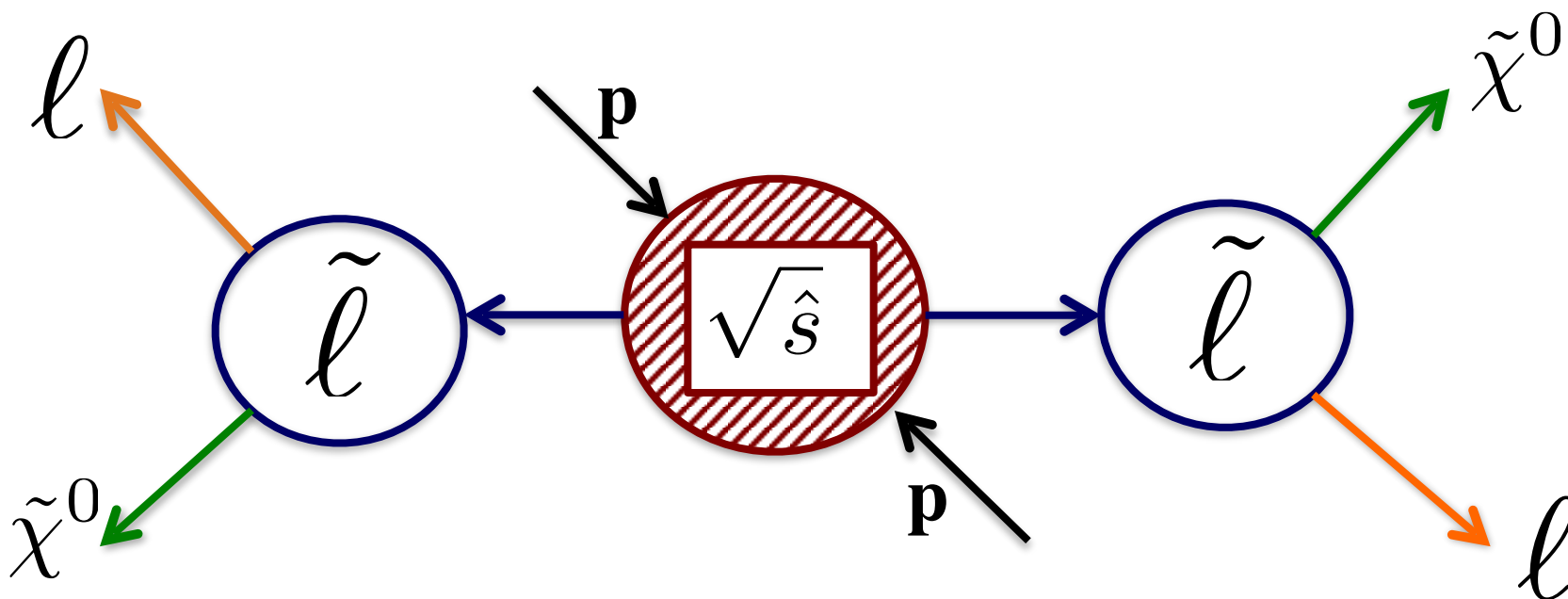


Main background:





Example: slepton pair-production

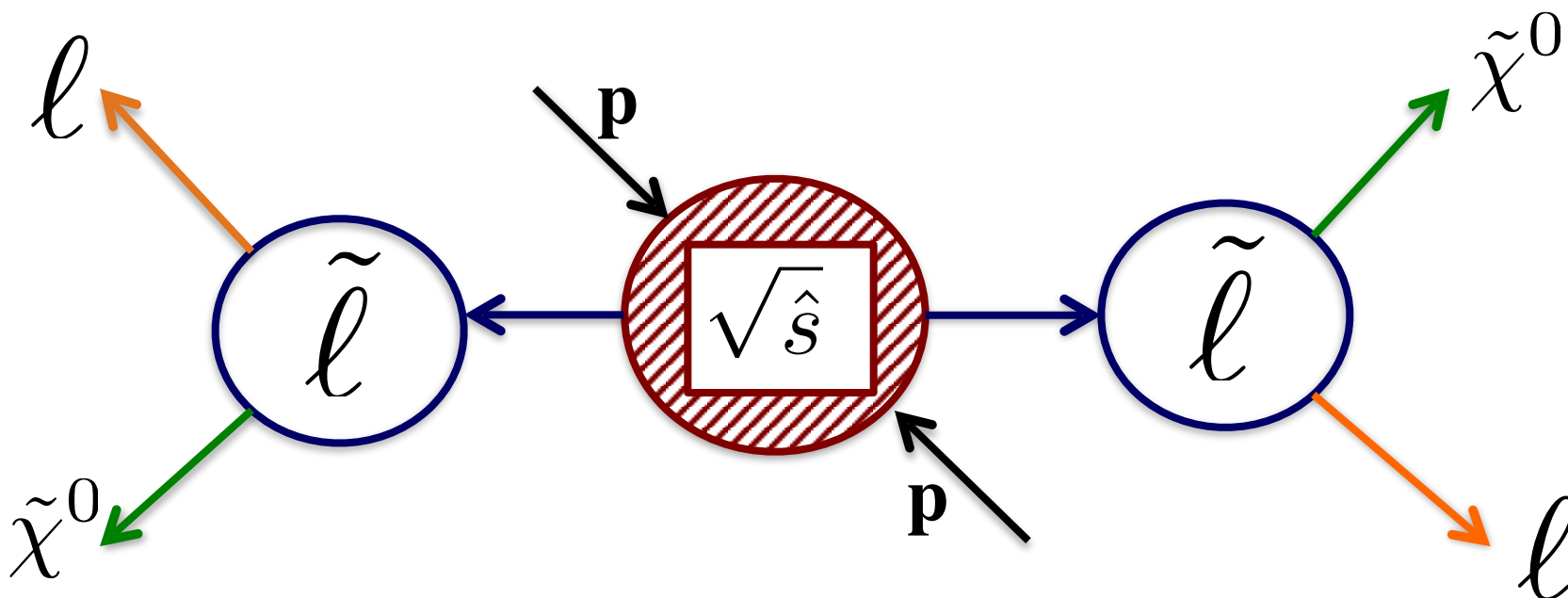


What quantities, if we could calculate them, could help us distinguish between signal and background events?

$$\sqrt{\hat{s}} = 2\gamma^{decay} m_{\tilde{\ell}} \quad M_{\Delta} \equiv \frac{m_{\tilde{\ell}}^2 - m_{\tilde{\chi}^0}^2}{m_{\tilde{\ell}}}$$



Example: slepton pair-production

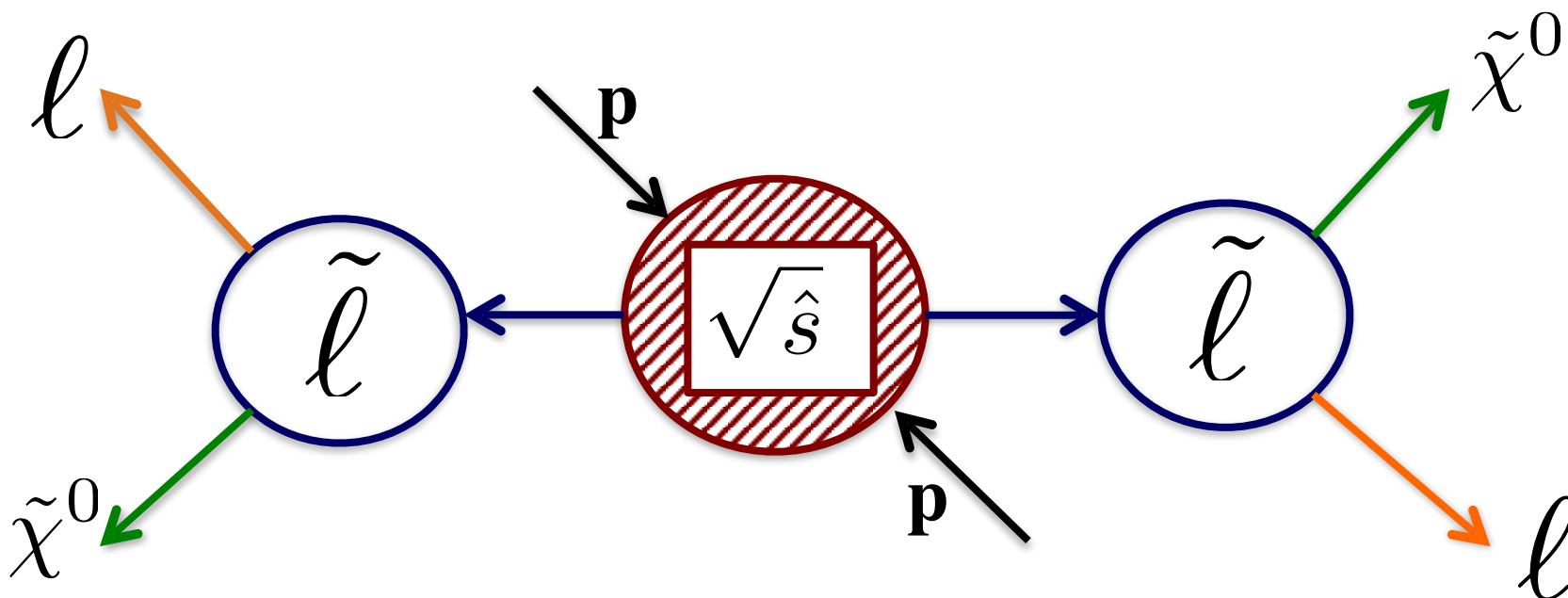


What information are we missing?

We don't observe the weakly interacting particles in the event. We can't measure their momentum or masses.



Example: slepton pair-production

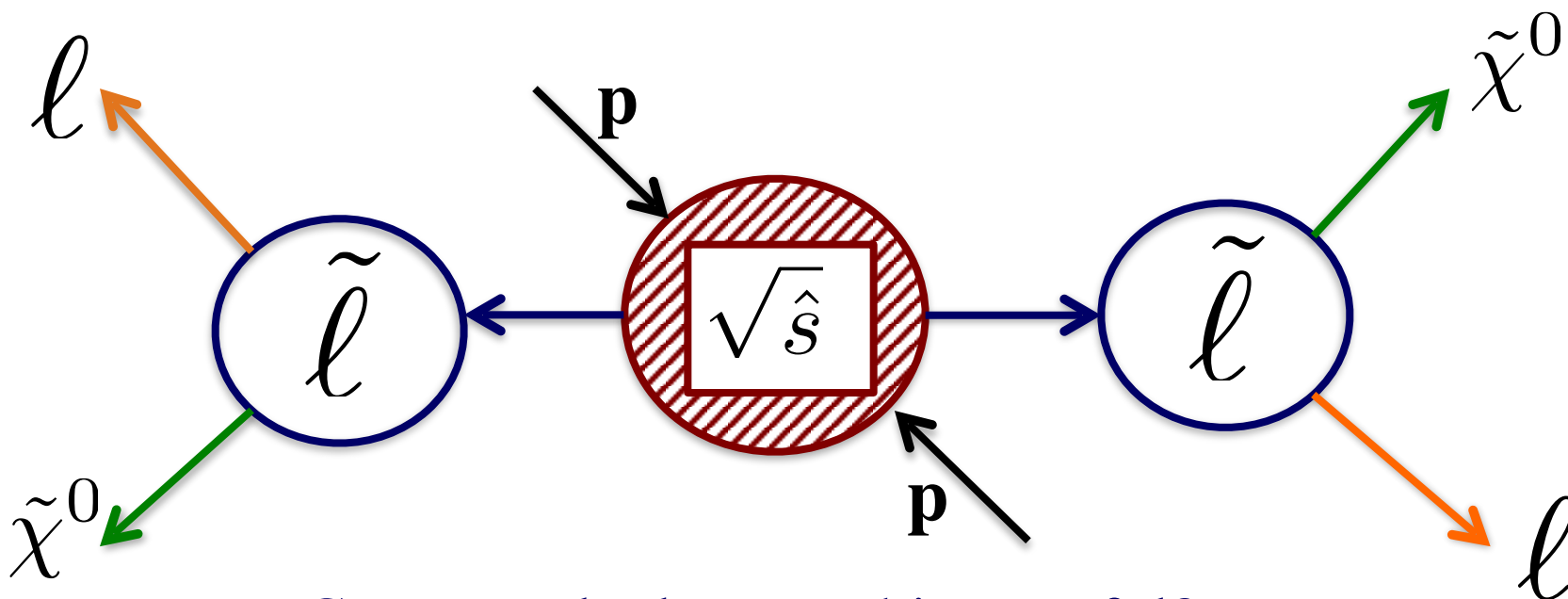


What do we know?

We can reconstruct the 4-vectors of the two leptons and the transverse momentum in the event



Example: slepton pair-production



Can we calculate anything useful?

With a number of simplifying assumptions...

$$\vec{E}_T^{miss} = \sum \vec{p}_T^{\tilde{\chi}^0} \quad m_{\tilde{\chi}^0} = 0$$

...we are still 4 d.o.f. short of reconstructing any masses of interest



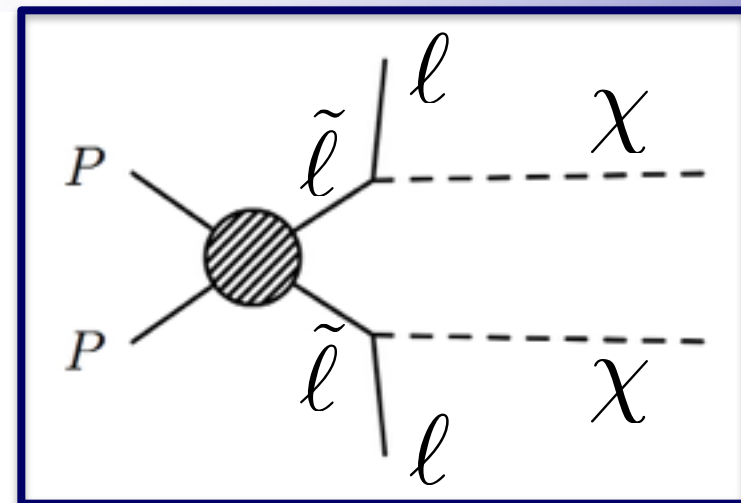
Singularity Variables

- State-of-the-art for LHC Run I was to use **singularity variables** as observables in searches
- Derive observables that bound a mass or mass-splitting of interest by
 - Assuming knowledge of event decay topology
 - Extremizing over under-constrained kinematic degrees of freedom associated with weakly interacting particles



Singularity Variable Example: M_{T2}

Generalization of transverse mass to two weakly interacting particle events



LSP 'test mass' Extremization of unknown degrees of freedom

$$M_{T2}^2(m_\chi) = \min_{\vec{p}_T^{\chi 1} + \vec{p}_T^{\chi 2} = \vec{E}_T^{miss}} \max \left[m_T^2(\vec{p}_T^{\ell 1}, \vec{p}_T^{\chi 1}, m_\chi), m_T^2(\vec{p}_T^{\ell 2}, \vec{p}_T^{\chi 2}, m_\chi) \right]$$

Subject to constraints

with: $m_T^2(\vec{p}_T^{\ell i}, \vec{p}_T^{\chi i}, m_\chi) = m_\chi^2 + 2 \left(E_T^{\ell i} E_T^{\chi i} - \vec{p}_T^{\ell i} \cdot \vec{p}_T^{\chi i} \right)$

Constructed to have a kinematic endpoint

(with the right test mass) at: $M_{T2}^{\max}(m_\chi) = m_{\tilde{\ell}}$ $M_{T2}^{\max}(0) = M_\Delta \equiv \frac{m_{\tilde{\ell}}^2 - m_{\tilde{\chi}}^2}{m_{\tilde{\ell}}}$

From:

C.G. Lester and D.J. Summers. Measuring masses of semiinvisibly decaying particles pair produced at hadron colliders. *Phys.Lett.*, B463:99–103, 1999.



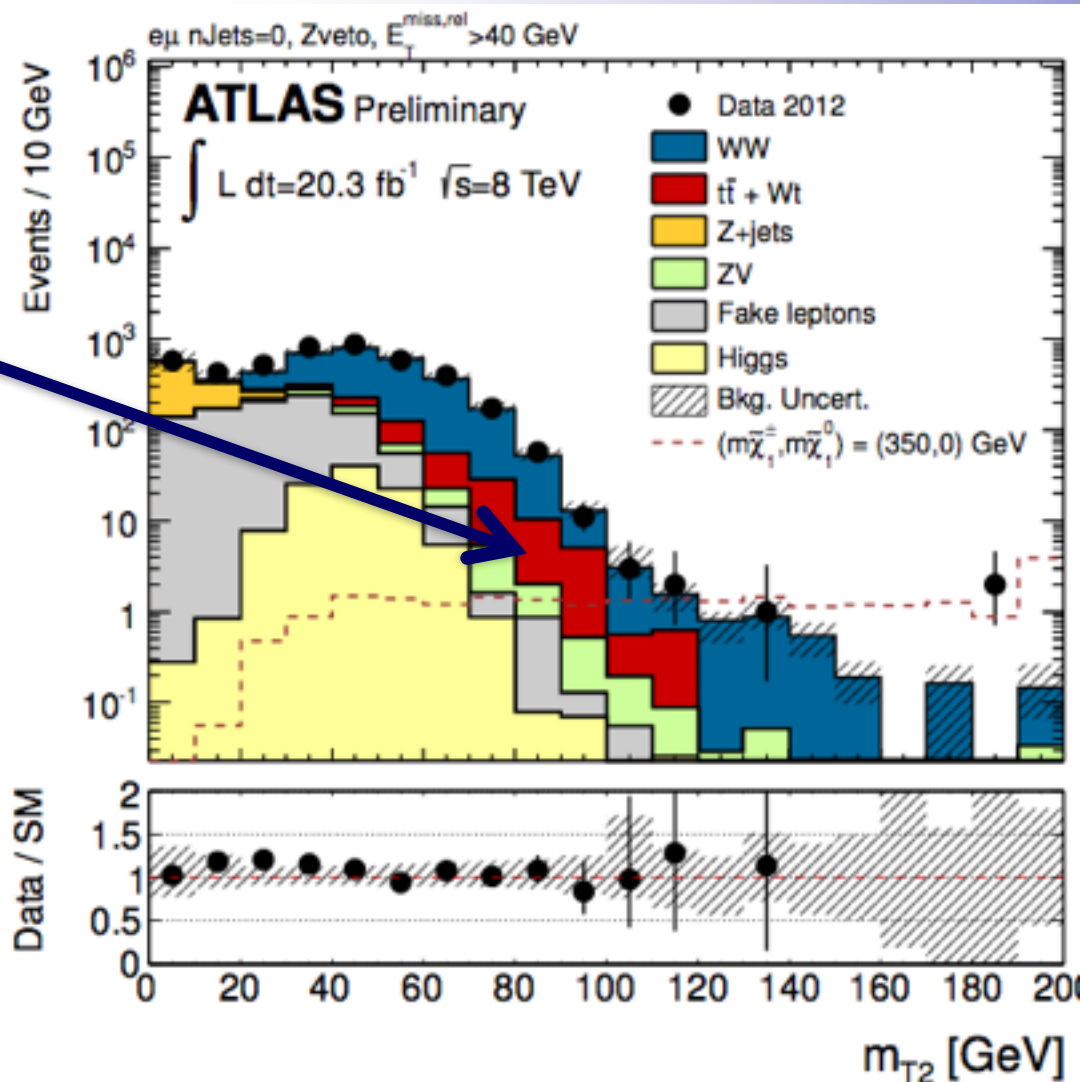
M_{T2} in practice

From:

ATLAS-CONF-2013-049

Backgrounds with leptonic W decays fall steeply once M_{T2} exceeds the W mass

Searches based on singularity variables have sensitivity to new physics signatures with mass splittings larger than the analogous SM ones





Recursive Jigsaw Reconstruction

New approach to reconstructing final states with weakly interacting particles:

- The strategy is to transform observable momenta iteratively *reference-frame to reference-frame*, traveling through each of the reference frames relevant to the topology
- At each step, *extremize only the relevant d.o.f. related to that transformation*
- Repeat procedure recursively according to particular rules defined for each topology (the topology relevant to each reference frame)
- Rather than obtaining one observable, get a complete basis of useful observables for each event



Recursive rest-frame reconstruction

For two lepton case, these are the ‘super-razor variables’:

M. Buckley, J. Lykken, CR, M. Spiropulu, PRD 89, 055020 (2014)

$\ell_1^{lab}, \ell_2^{lab}$ Begin with reconstructed lepton 4-vectors in lab frame



Recursive rest-frame reconstruction

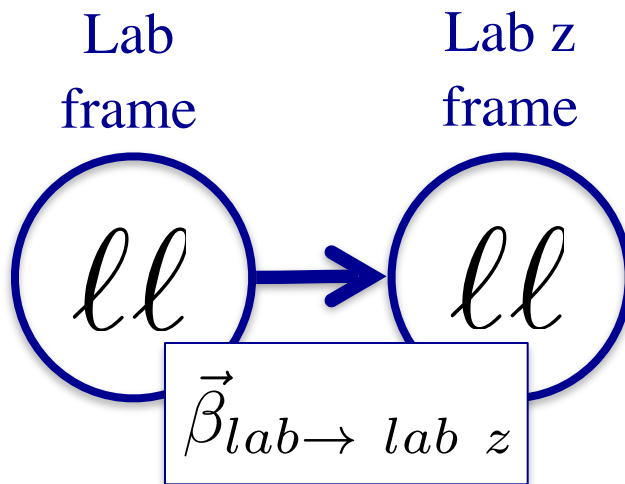
For two lepton case, these are the ‘super-razor variables’:

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$\ell_1^{lab}, \ell_2^{lab}$ Begin with reconstructed lepton 4-vectors in lab frame

$$\frac{\partial(E_{\ell_1}^{lab} z + E_{\ell_2}^{lab} z)}{\partial \beta_z} = 0 \rightarrow \beta_z$$

Remove dependence on unknown longitudinal boost by moving from ‘lab’ to ‘lab z’ frames





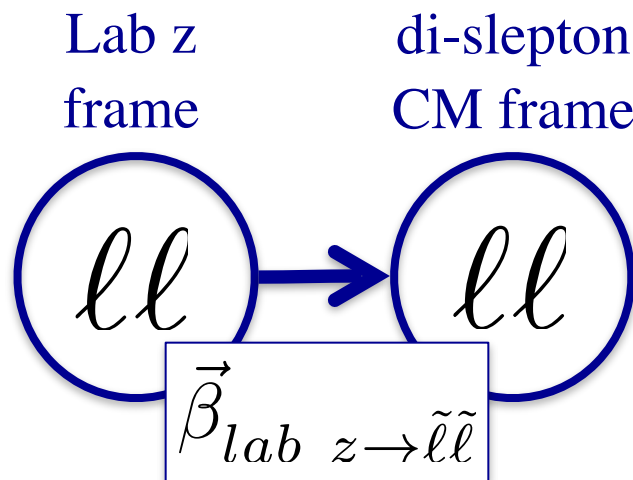
Recursive rest-frame reconstruction

For two lepton case, these are the ‘**super-razor variables**’:

M. Buckley, J. Lykken, CR, M. Spiropulu, PRD 89, 055020 (2014)

$$(\tilde{\chi}_1 + \tilde{\chi}_2)^2 = (\ell_1 + \ell_2)^2$$

Determine boost from ‘**lab z**’ to ‘**CM** ($\tilde{\ell}\tilde{\ell}$)’ frame by specifying Lorentz-invariant choice for invisible system mass

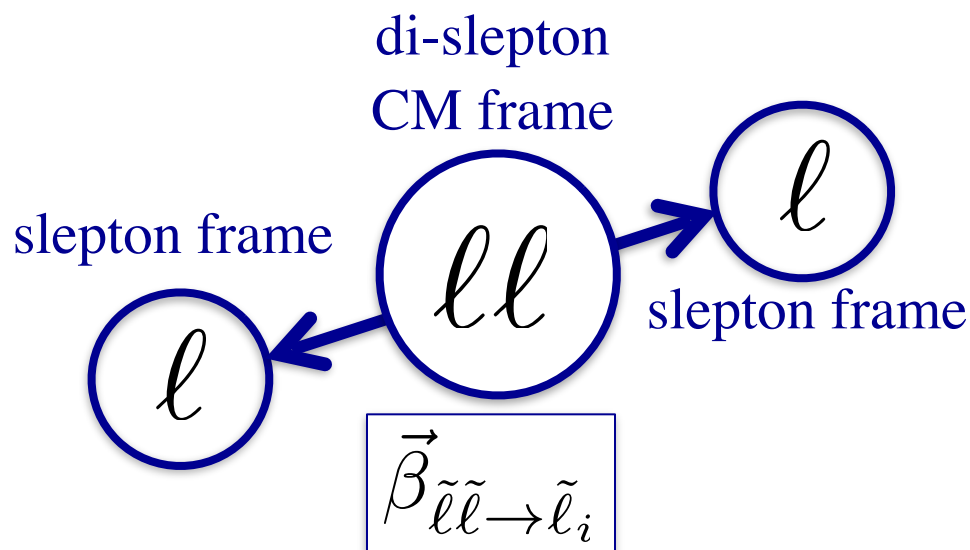




Recursive rest-frame reconstruction

For two lepton case, these are the ‘**super-razor variables**’:

M. Buckley, J. Lykken, CR, M. Spiropulu, PRD 89, 055020 (2014)



$$\frac{\partial(E_{\ell_1}^{\tilde{l}} + E_{\ell_2}^{\tilde{l}})}{\partial \vec{\beta}_{\tilde{l}\tilde{l} \rightarrow \tilde{l}_i}} = 0 \rightarrow \vec{\beta}_{\tilde{l}\tilde{l} \rightarrow \tilde{l}_i}$$

Determine asymmetric boost from
CM to slepton rest frames by minimizing
lepton energies in those frames



Recursive rest-frame reconstruction

For two lepton case, these are the ‘super-razor variables’:

M. Buckley, J. Lykken, CR, M. Spiropulu, PRD 89, 055020 (2014)

$\ell_1^{lab}, \ell_2^{lab}$ Begin with reconstructed lepton 4-vectors in lab frame

$$\frac{\partial(E_{\ell_1}^{lab} z + E_{\ell_2}^{lab} z)}{\partial \beta_z} = 0 \rightarrow \beta_z$$

Remove dependence on unknown longitudinal boost by moving from ‘lab’ to ‘lab z’ frames

$$(\tilde{\chi}_1 + \tilde{\chi}_2)^2 = (\ell_1 + \ell_2)^2$$

Determine boost from ‘lab z’ to ‘CM ($\tilde{\ell}\tilde{\ell}$)’ frame by specifying Lorentz-invariant choice for invisible system mass

$$\frac{\partial(E_{\ell_1}^{\tilde{\ell}} + E_{\ell_2}^{\tilde{\ell}})}{\partial \vec{\beta}_{\tilde{\ell}\tilde{\ell} \rightarrow \tilde{\ell}_i}} = 0 \rightarrow \vec{\beta}_{\tilde{\ell}\tilde{\ell} \rightarrow \tilde{\ell}_i}$$

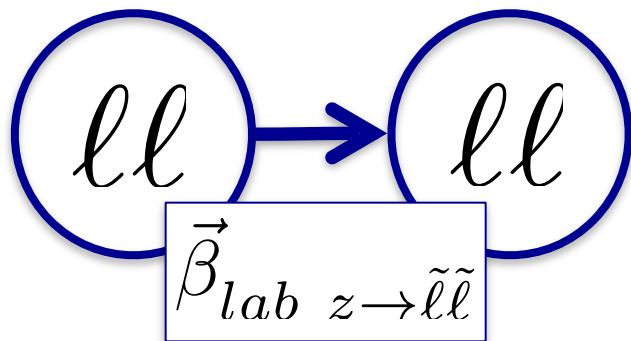
Determine asymmetric boost from CM to slepton rest frames by minimizing lepton energies in those frames



Recursive rest-frame reconstruction

Lab
frame

di-slepton
CM frame

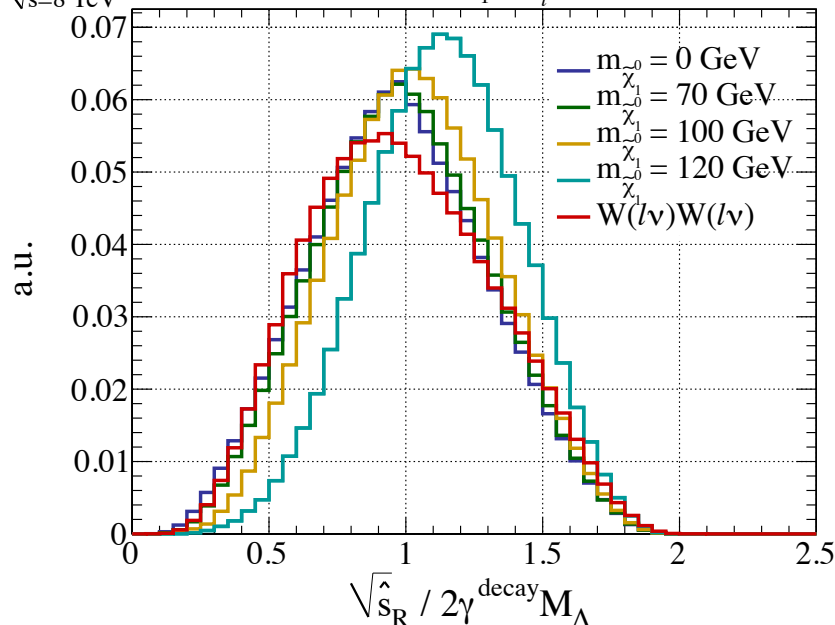


1st transformation: extract variable
sensitive to invariant mass of total
event: $\sqrt{\hat{s}_R}$

Resulting variable is invariant
under p_T of di-slepton system

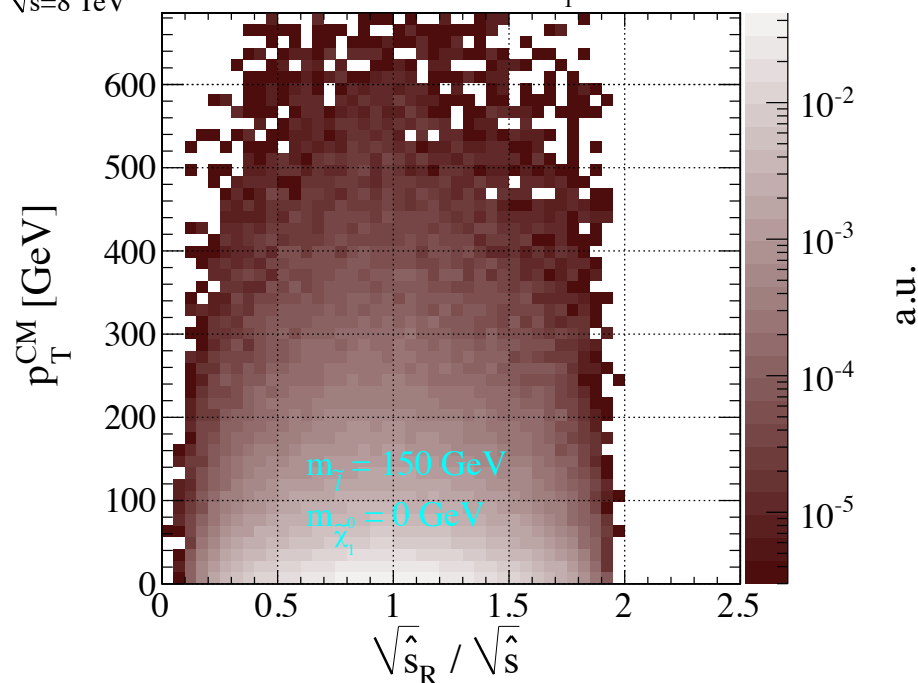
MadGraph+PGS
 $\sqrt{s}=8$ TeV

$pp \rightarrow \tilde{t}\tilde{t}; \tilde{t} \rightarrow t\tilde{\chi}_1^0; m_{\tilde{t}} = 150$ GeV



MadGraph+PGS
 $\sqrt{s}=8$ TeV

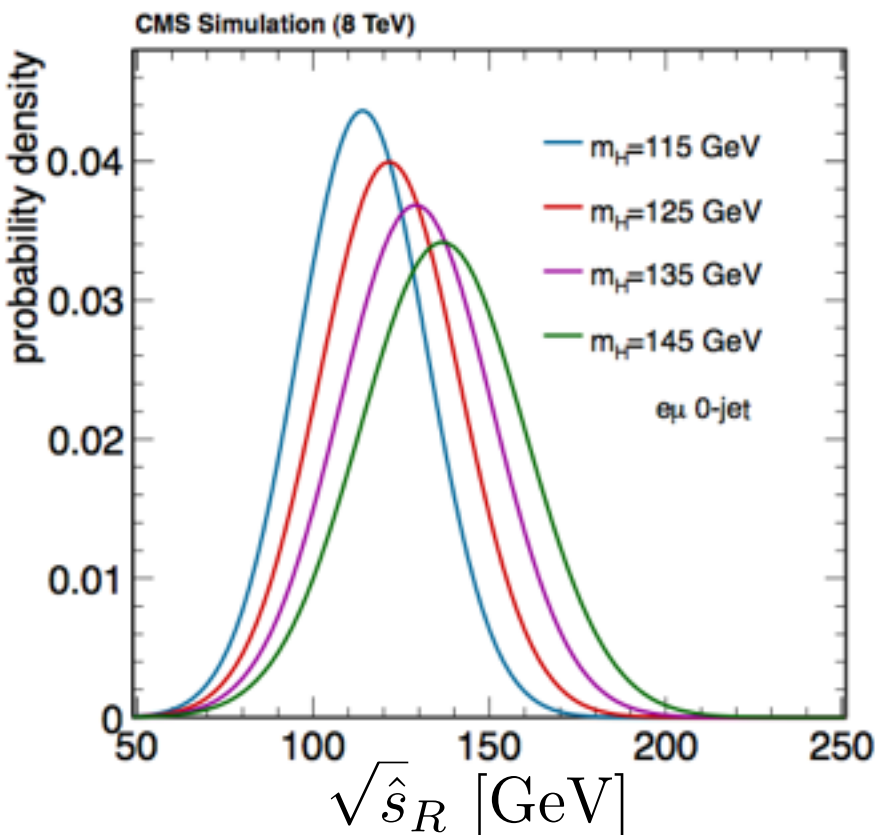
$pp \rightarrow \tilde{t}\tilde{t}; \tilde{t} \rightarrow t\tilde{\chi}_1^0$





Resonant Higgs production

$$H \rightarrow WW^* \rightarrow 2\ell 2\nu$$



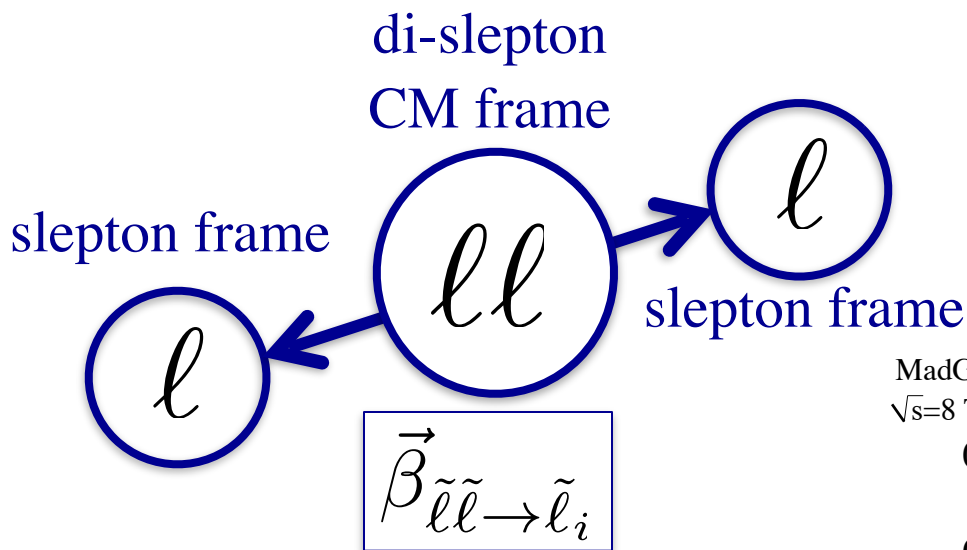
Using information from the two leptons, and the missing transverse momentum, the observable $\sqrt{\hat{s}}_R$ is directly sensitive to the Higgs mass

From:

CMS Collaboration, *Measurement of Higgs boson production and properties in the WW decay channel with leptonic final states*, arXiv:1312.1129v1 [hep-ex]



Recursive rest-frame reconstruction



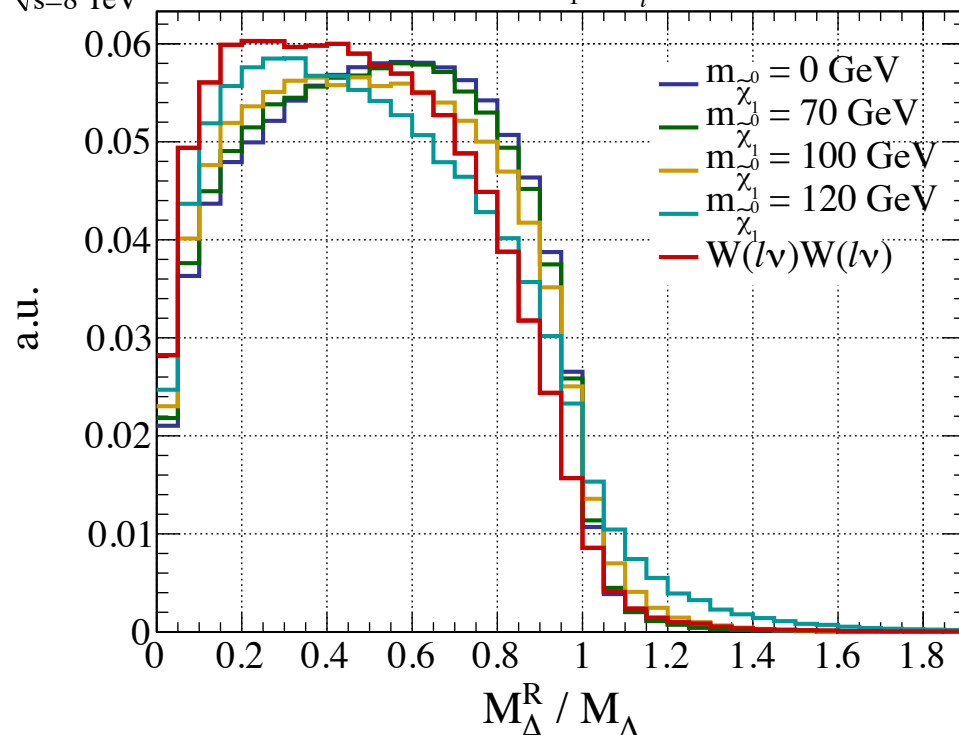
Resulting variable has kinematic endpoint at:

$$M_{\Delta} \equiv \frac{m_{\tilde{l}}^2 - m_{\tilde{\chi}^0}^2}{m_{\tilde{l}}}$$

2nd transformation(s): extract variable sensitive to invariant mass of squark: M_{Δ}^R

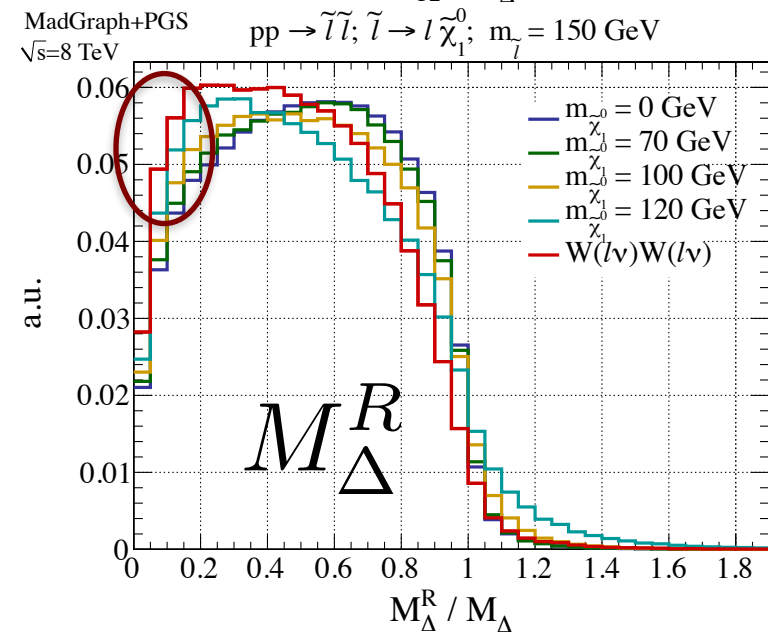
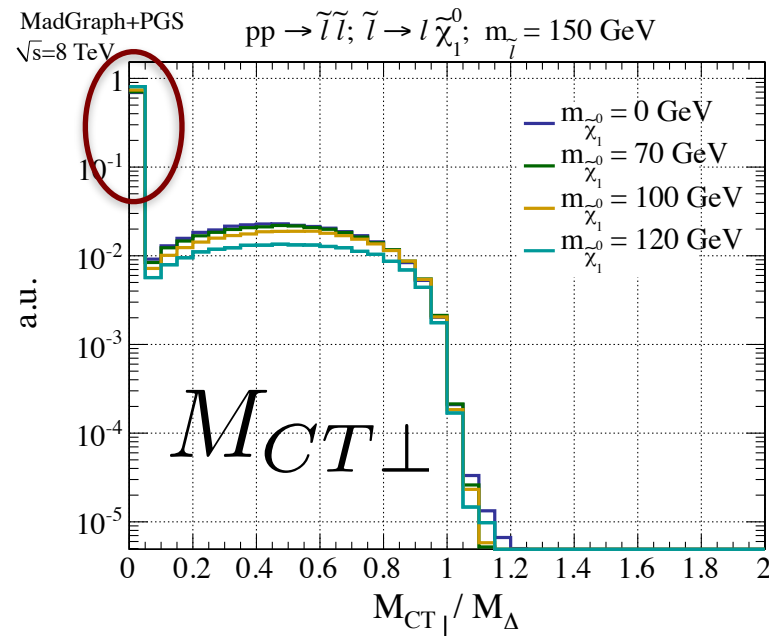
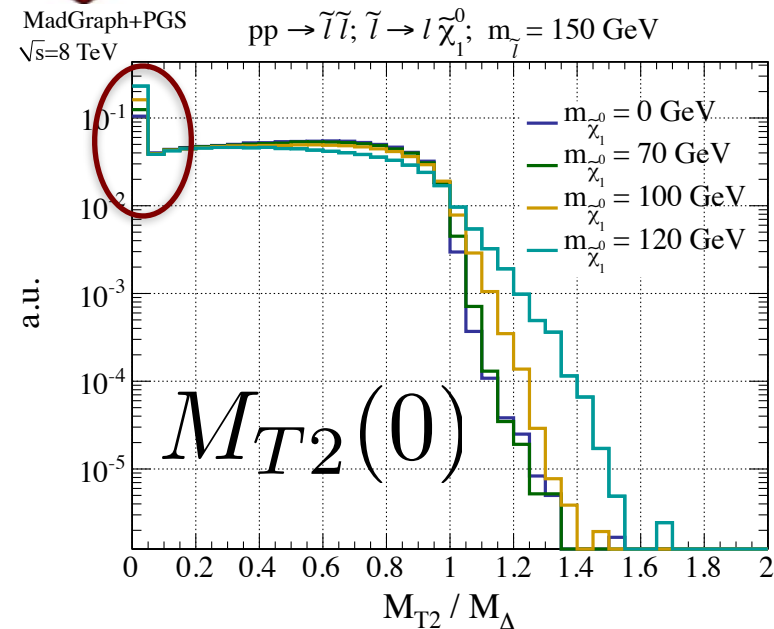
MadGraph+PGS
 $\sqrt{s}=8$ TeV

$pp \rightarrow \tilde{l}\tilde{l}; \tilde{l} \rightarrow l\tilde{\chi}_1^0; m_{\tilde{l}} = 150$ GeV





Variable comparison



Three different singularity variables, all attempting to measure the same thing

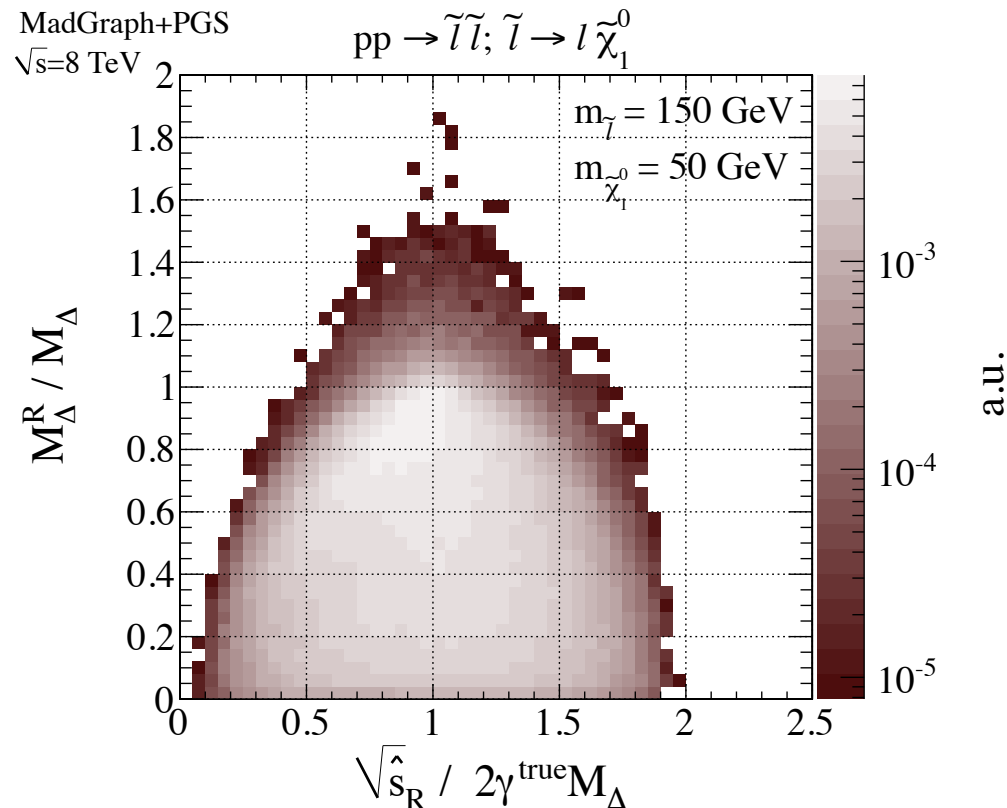
$$M_{\Delta}^R \geq M_{T2}(0) \geq M_{CT\perp}$$

More details about variable comparisons in PRD 89, 055020 (arXiv:1310.4827) and backup slides 32



But what else can we calculate?

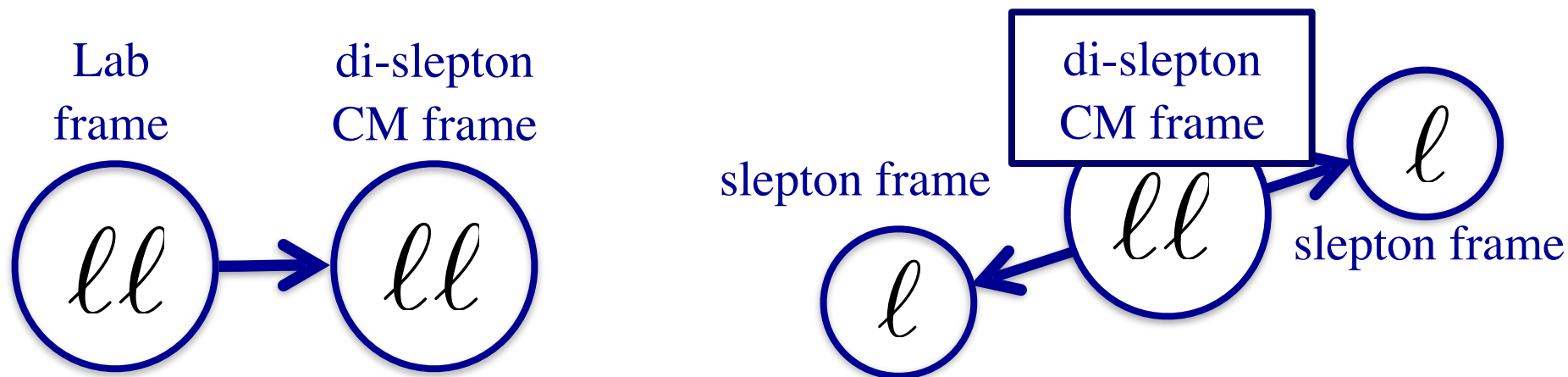
With recursive scheme can extract the two mass scales $\sqrt{\hat{s}_R}$ and M_Δ^R almost completely independently





Angles, angles, angles...

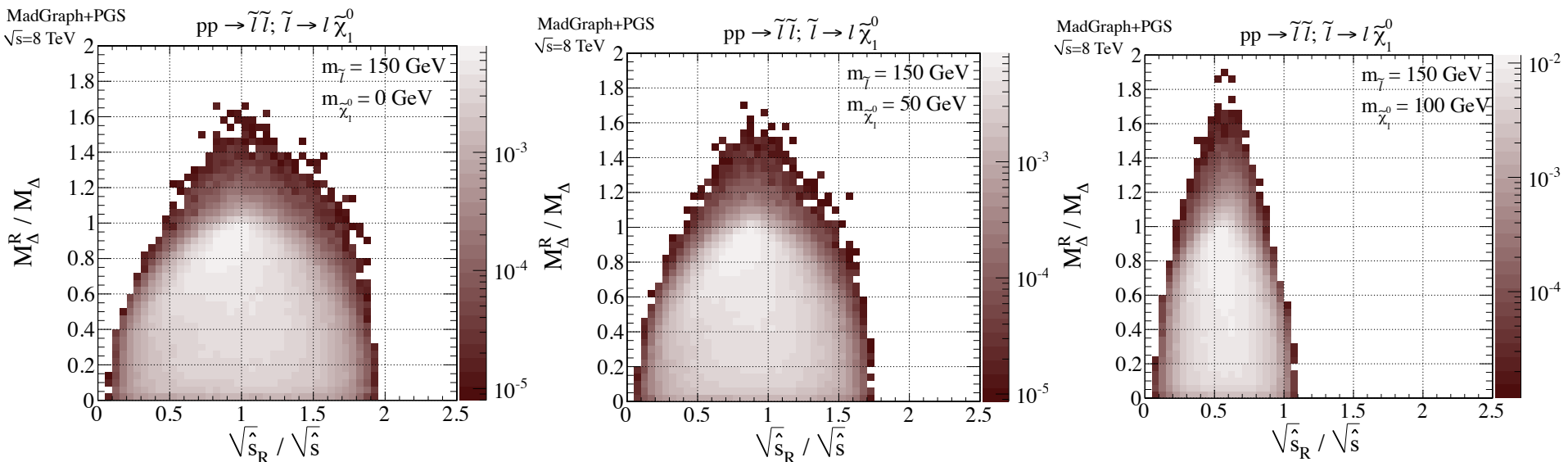
Recursive scheme fully specifies approximate event decay chain, also yielding angular observables



Two transformations mean at least two independent angles of interest (essentially the decay angle of the state whose rest-frame you are in)



Towards a kinematic basis



but $\sqrt{\hat{s}_R} \sim 2\gamma^{decay} M_\Delta$

while $\sqrt{\hat{s}} = 2\gamma^{decay} m_{\tilde{l}}$

Underestimating the real mass means
over-estimating the boost magnitude:

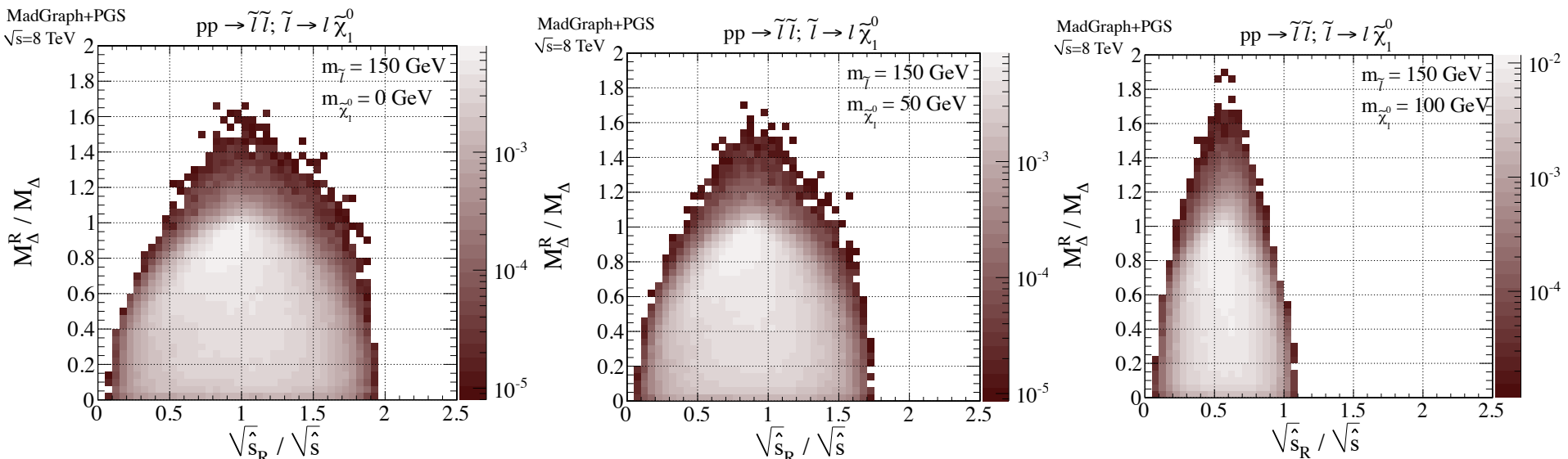
$$\vec{p}_T^{CM} = \vec{p}_T^{\ell_1} + \vec{p}_T^{\ell_2} + \vec{E}_T^{\text{miss}}$$

$$\vec{\beta}_{lab \rightarrow CM} = \frac{\vec{p}_T^{CM}}{\sqrt{|\vec{p}_T^{CM}|^2 + \hat{s}}}$$

From PRD 89, 055020 (2014)



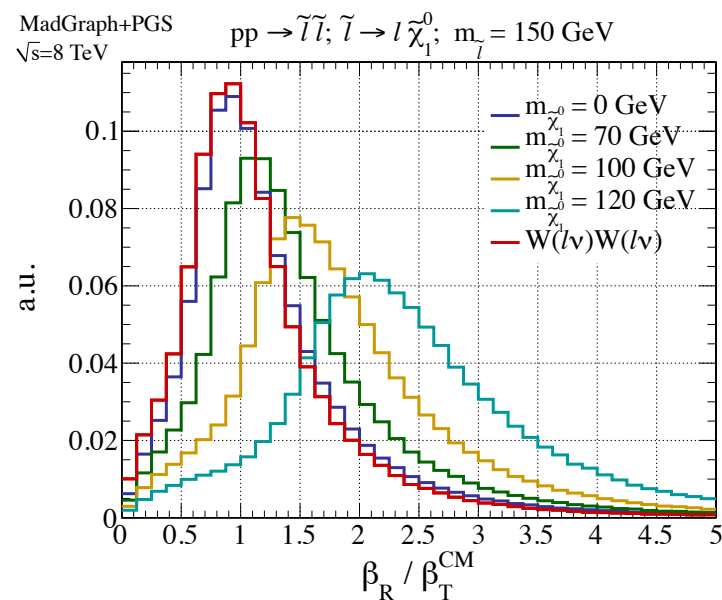
Towards a kinematic basis



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while $\sqrt{\hat{s}} = 2\gamma^{decay} m_{\tilde{\ell}}$

Underestimating the real mass means
over-estimating the boost magnitude:



From PRD 89, 055020 (2014)



Angular Variables

Incorrect boost magnitude
induces correlation

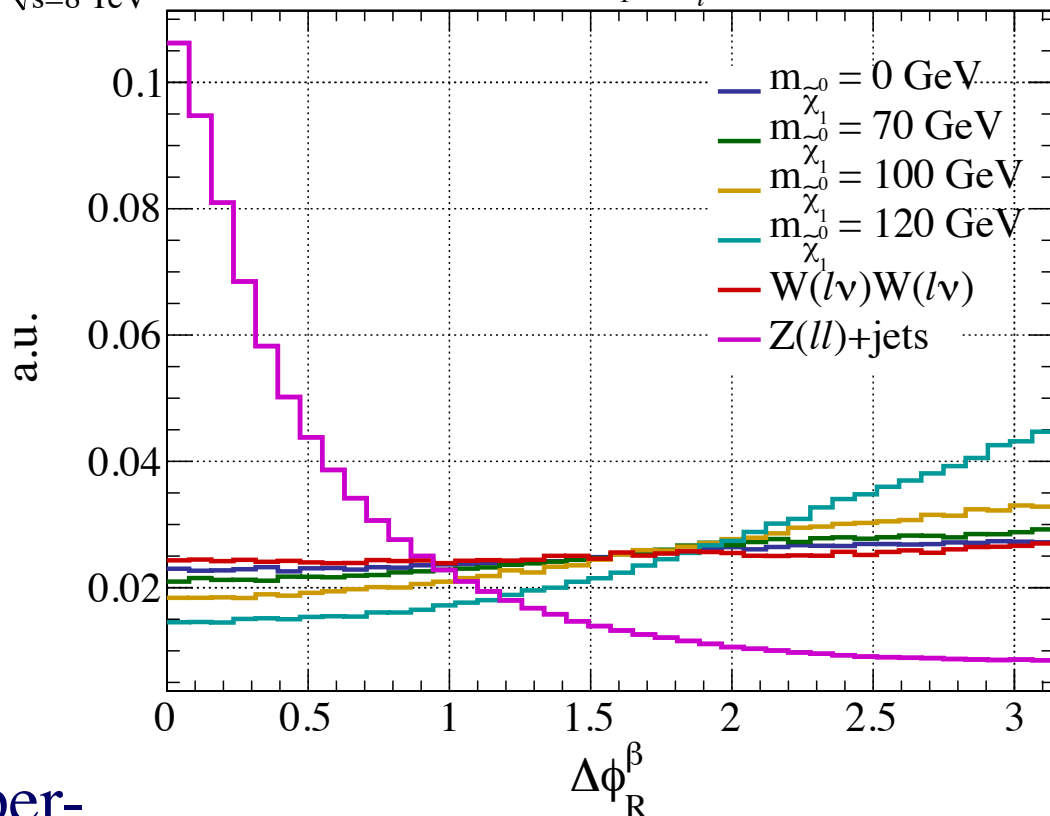
Angle between
lab $\rightarrow$ CM frame boost and
di-leptons in CM frame is
sensitive to

$$\frac{m_\chi}{m_{\tilde{\ell}}} \text{ rather than } M_\Delta$$

$\sim$ Uncorrelated with other super-
razor variables

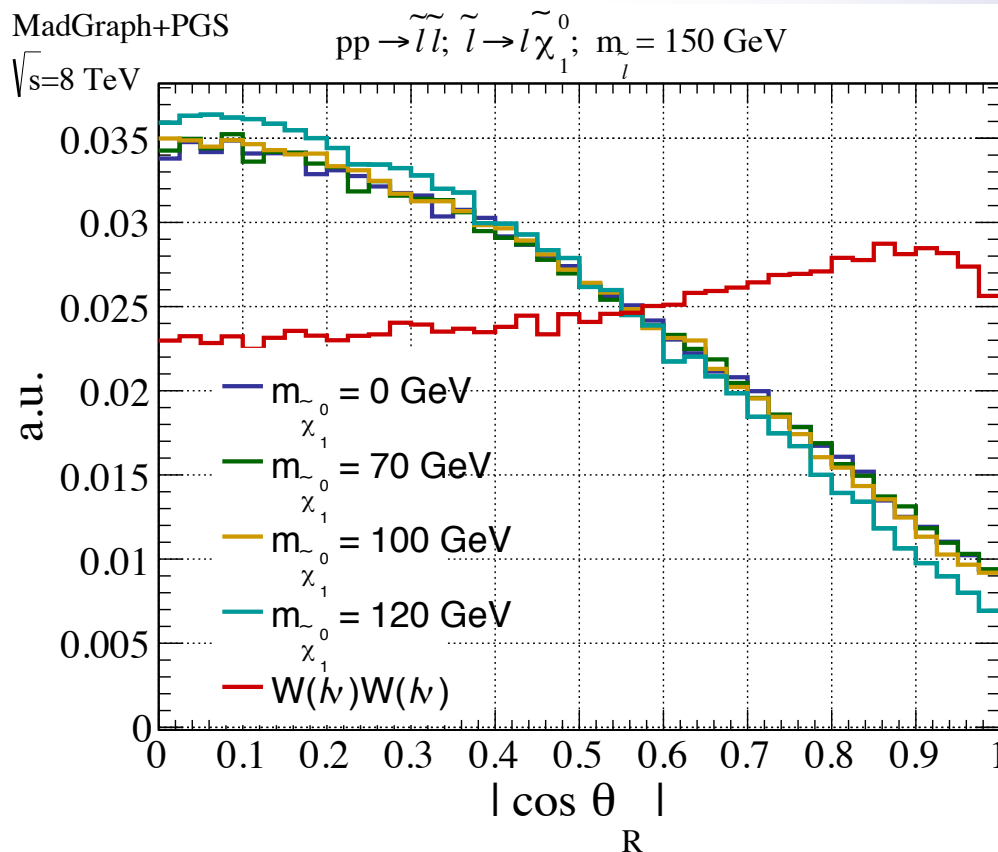
MadGraph+PGS
 $\sqrt{s}=8$ TeV

$pp \rightarrow \tilde{l}\tilde{l}; \tilde{l} \rightarrow l\tilde{\chi}_1^0; m_{\tilde{l}} = 150$ GeV





Angular Variables



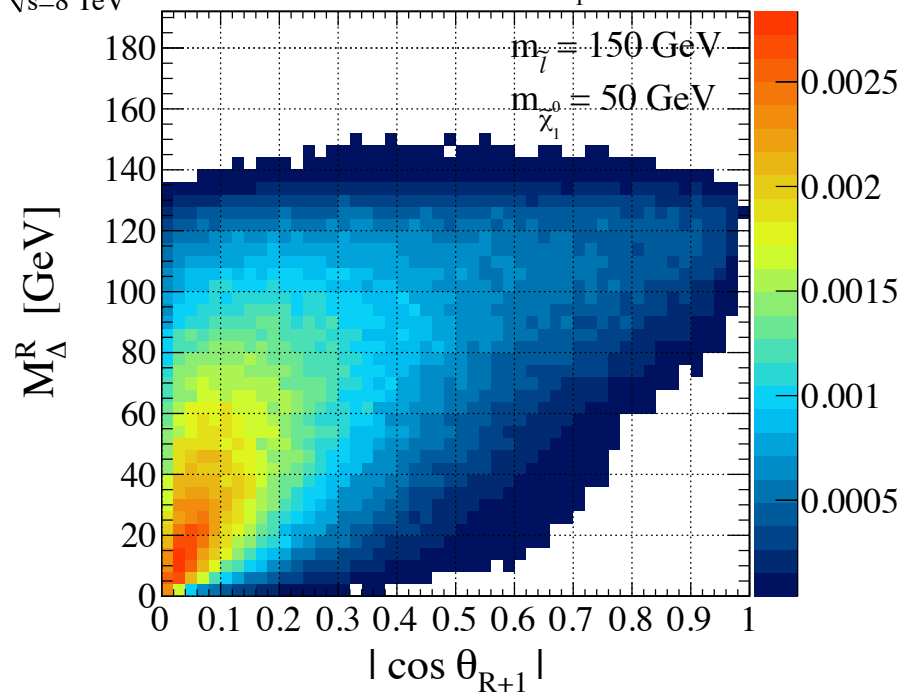
In the approximate di-slepton rest frame,
 reconstructed decay angle sensitive to particle
 spin and production



Angular Variables

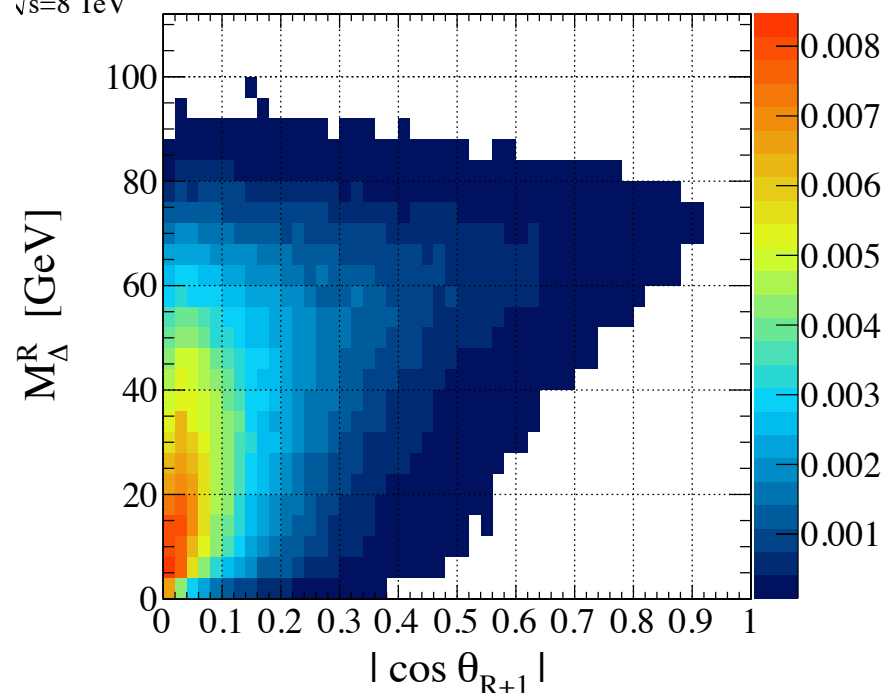
MadGraph+PGS
 $\sqrt{s}=8$ TeV

$pp \rightarrow \tilde{l}\tilde{l}; \tilde{l} \rightarrow l\tilde{\chi}_1^0$



MadGraph+PGS
 $\sqrt{s}=8$ TeV

$pp \rightarrow W^+W^-; W^\pm \rightarrow l^\pm \nu$



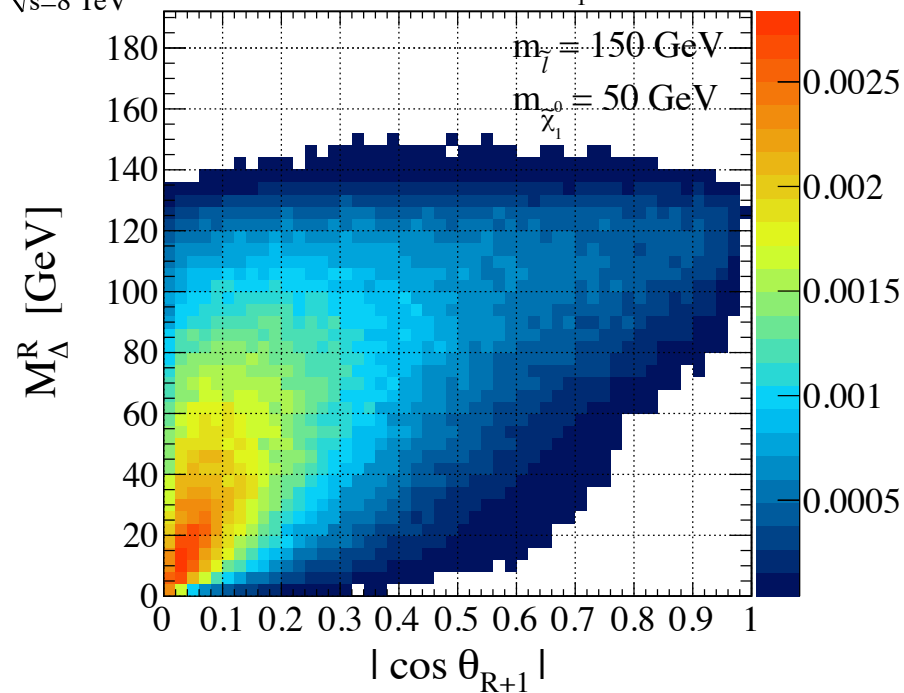
In the approximate slepton rest frames,
reconstructed slepton decay angle sensitive to
particle spin correlations



Angular Variables

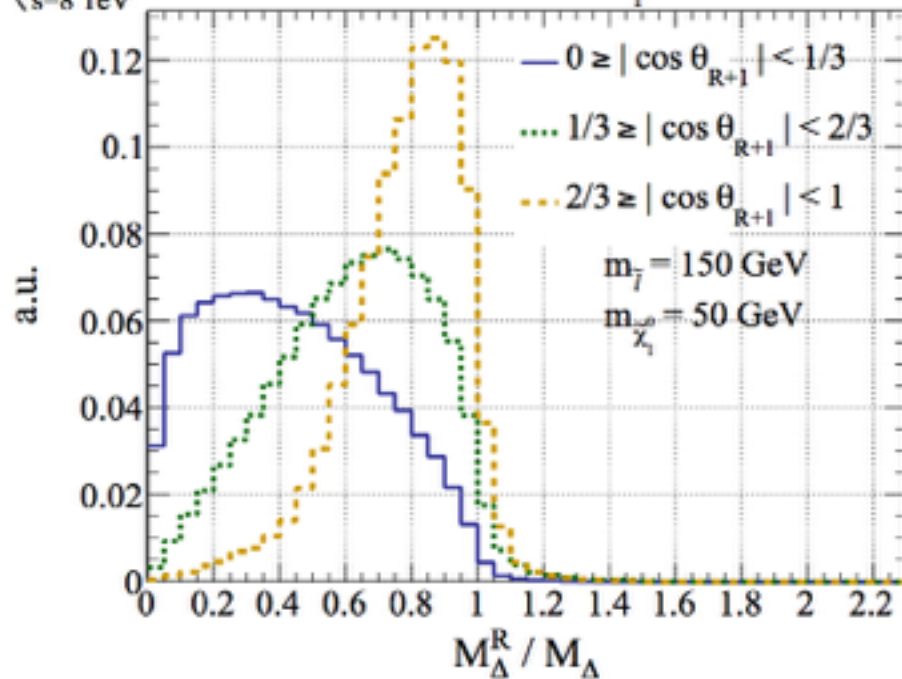
MadGraph+PGS
 $\sqrt{s}=8$ TeV

$pp \rightarrow \tilde{t}\tilde{t}^*; \tilde{t} \rightarrow l\tilde{\chi}_1^0$



MadGraph+PGS
 $\sqrt{s}=8$ TeV

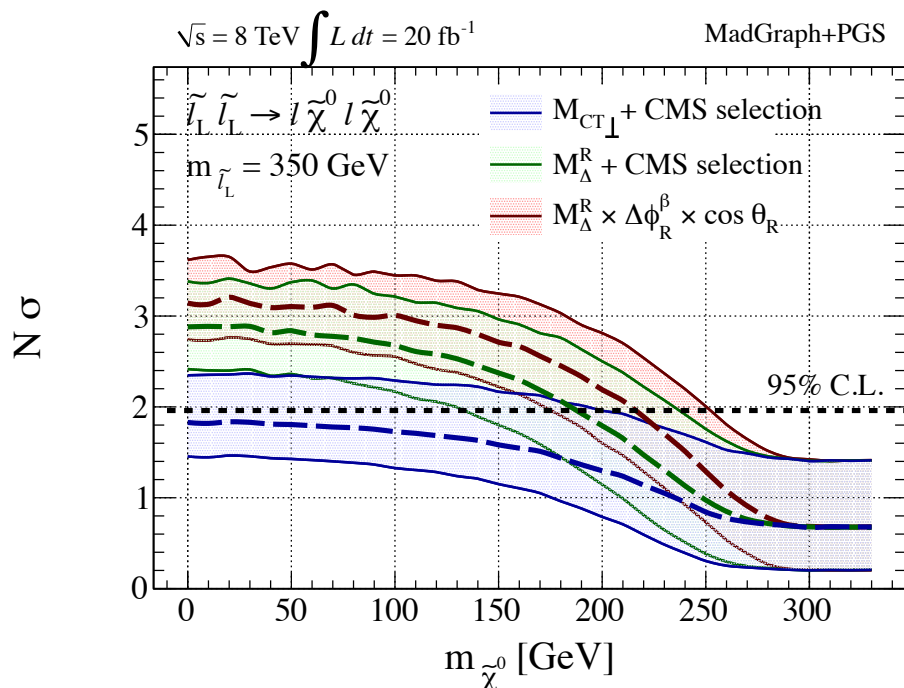
$pp \rightarrow \tilde{t}\tilde{t}^*; \tilde{t} \rightarrow l\tilde{\chi}_1^0$



Also allows us to better resolve the kinematic endpoint of interest



Super-razor variable basis



Can re-imagine a di-lepton analysis
in new basis of variables

Can improve sensitivity while
removing MET cuts!

$\Delta\phi_R^{\beta}$ Sensitive to ratio of invisible
and visible masses

$\sqrt{\hat{s}}_R$ Sensitive to mass of CM
Good for resonant production
of heavy parents

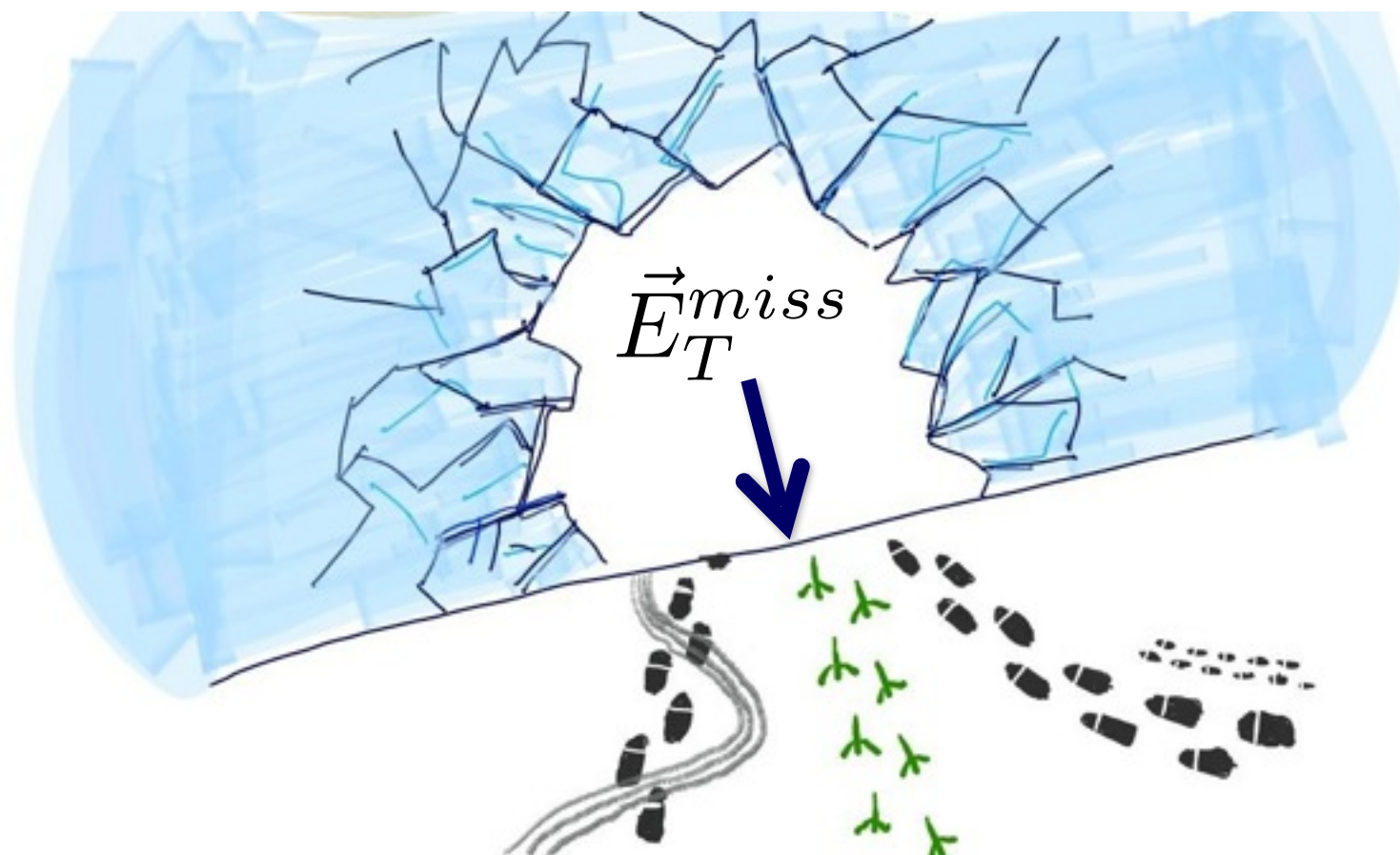
$|\cos\theta_R|$ Spin and production

M_{Δ}^R Mass-squared difference
resonant/non-resonant prod.

$|\cos\theta_{R+1}|$ Spin correlations, better
resolution of mass edge



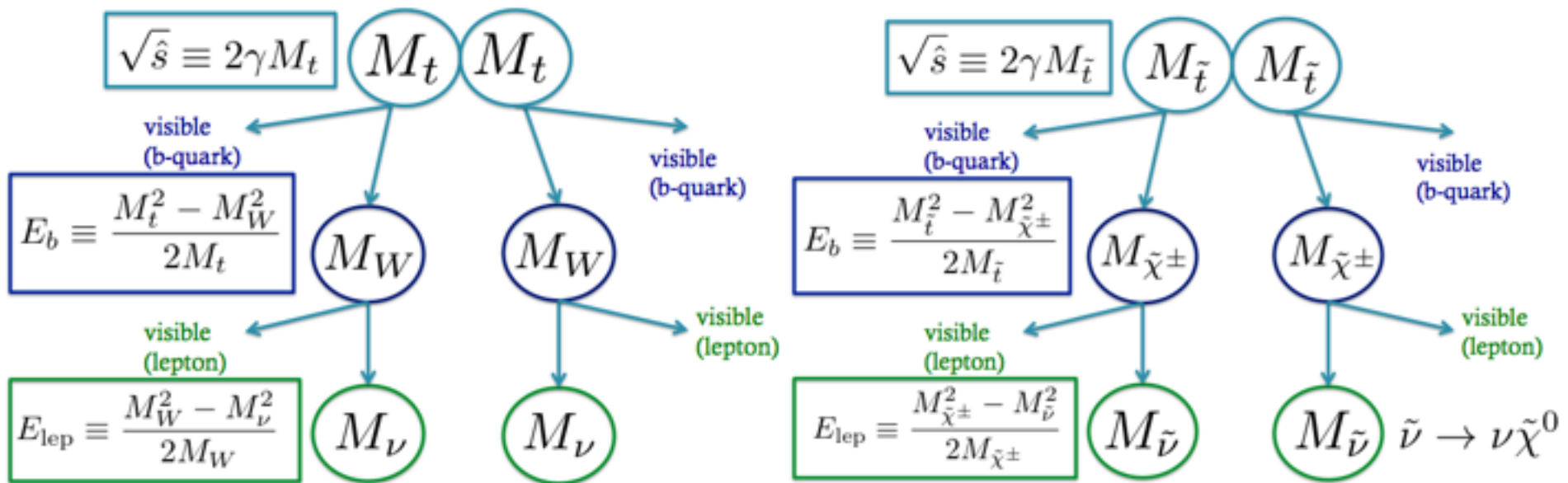
Generalizing further



Recursive Jigsaw approach can be generalized to arbitrarily complex final states with weakly interacting particles

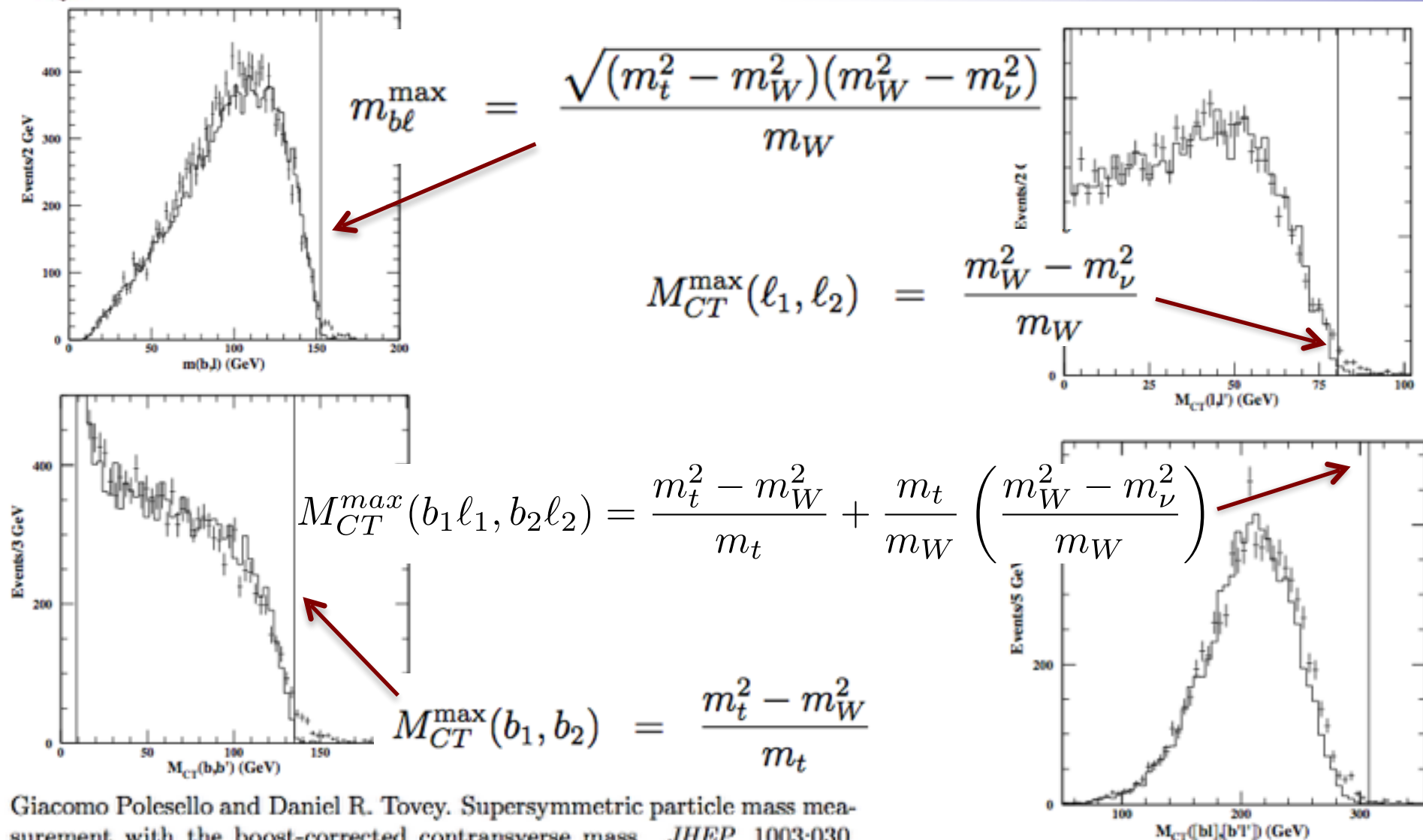


Example: the di-leptonic top basis



In more complicated decay topologies there can be many masses/mass-splittings, spin-sensitive angles and other observables of interest that can be used to distinguish between the SM and SUSY signals

Singularity Variables for Tops



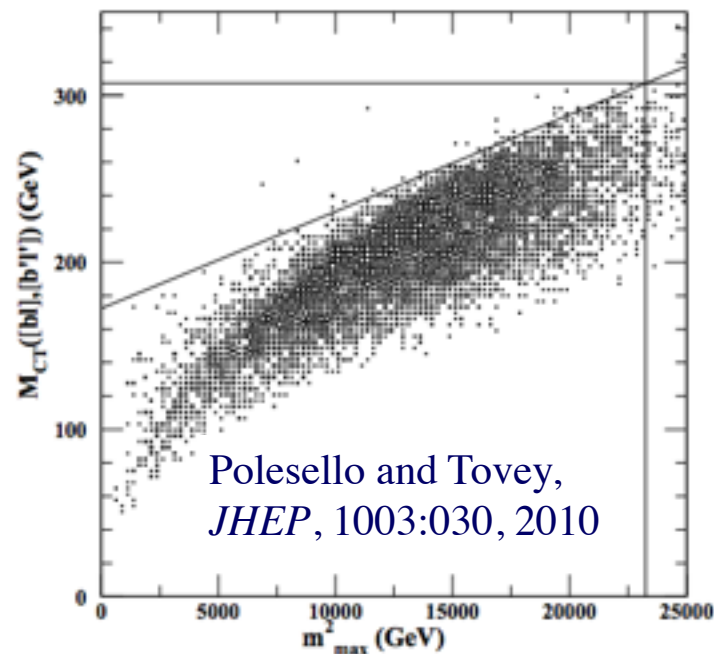
Giacomo Polesello and Daniel R. Tovey. Supersymmetric particle mass measurement with the boost-corrected contranverse mass. *JHEP*, 1003:030, 2010.



Singularity Variables for Tops

If there are multiple masses or mass splittings of interest in the event, appearing at different points in the decay chains, then singularity variables correspond to different extremizations of missing d.o.f.

What are the correlations between the different singularity variables estimating different mass splittings? Can be large/complicated when using M_{CT} or M_{T2} in this case

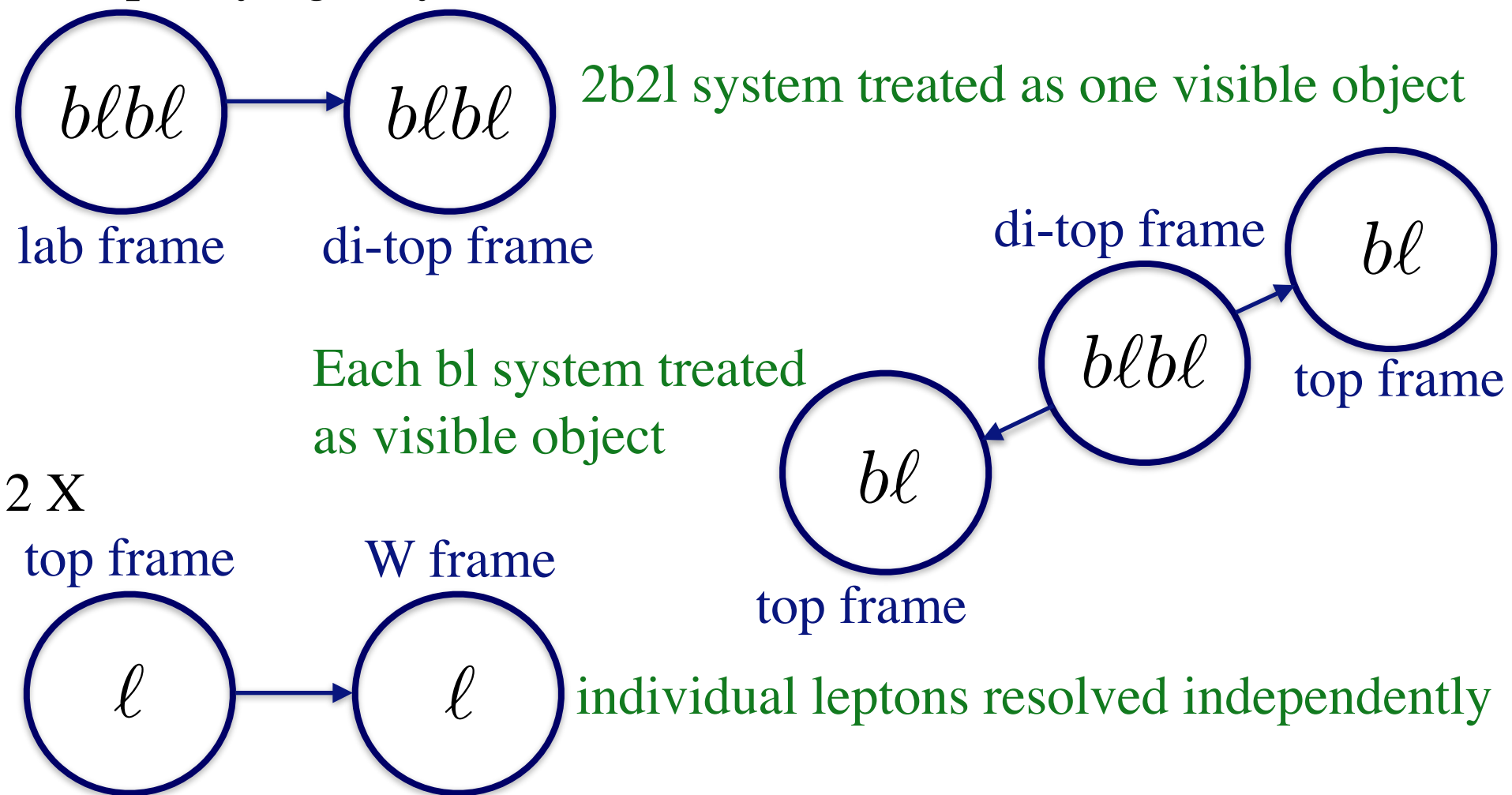


What if the two tops/stops are decaying differently through different particles?



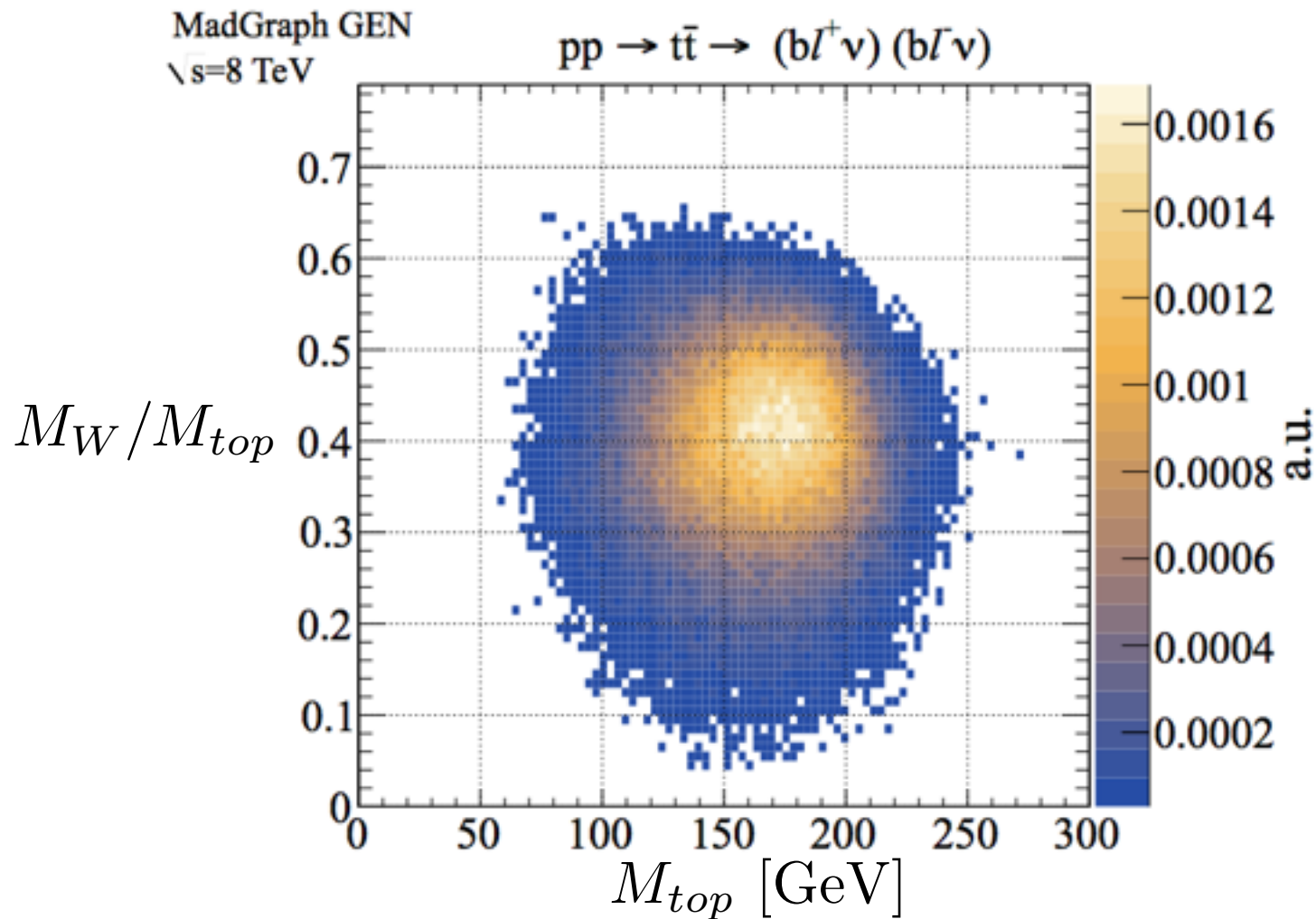
Recursive Jigsaw reconstruction

Move through each reference frame of interest in the event, specifying only d.o.f. relevant to each transformation:





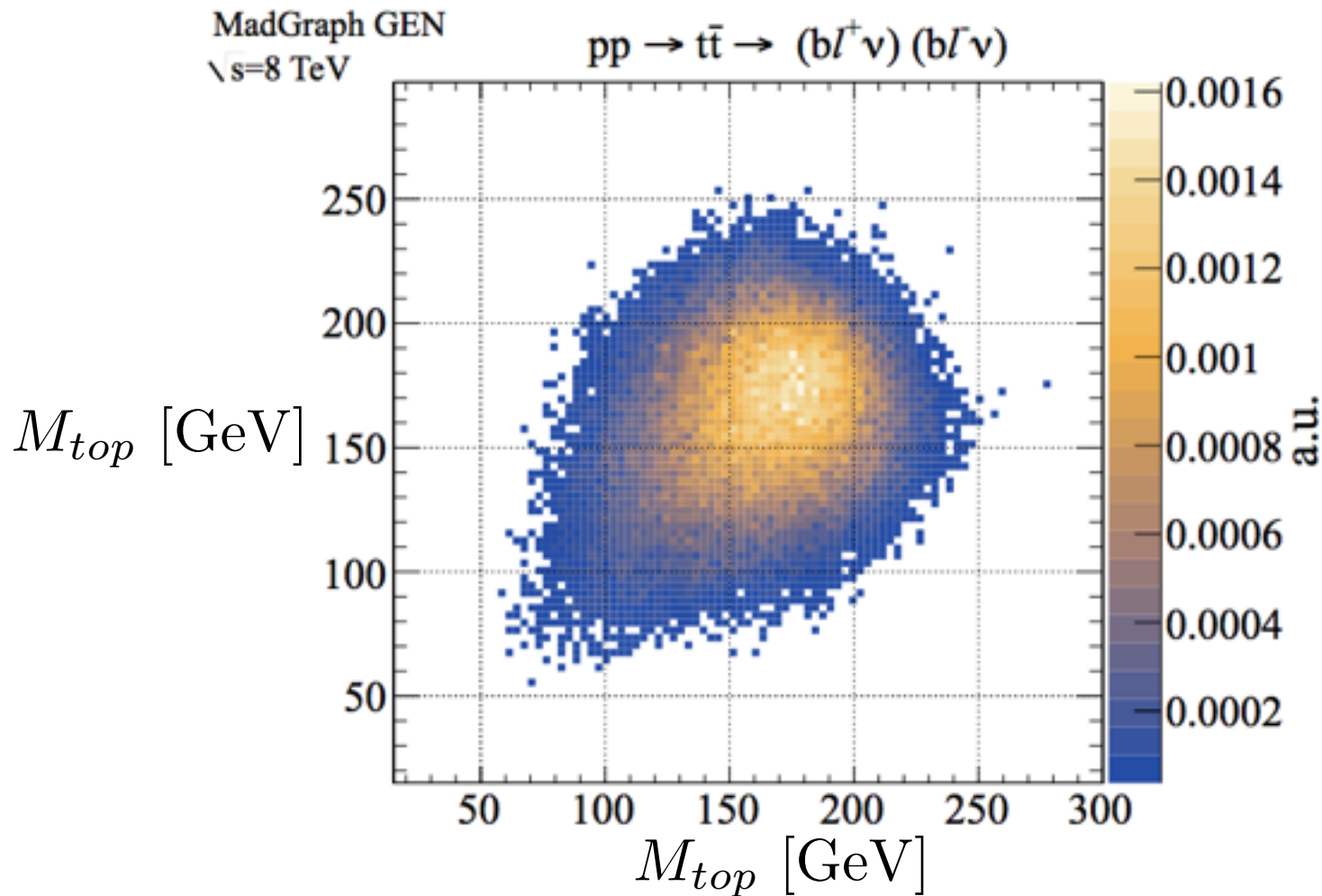
Recursive Jigsaw reconstruction



The scales can be extracted independently



Recursive Jigsaw reconstruction



In fact the scales can be extracted independently for each top –
the reconstruction chains are *decoupled*



The di-leptonic top basis

$$M_{t\bar{t}}, \vec{p}_{t\bar{t}}, \cos \theta_{TT}$$

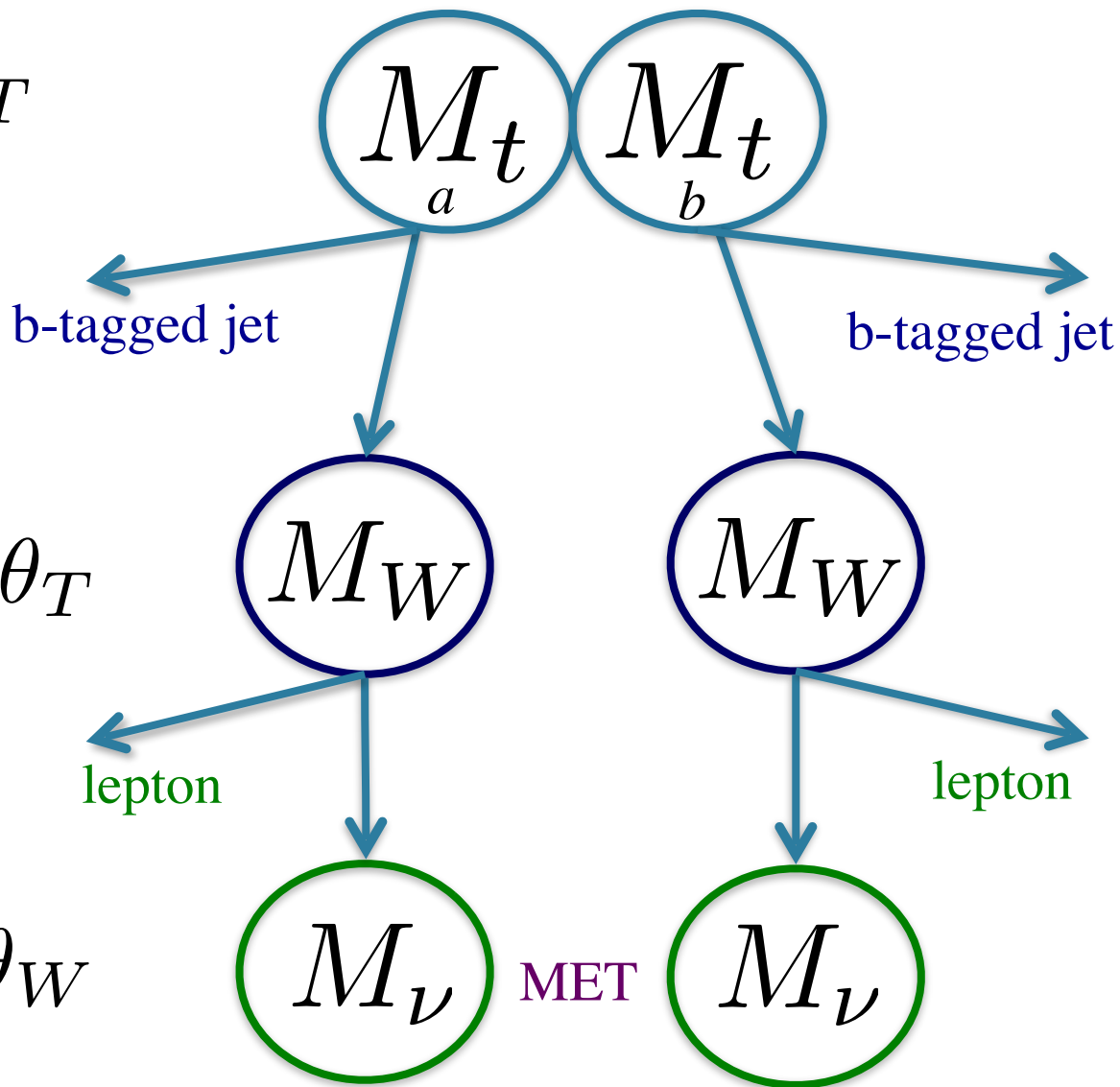
$$\Delta\phi_{T1,T2}$$

2 X

$$E_b^{\text{top-frame}}, \cos \theta_T$$

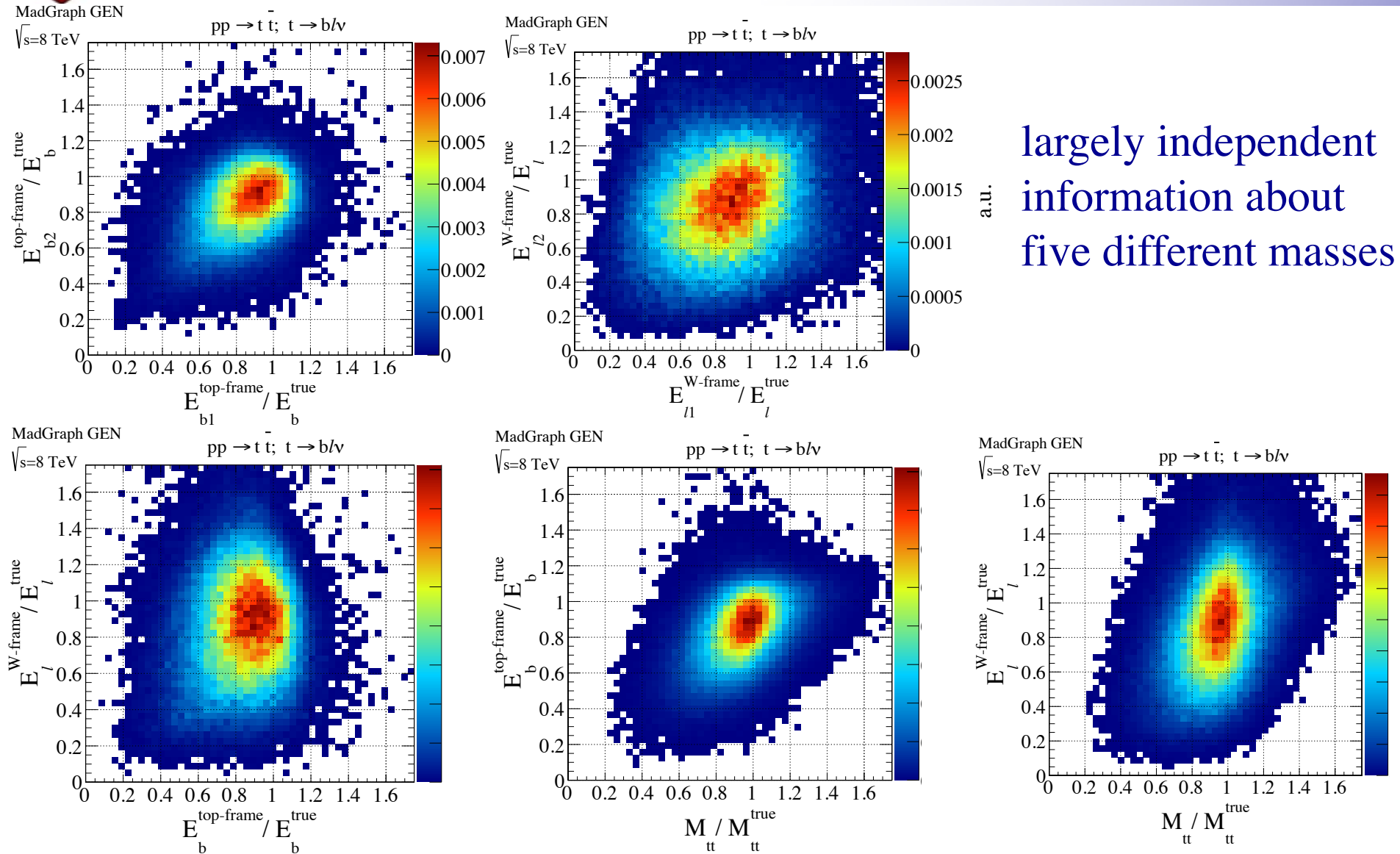
$$\Delta\phi_{T,W}$$

$$E_\ell^{W\text{-frame}}, \cos \theta_W$$





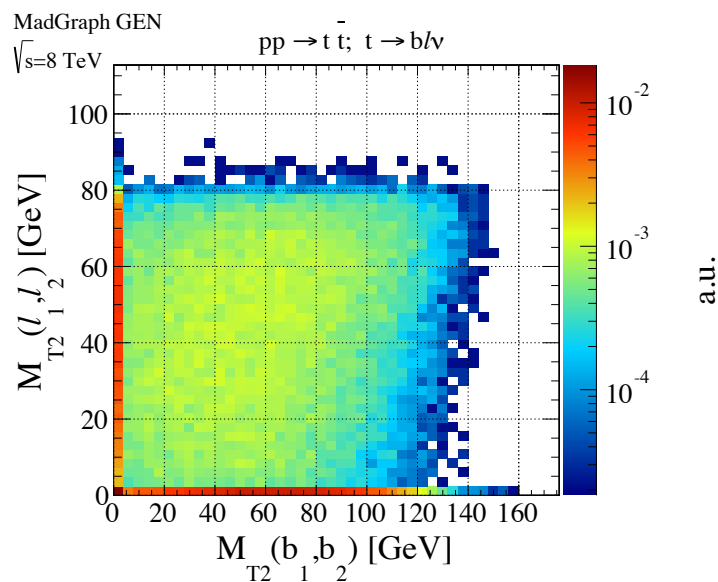
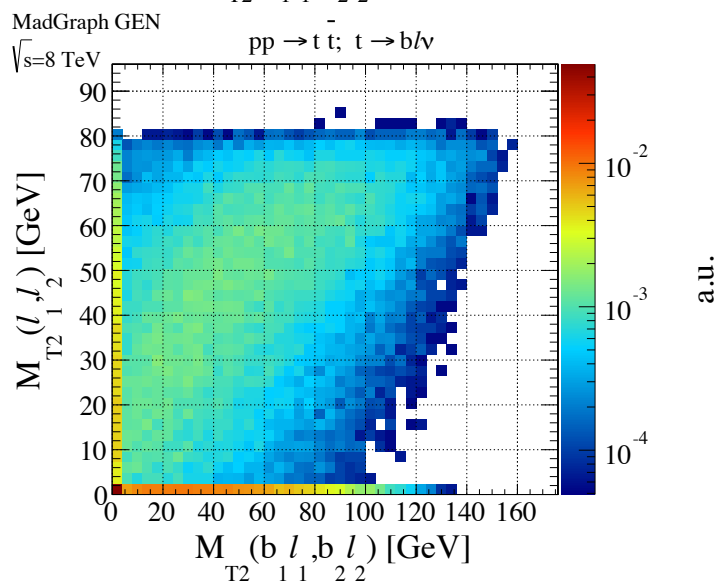
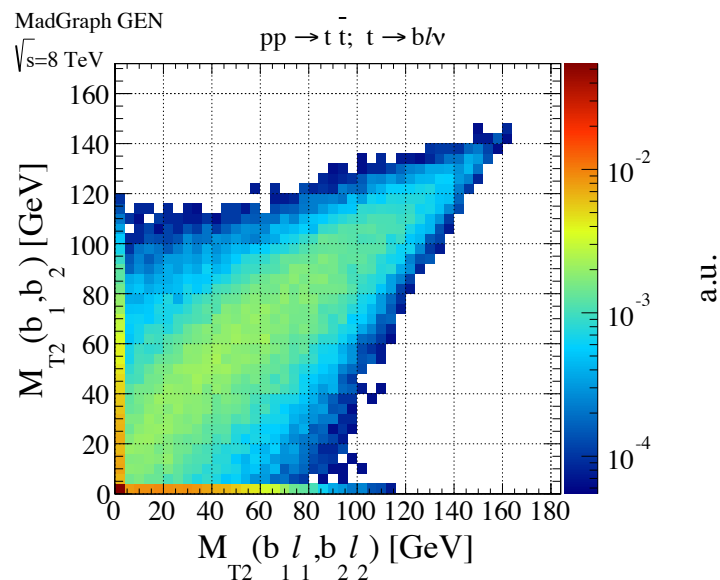
The di-leptonic top basis





Previous state-of-the-art

Mass-sensitive
singularity variables
are not necessarily
independent

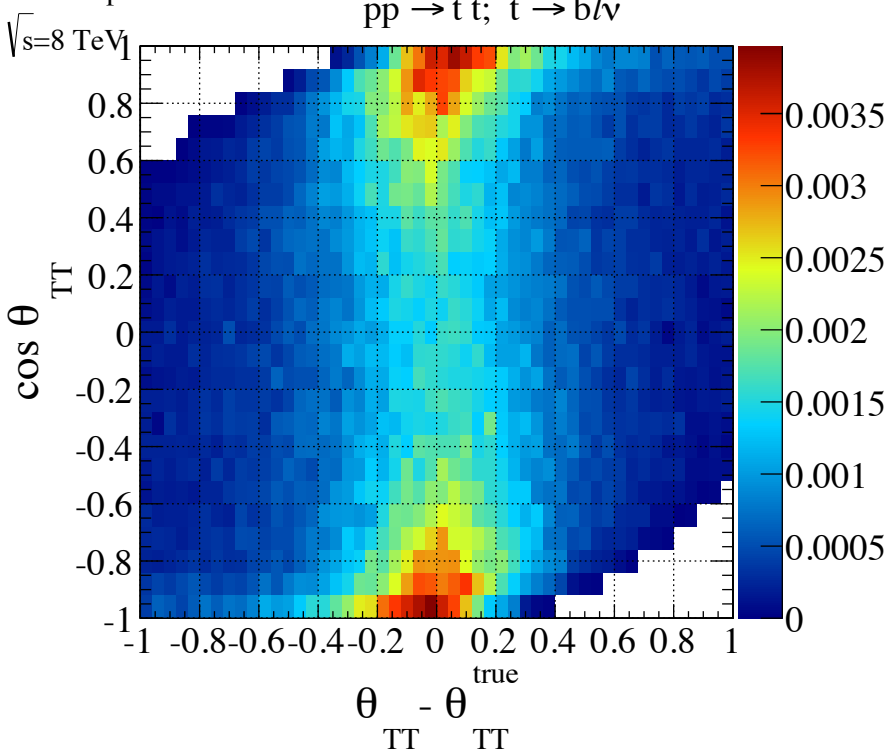




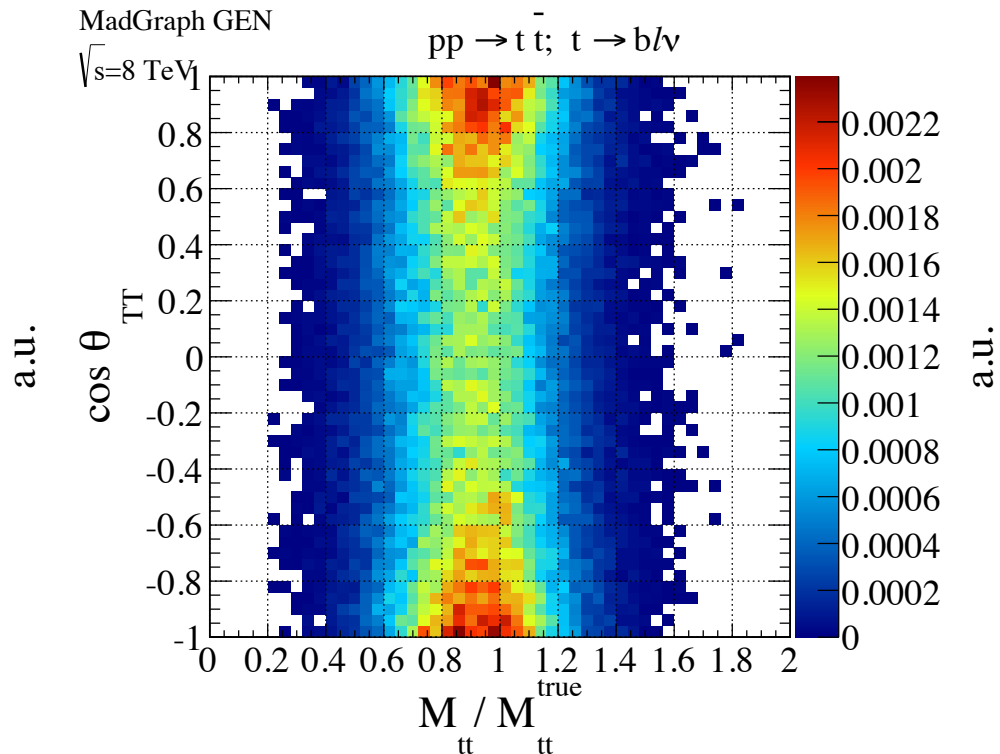
The di-leptonic top basis

largely independent information about decay angles

MadGraph GEN



MadGraph GEN

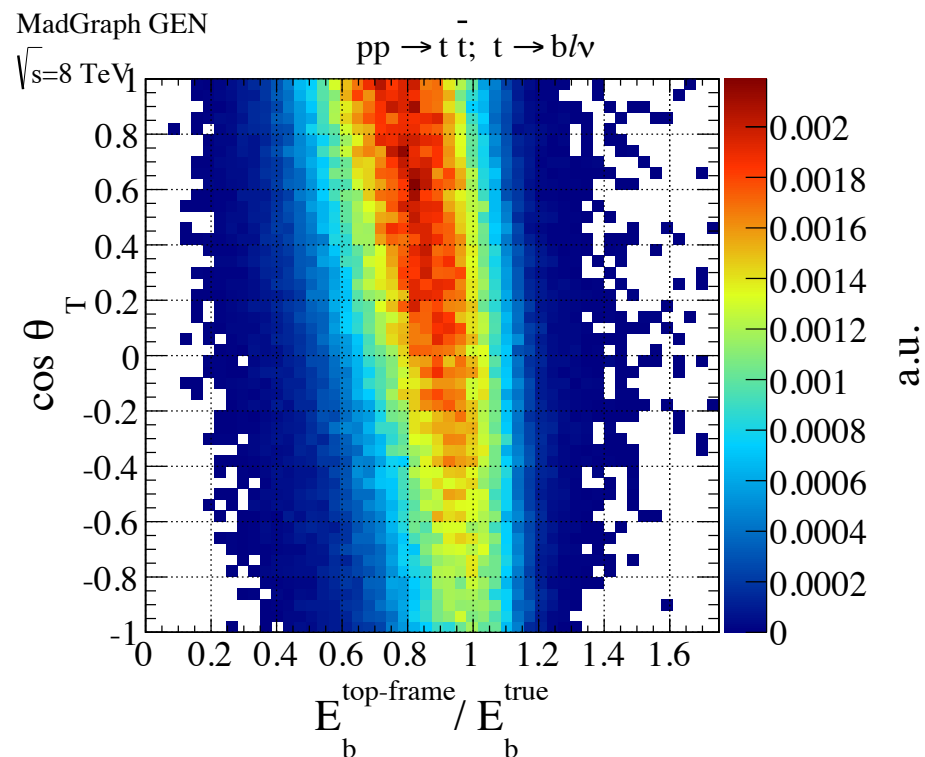
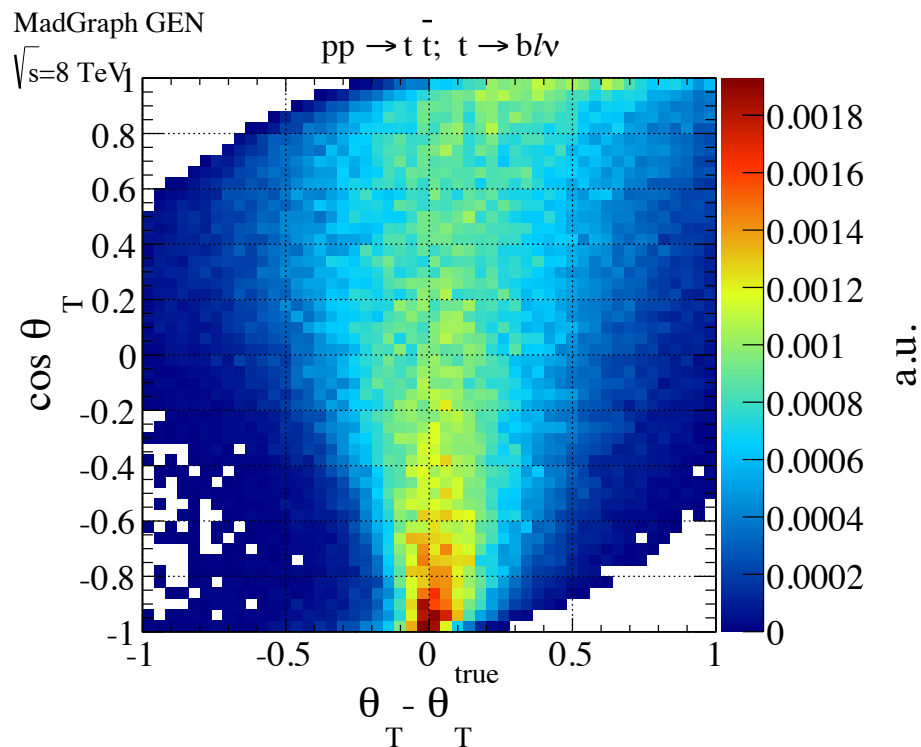


Here, the decay angle of the top/anti-top system



The di-leptonic top basis

largely independent information about decay angles

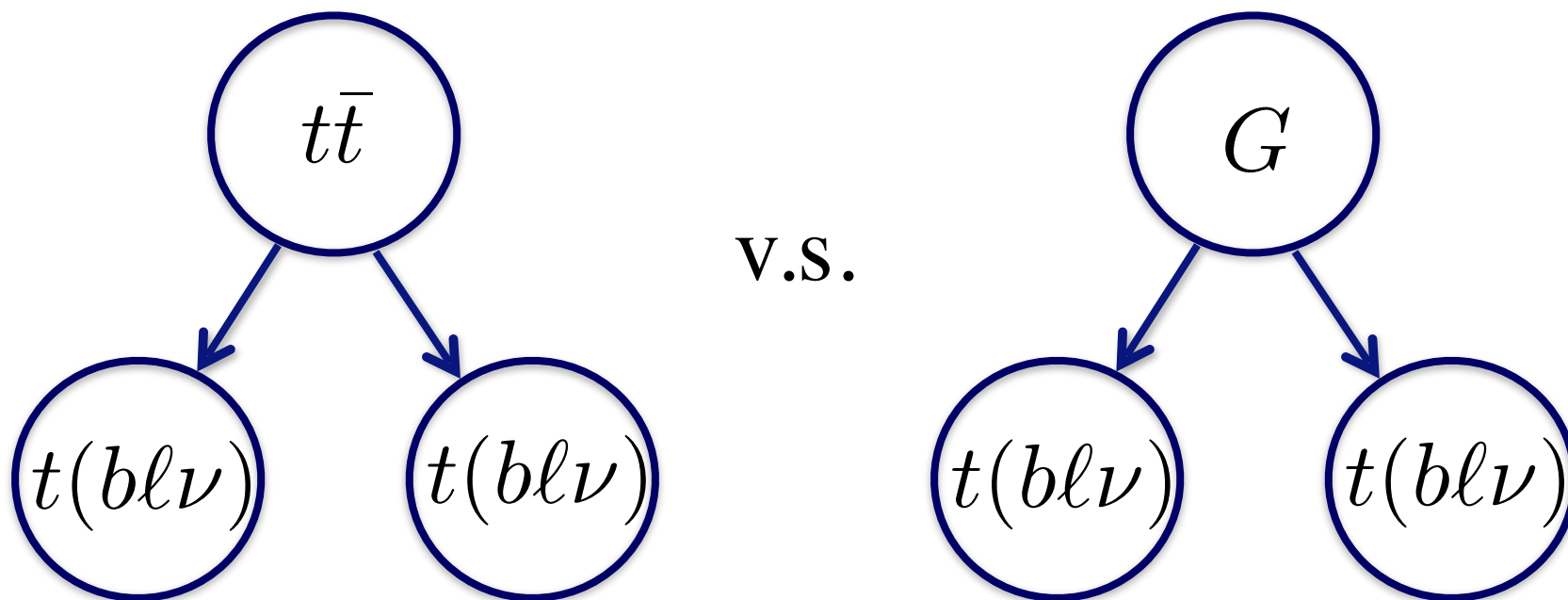


Here, the decay angle of one of the top quarks



The di-leptonic top basis vs. signals

Different variables in the basis are useful for different signals



First, we consider resonant $t\bar{t}$ production through a graviton



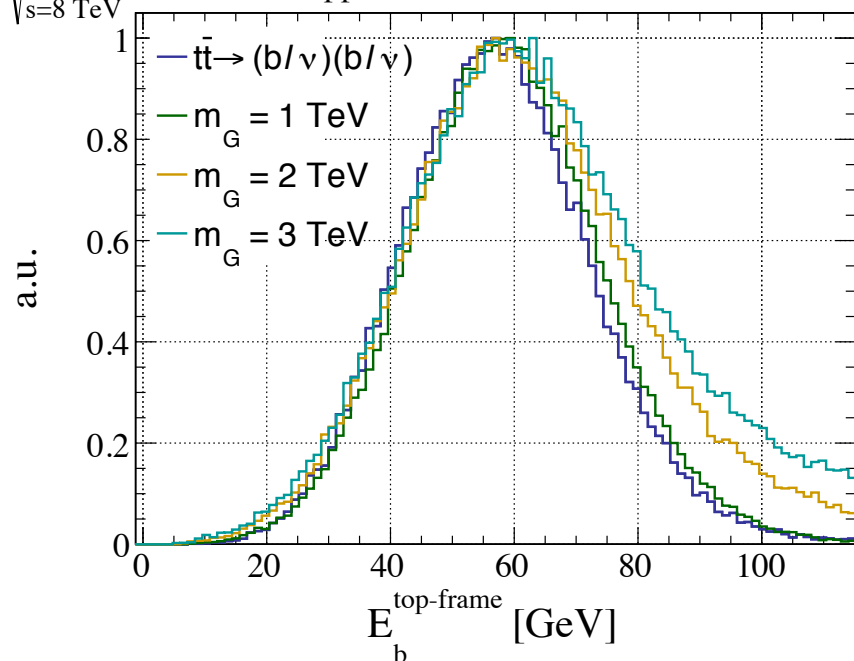
The di-leptonic top basis vs. gravitons

Different variables in the basis are useful for different signals

MadGraph GEN

$\sqrt{s}=8$ TeV

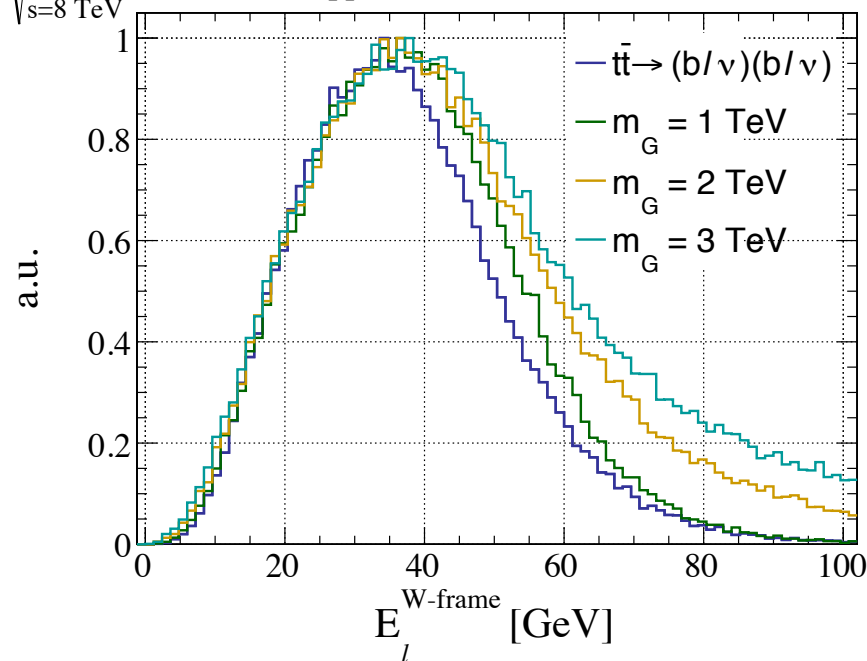
$pp \rightarrow G \rightarrow t \bar{t}; t \rightarrow b \nu$



MadGraph GEN

$\sqrt{s}=8$ TeV

$pp \rightarrow G \rightarrow t \bar{t}; t \rightarrow b \nu$

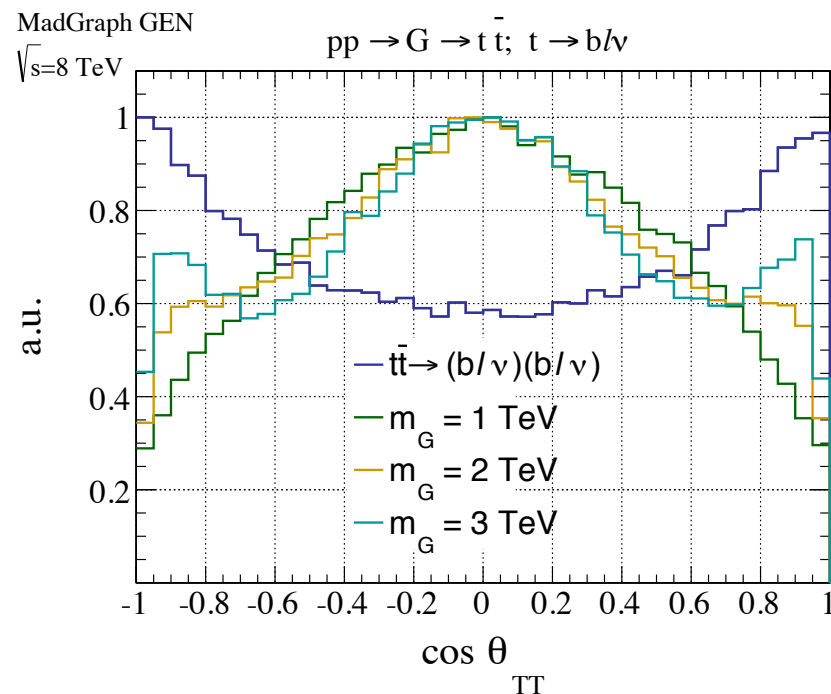
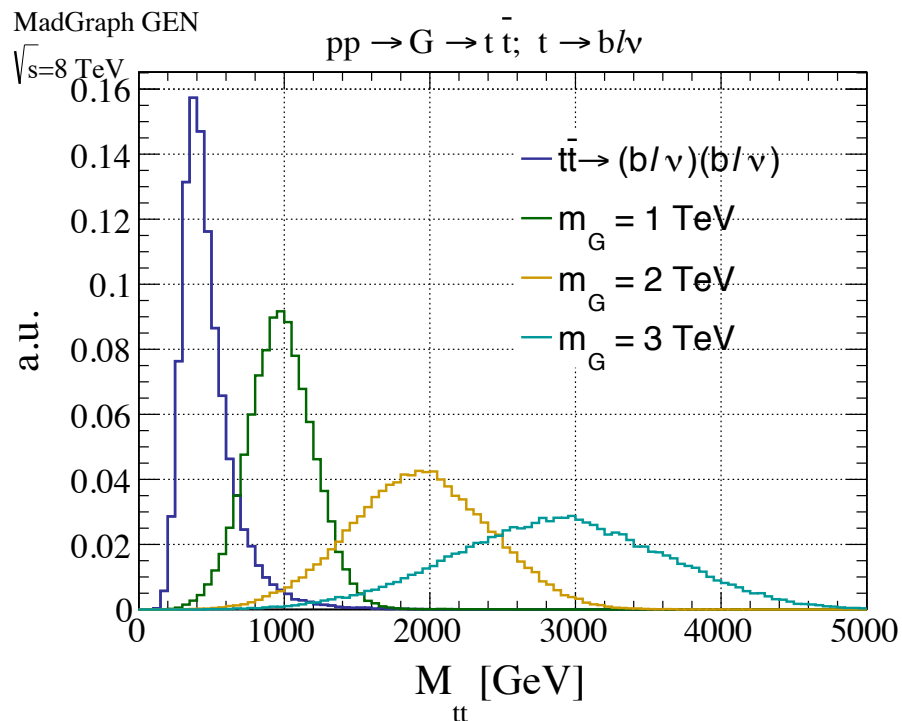


Distributions of top/W/neutrino mass-splitting-sensitive observables are nearly identical since graviton signal and non-resonant background both contain on-shell tops



The di-leptonic top basis vs. gravitons

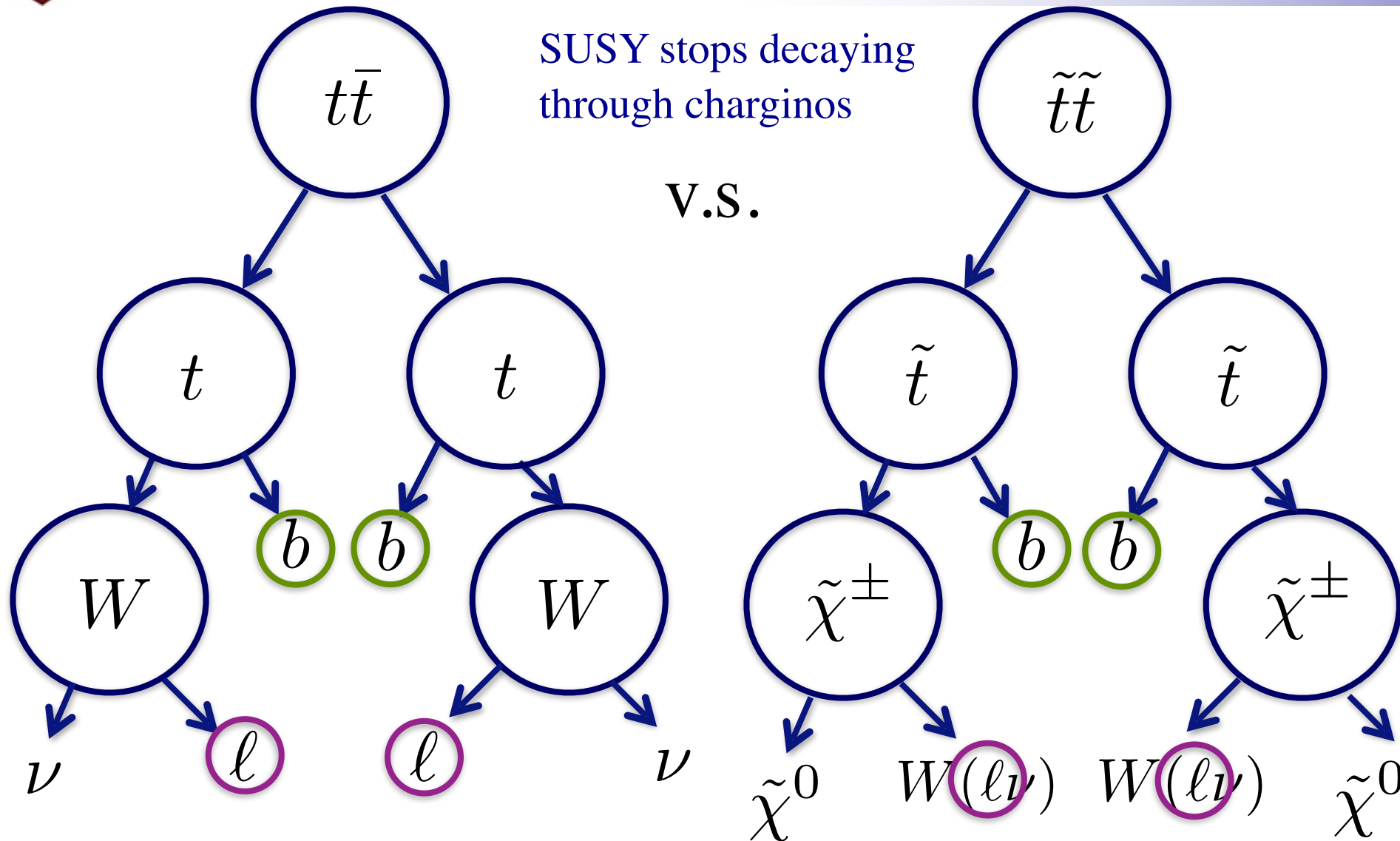
Different variables in the basis are useful for different signals



Instead, observables related to the production of the two tops are sensitive to the intermediate resonance

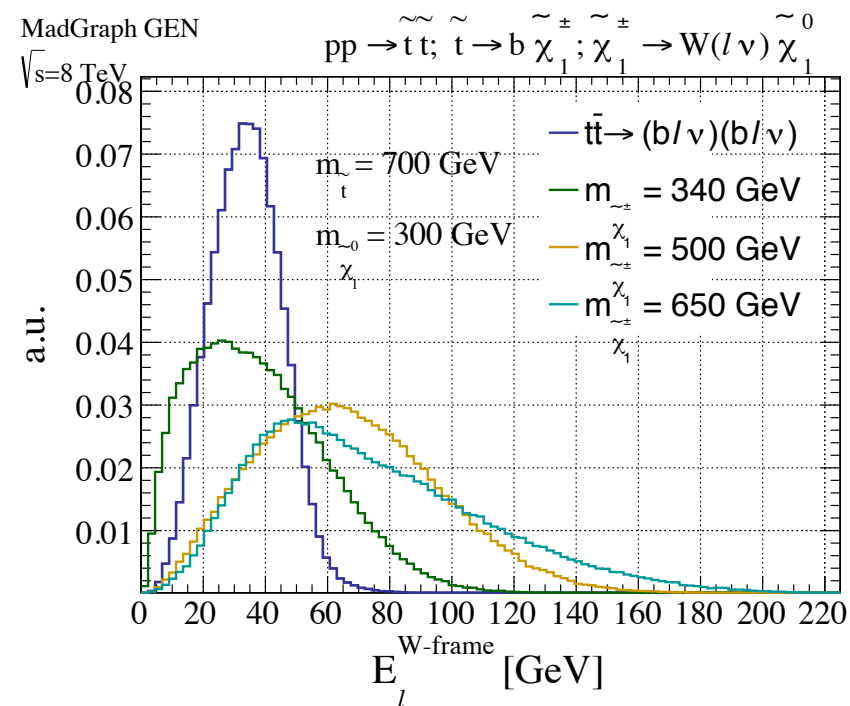
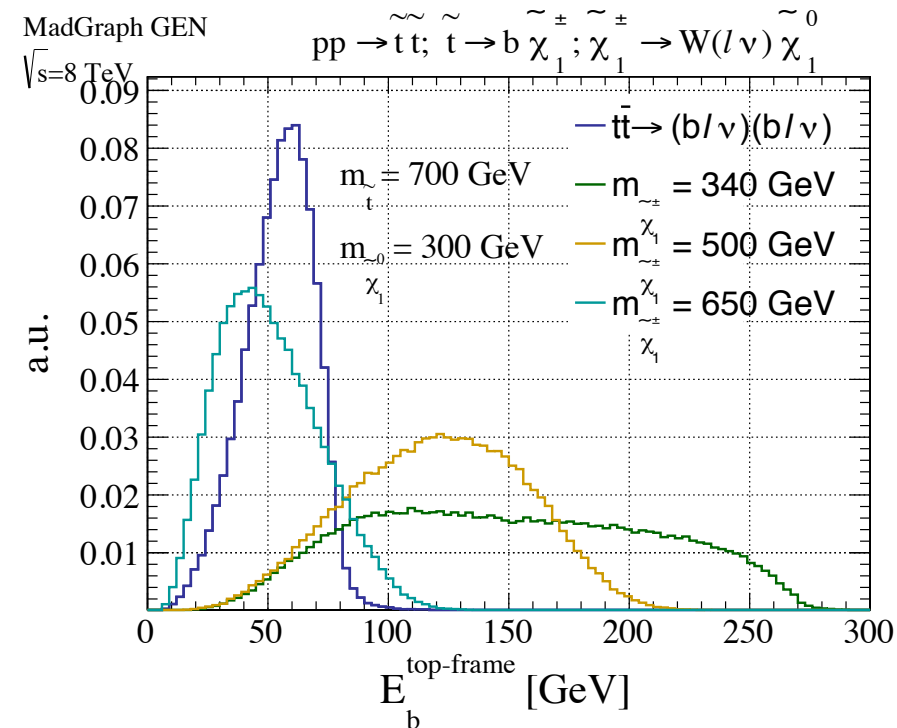


The di-leptonic top basis vs. stops





The di-leptonic top basis vs. stops

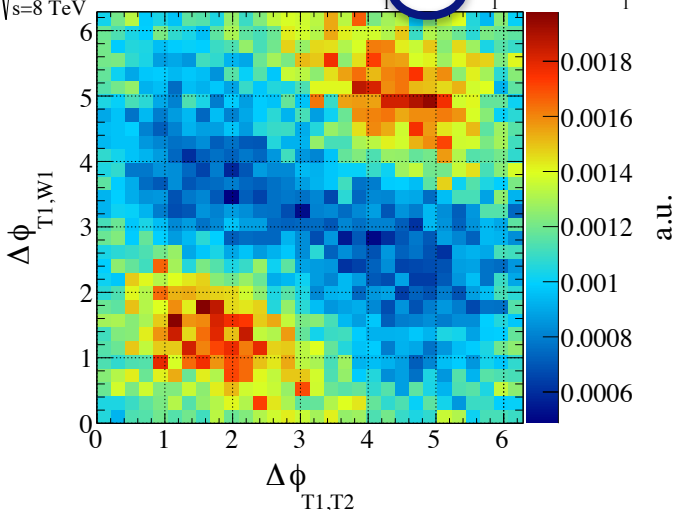


Mass-splitting-sensitive observables can be used to distinguish presence of signals

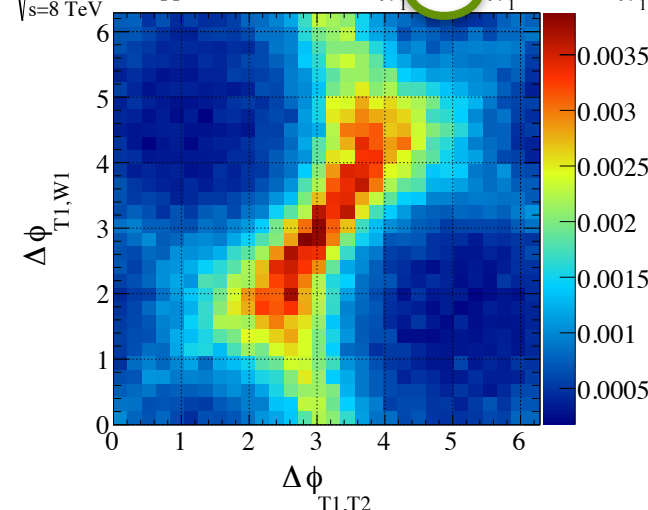


The di-leptonic top basis vs. stops

MadGraph GEN $\sqrt{s}=8$ TeV $pp \rightarrow t\bar{t} (700); \tilde{t} \rightarrow b\tilde{\chi}_1^\pm (650) \tilde{\chi}_1^\pm \rightarrow W(l\nu)\tilde{\chi}_1^0 (300)$



MadGraph GEN $\sqrt{s}=8$ TeV $pp \rightarrow t\bar{t} (700); \tilde{t} \rightarrow b\tilde{\chi}_1^\pm (500) \tilde{\chi}_1^\pm \rightarrow W(l\nu)\tilde{\chi}_1^0 (300)$



$$m_{\tilde{t}} = 700 \text{ GeV}$$

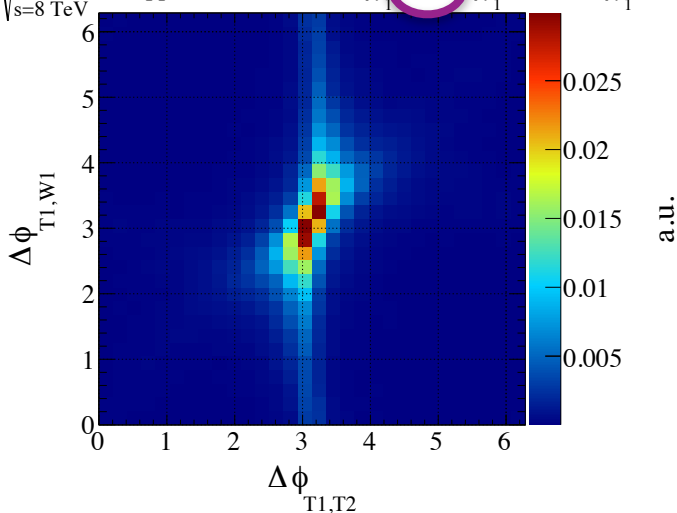
$$650$$

$$m_{\tilde{\chi}^\pm} = 500 \text{ GeV}$$

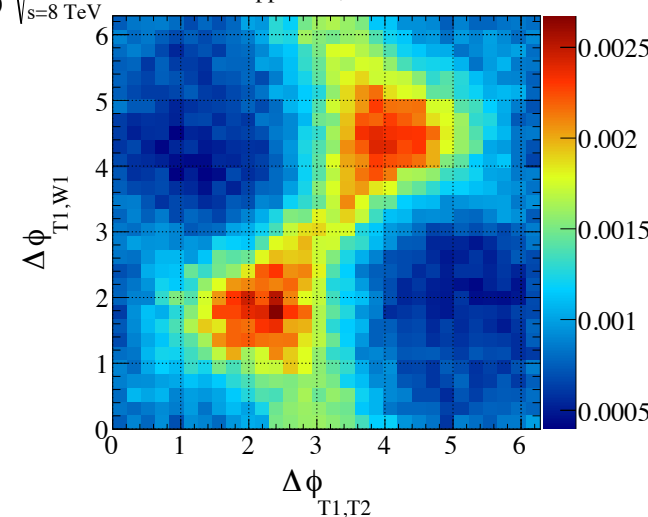
$$340$$

$$m_{\tilde{\chi}^0} = 300 \text{ GeV}$$

MadGraph GEN $\sqrt{s}=8$ TeV $pp \rightarrow t\bar{t} (700); \tilde{t} \rightarrow b\tilde{\chi}_1^\pm (340) \tilde{\chi}_1^\pm \rightarrow W(l\nu)\tilde{\chi}_1^0 (300)$



MadGraph GEN $\sqrt{s}=8$ TeV



Here, the azimuthal angle between the the top and W decay planes $\Delta\phi_{T1,W1}$ and the angle between the two top decay planes

$$\Delta\phi_{T1,T2}$$



Summary

- The strategy of **Recursive Jigsaw Reconstruction** is to not only develop ‘good’ mass estimator variables, but to decompose each event into a *basis of kinematic variables*
- Through the recursive procedure, each variable is (as much as possible) *independent of the others*
- The interpretation of variables is straightforward; they each correspond to an *actual, well-defined, quantity in the event*
- Can be generalized to arbitrarily complex final states with *many weakly interacting particles*
- Code package to be released early this month (***RestFrames***)
- First papers nearing completion
(***Recursive Jigsaw Reconstruction***,
The Di-leptonic $t\bar{t}$ basis - with Paul Jackson)

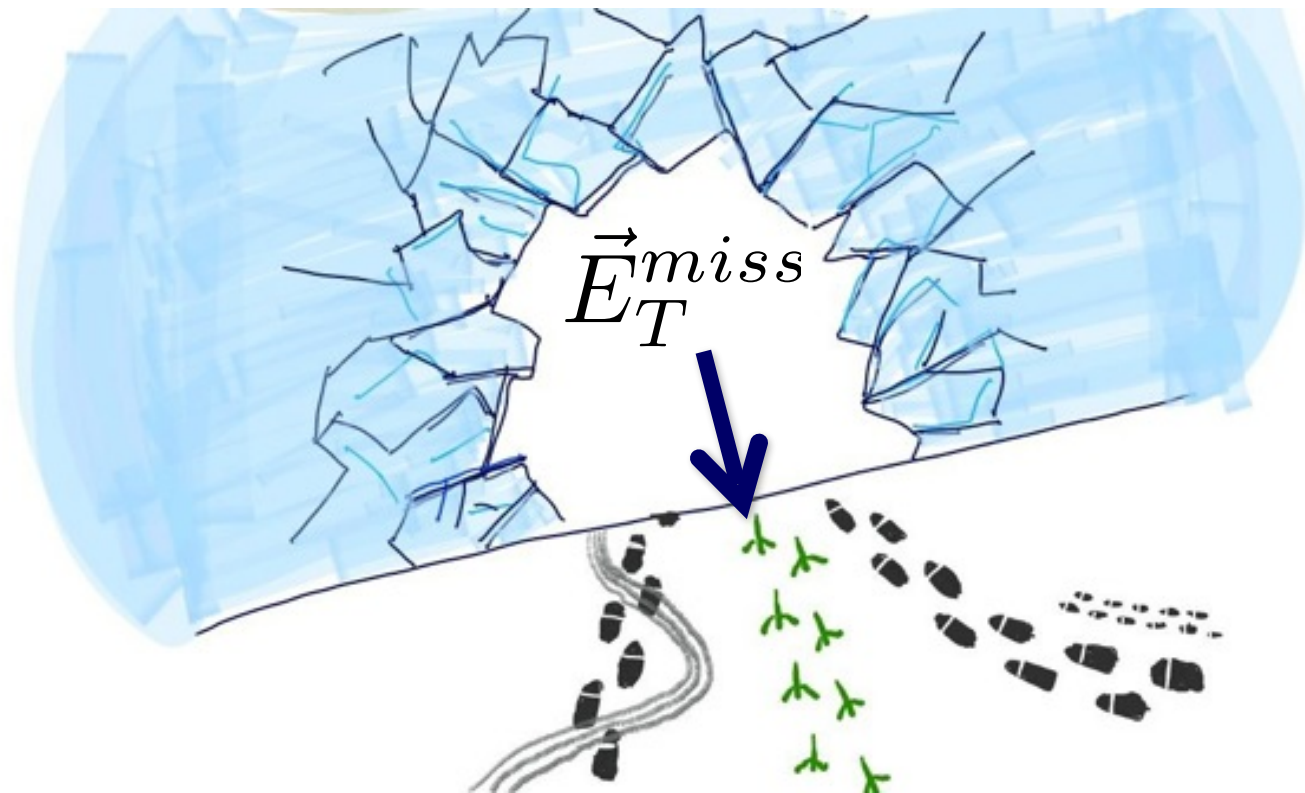


Outlook

- Recursive rest-frame reconstruction is a **systematic recipe** for deriving a kinematic basis for any open final state
- **Mixed decay cases can now be treated in a sensible way**

$$\begin{array}{lcl} \tilde{t} \rightarrow t\tilde{\chi}^0 & \text{vs.} & \tilde{t} \rightarrow b\tilde{\chi}^\pm \\ & & \tilde{\chi}^\pm \rightarrow W^\pm \tilde{\chi}^0 \text{ vs.} \\ & & \tilde{\chi}^\pm \rightarrow \ell^\pm \tilde{\nu} \text{ vs.} \\ & & \tilde{\chi}^\pm \rightarrow \nu \tilde{\ell}^\pm \end{array}$$

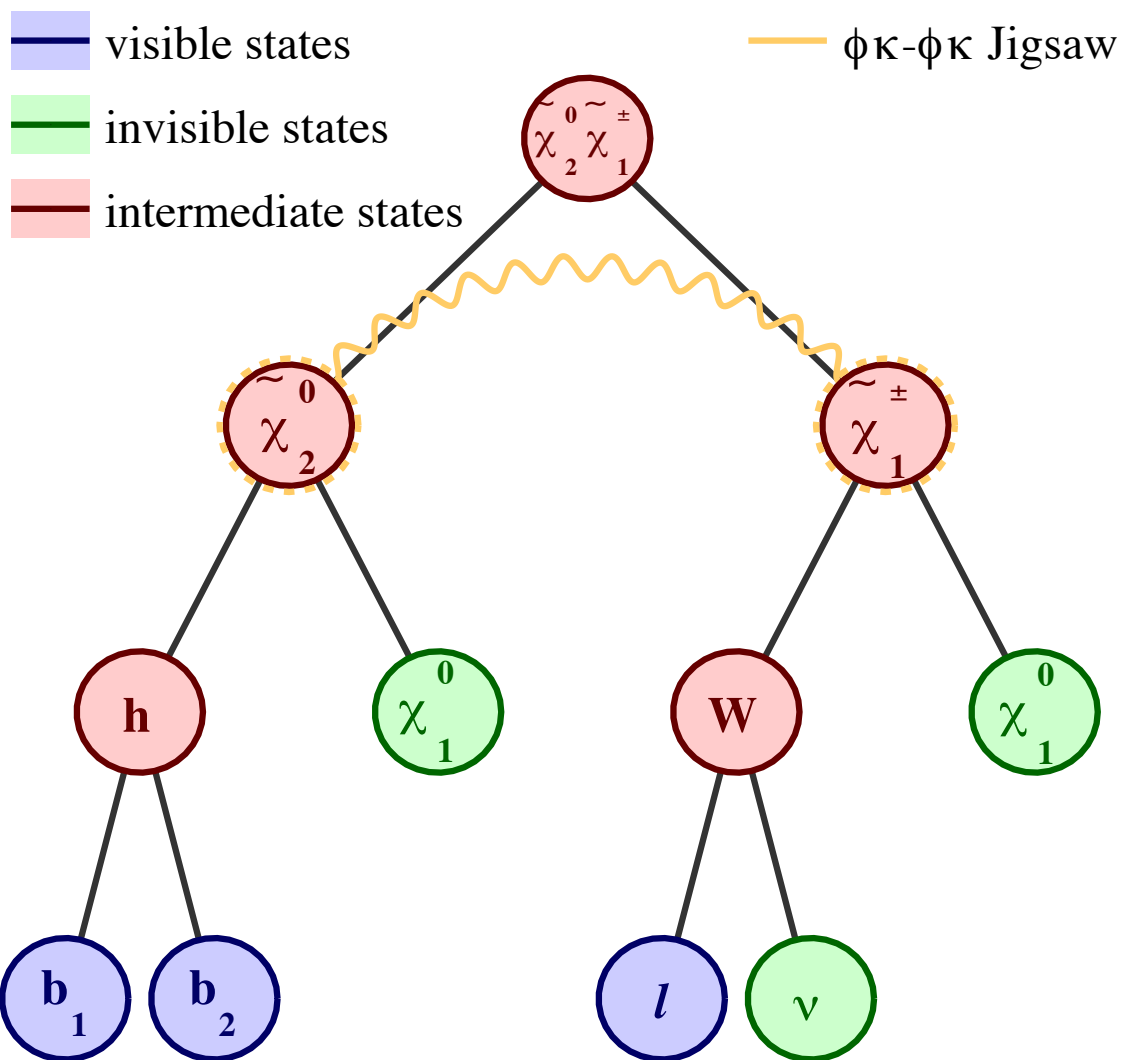
- Lots of potential applications – some I'm currently thinking about:
 - Di-lepton searches and differential measurements
 - stop / top-quark partner and resonant $t\bar{t}$ searches
 - other $\mathbb{Z}_2$ signatures (SUSY, exotica)
 - $H \rightarrow WW^* \rightarrow \ell\nu\ell\nu$ spin measurements
 - ...



BACKUP SLIDES

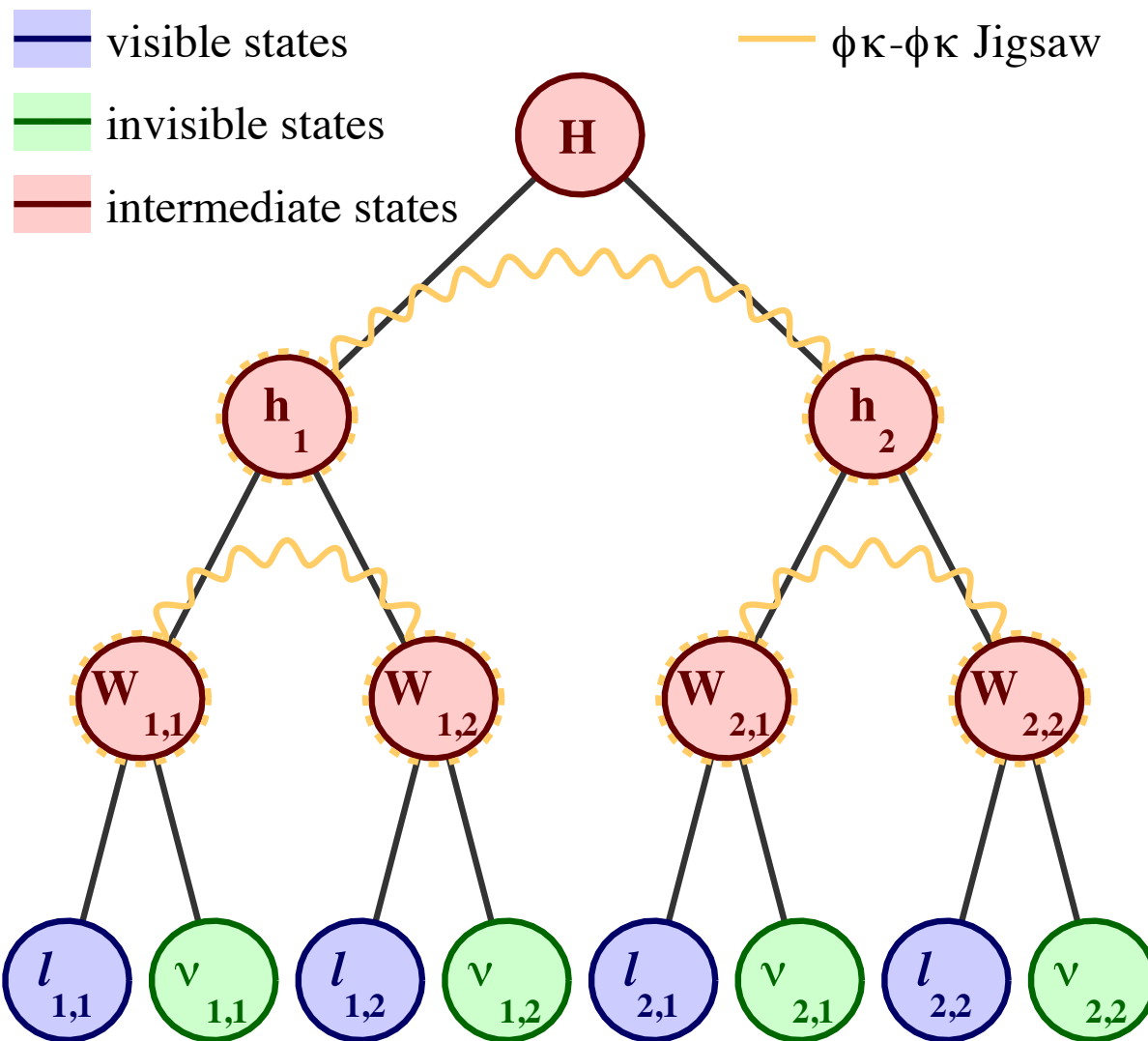


Possibilities





Possibilities





RestFrames Code

RestFrames: Soon-to-be-public code that can be used to calculate kinematic variables associated with any decay chain and can implement “Recursive Jigsaw” rules

Example: Di-leptonic $t\bar{t}b\bar{a}r$ decays - how to use the code to initialize a decay tree, implement a jigsaw rule for determining the neutrino four-momenta, and analyze events



RestFrames Code

```
//Initialize lists of visible, invisible particles and intermediate states
```

```
LabFrame *LAB = new LabFrame("LAB", "lab");  
DecayFrame *TT = new DecayFrame("TT", "t #bar{t}");  
DecayFrame *T1 = new DecayFrame("T1", "t_{1}");  
DecayFrame *T2 = new DecayFrame("T2", "t_{2}");  
VisibleFrame *B1 = new VisibleFrame("B1", "b_{1}");  
VisibleFrame *B2 = new VisibleFrame("B2", "b_{2}");  
DecayFrame *W1 = new DecayFrame("W1", "W_{1}");  
DecayFrame *W2 = new DecayFrame("W2", "W_{2}");  
VisibleFrame *L1 = new VisibleFrame("L1", "#it{l}_{1}");  
VisibleFrame *L2 = new VisibleFrame("L2", "#it{l}_{2}");  
InvisibleFrame *NU1 = new InvisibleFrame("NU1", "#nu_{1}");  
InvisibleFrame *NU2 = new InvisibleFrame("NU2", "#nu_{2}");
```

initialize all your reference
frames of interest

```
// connect the different  
// reference frames  
// according to decays  
LAB->SetChildFrame(TT);
```

```
TT->SetChildFrame1(T1);  
TT->SetChildFrame2(T2);
```

```
T1->SetChildFrame1(B1);  
T1->SetChildFrame2(W1);  
T2->SetChildFrame1(B2);  
T2->SetChildFrame2(W2);
```

```
W1->SetChildFrame1(L1);  
W1->SetChildFrame2(NU1);  
W2->SetChildFrame1(L2);  
W2->SetChildFrame2(NU2);
```

connect them according to
the decay tree you want to
impose on the event

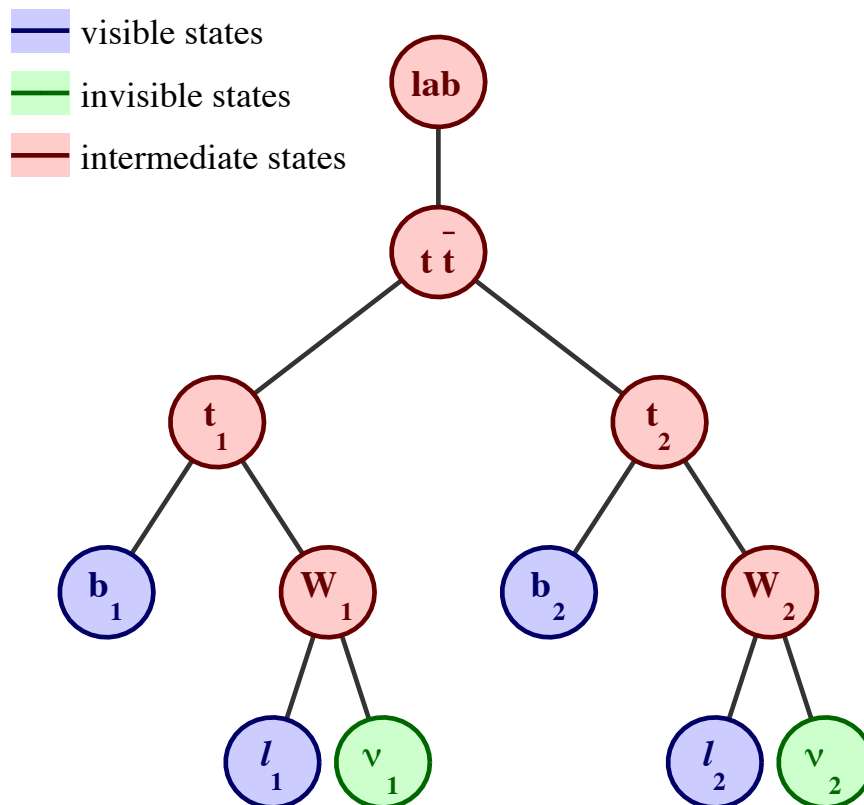


RestFrames Code

```
// draw decay tree
RPlot *ttbar_plot = new RPlot();
ttbar_plot->SetCanvas("ttbar", "ttbar", 600, 600);
ttbar_plot->DrawTree(LAB);
TCanvas *canvas = ttbar_plot->GetCanvas();
canvas->Draw();
```

draw your decay tree...

...and you'll see this





RestFrames Code

```
// check that tree topology is consistent
bool is_consistent = LAB->IsConsistentTopology();
ContraBoostInvariantJigsaw *TopJigsaw = new ContraBoostInvariantJigsaw("Top-Top Jigsaw");
// tell tree how to resolve neutrino four-momenta
TopJigsaw->AddFrame1(T1);
TopJigsaw->AddFrame2(T2);
// add jigsaw rule to tree
bool ok_jigsaw = TT->AddJigsaw(TopJigsaw) << endl;
// initialize tree for analysis
bool is_ready = LAB->InitializeAnalysis();
```

Check to make sure that
your tree makes sense

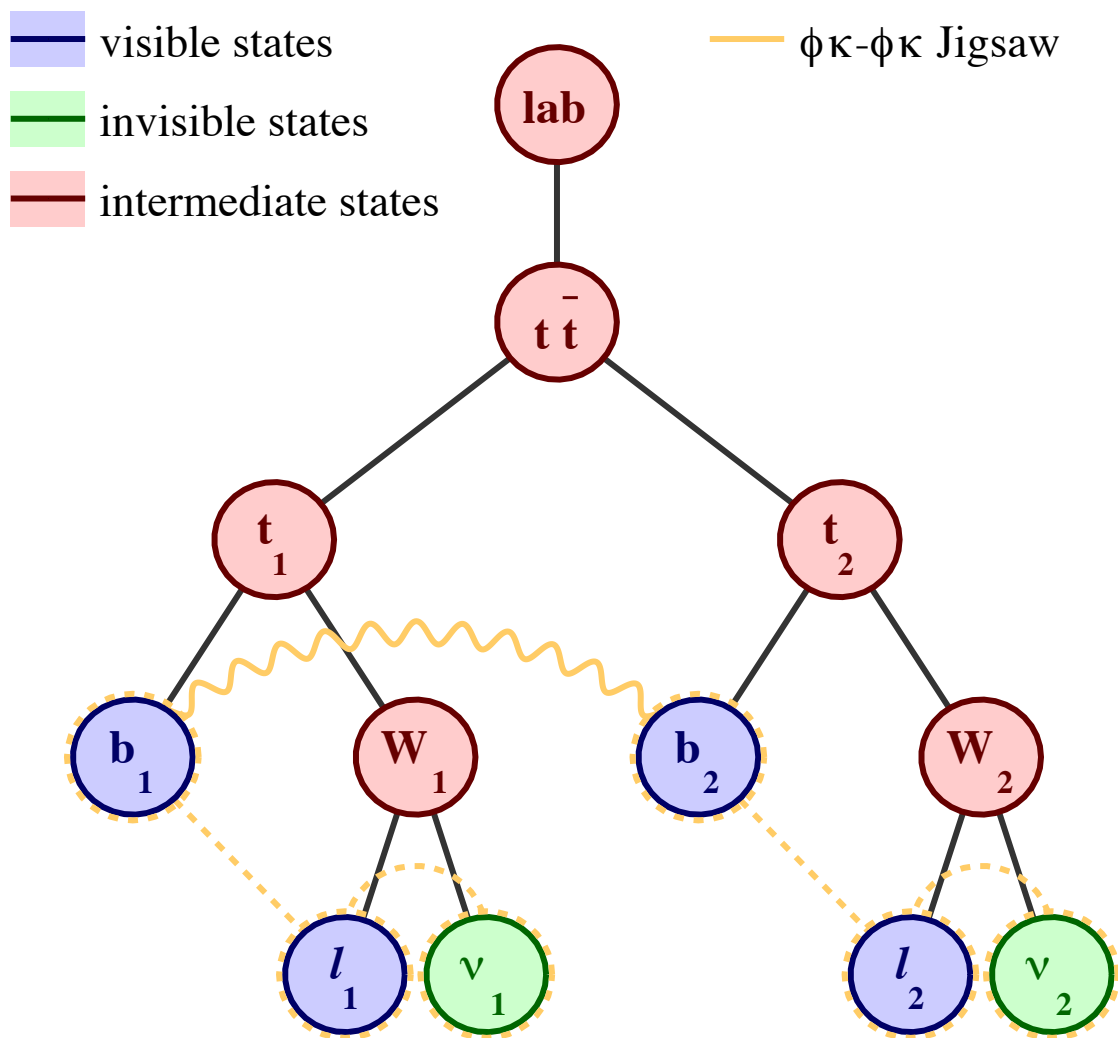
Tell your tree how to calculate the neutrino
four-vectors by giving it a jigsaw rule

Initialize it to prepare to analyze events



RestFrames Code

If you draw your tree with the jigsaw rule added it will now look like this:





RestFrames Code

Now we're ready to analyze events!

```
B1->SetLabFrameFourVector(Bjet1);  
B2->SetLabFrameFourVector(Bjet2);  
L1->SetLabFrameFourVector(Lep1);  
L2->SetLabFrameFourVector(Lep2);  
LAB->AnalyzeEvent(MET);
```

For each event we set the lab-frame four-momenta of all the visible states in the tree that we have measured and give it the MET to analyze the event

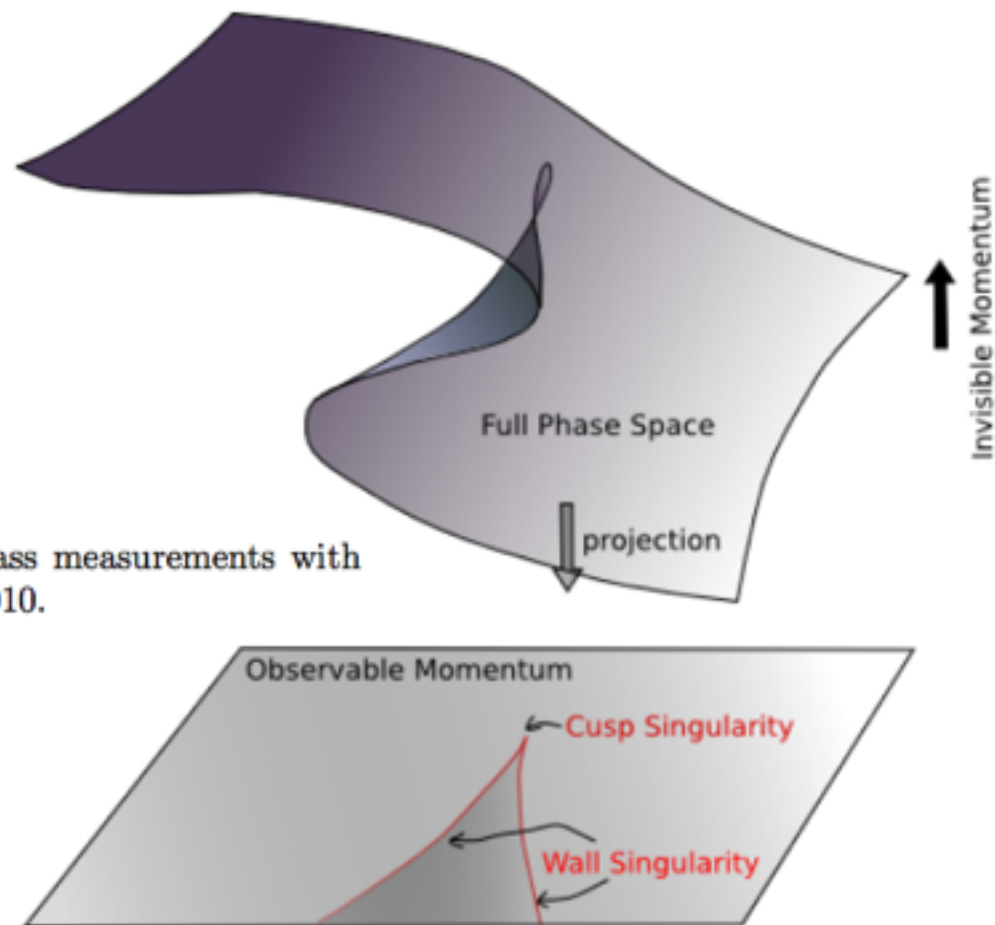
```
// retrieve a mass  
double Mtop1 = T1->GetMass();  
// get a polar decay angle  
double costhetaW2 = W2->GetCostheta();  
// get the azimuthal angle between two decay planes  
double dphi_T1_T2 = T1->GetDeltaPhi(T2);  
// evaluate an energy in a particular reference frame  
double E_L2_W2frame = L2->GetEnergy(W2);  
// get a four-vector in a particular reference frame  
TLorentzVector B1_T1frame = B1->GetFourVector(T1);
```

Finally, we can retrieve any observables of interest from the tree and its associated reference frames



Singularity variables

Kinematic Singularities. A singularity is a point where the local tangent space cannot be defined as a plane, or has a different dimension than the tangent spaces at non-singular points.



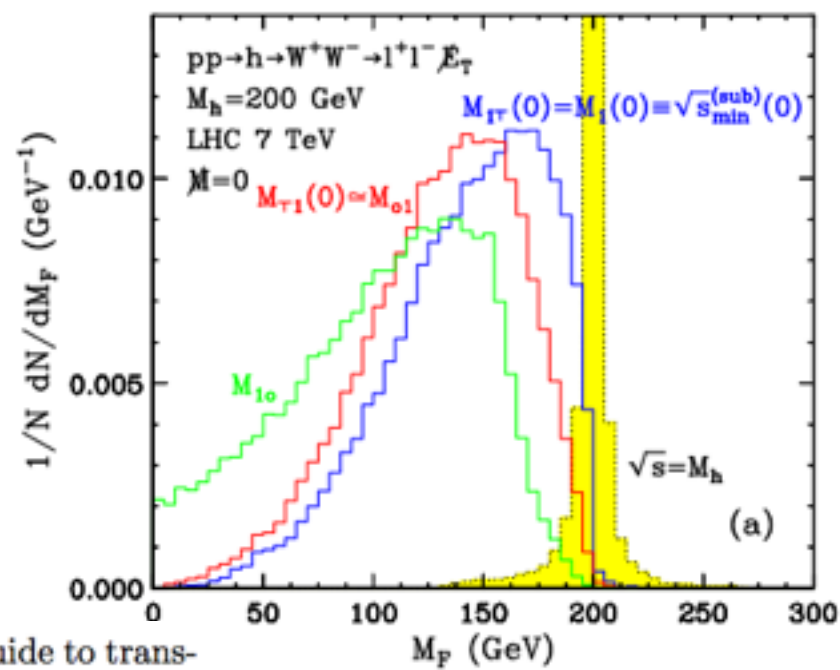
From:

Ian-Woo Kim. Algebraic singularity method for mass measurements with missing energy. *Phys. Rev. Lett.*, 104:081601, Feb 2010.



Singularity variables

The guiding principle we employ for creating useful hadron-collider event variables, is that: *we should place the best possible bounds on any Lorentz invariants of interest, such as parent masses or the center-of-mass energy $\hat{s}^{1/2}$, in any cases where it is not possible to determine the actual values of those Lorentz invariants due to incomplete event information.*



From:

A.J. Barr, T.J. Khoo, P. Konar, K. Kong, C.G. Lester, et al. Guide to transverse projections and mass-constraining variables. *Phys.Rev.*, D84:095031, 2011.

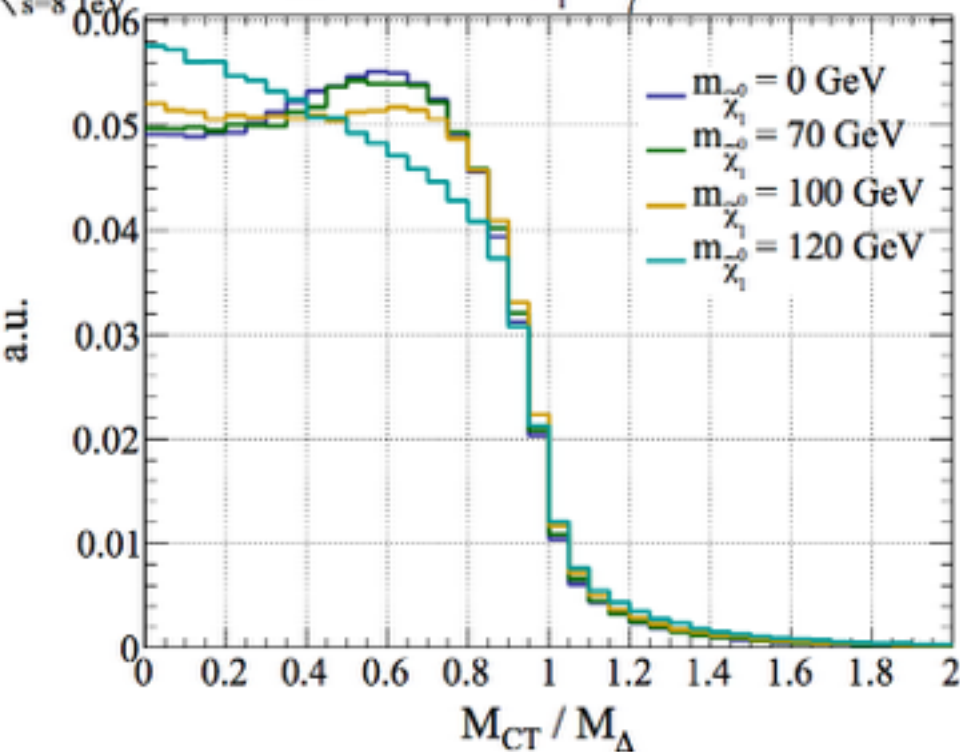


Example: M_{CT}

MadGraph+PGS

$\sqrt{s}=8\text{ TeV}$

$pp \rightarrow \tilde{l}\tilde{l}; \tilde{l} \rightarrow l\tilde{\chi}_1^0; m_{\tilde{l}} = 150\text{ GeV}$



assuming $\sim$ mass-less leptons

$$M_{CT}^2 = 2 \left(p_T^{\ell_1} p_T^{\ell_2} + \vec{p}_T^{\ell_1} \cdot \vec{p}_T^{\ell_2} \right)$$

Constructed to have a kinematic endpoint at:

$$M_{CT}^{\max} = \frac{m_{\tilde{\ell}}^2 - m_{\tilde{\chi}_1^0}^2}{m_{\tilde{\chi}_1^0}} = M_{\Delta}$$

From:

Daniel R. Tovey. On measuring the masses of pair-produced semi-invisibly decaying particles at hadron colliders. *JHEP*, 0804:034, 2008.

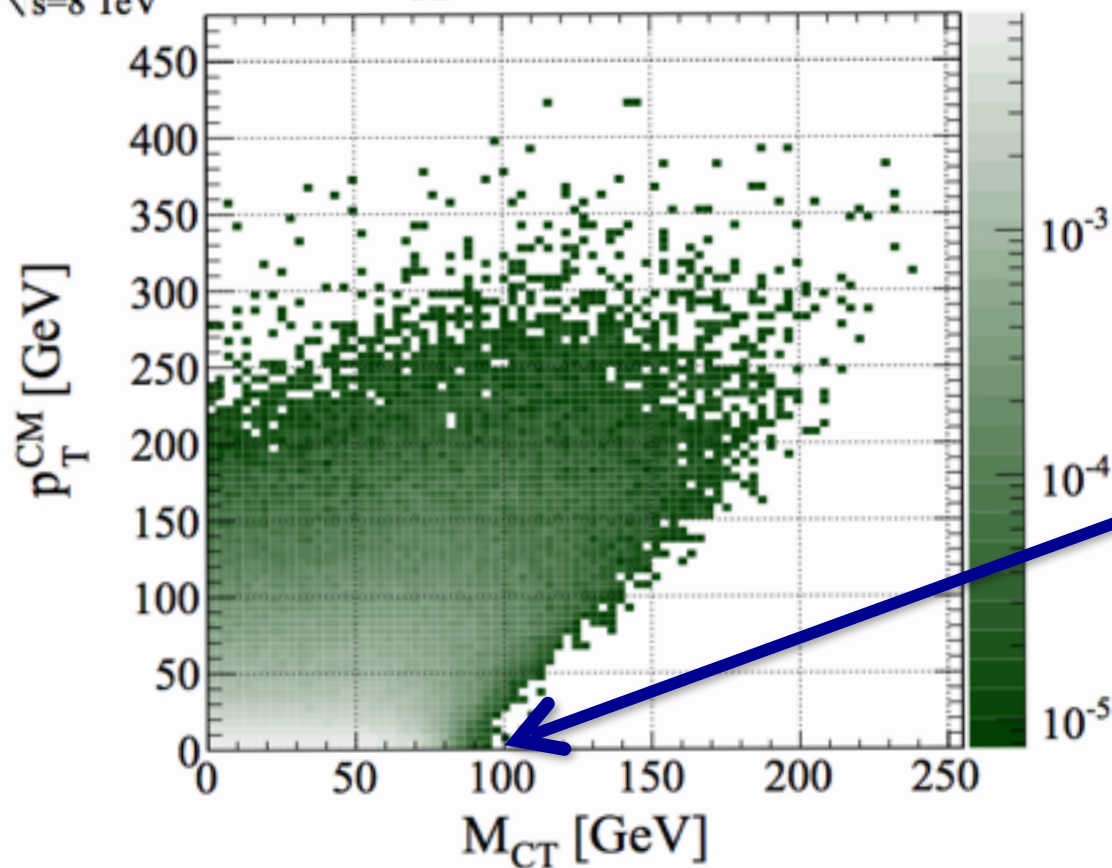


M_{CT} in practice

Singularity variables (like M_{CT}) can be sensitive to quantities that can vary dramatically event-by-event

MadGraph+PGS
 $\sqrt{s}=8$ TeV

$pp \rightarrow W^+ W^-; W^\pm \rightarrow l^\pm \nu$



Kinematic endpoint
'moves' with nonzero
CM system p_T



The mass challenge

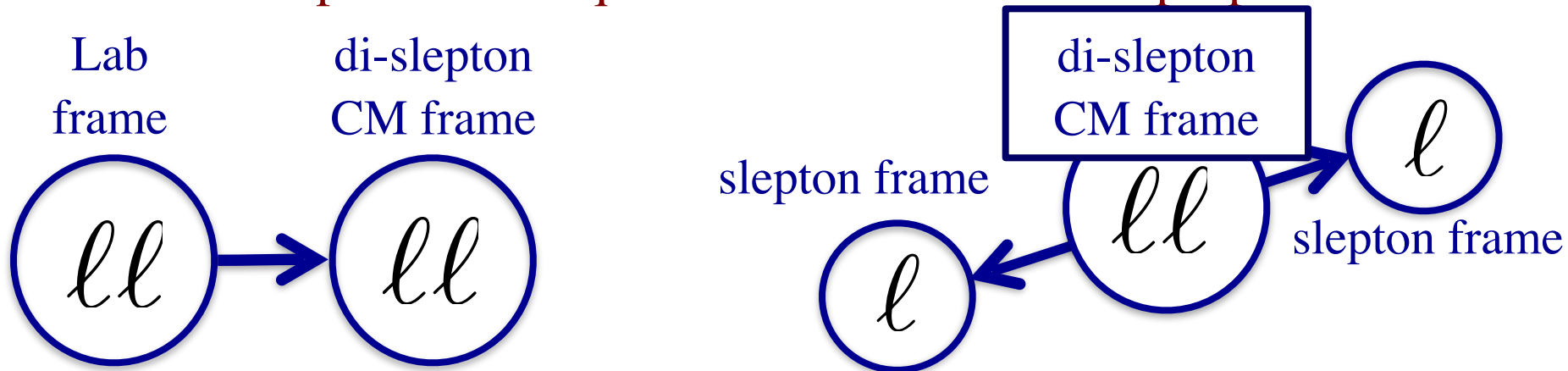
The invariant mass is invariant under coherent Lorentz transformations of two particles

$$m_{inv}^2(p_1, p_2) = m_1^2 + m_2^2 + 2(E_1 E_2 - \vec{p}_1 \cdot \vec{p}_2)$$

The Euclidean mass (or contra-variant mass) is invariant under anti-symmetric Lorentz transformations of two particles

$$m_{euc}^2(p_1, p_2) = m_1^2 + m_2^2 + 2(E_1 E_2 + \vec{p}_1 \cdot \vec{p}_2)$$

Even the simplest case requires variables with both properties!





Correcting for CM p_T

- Want to boost from lab-frame to CM-frame
- We know the transverse momentum of the CM-frame:

$$\vec{p}_T^{CM} = \vec{p}_T^{\ell_1} + \vec{p}_T^{\ell_2} + \vec{E}_T^{\text{miss}}$$

- But we don't know the energy, **or mass**, of the CM-frame:

$$\vec{\beta}_{lab \rightarrow CM} = \frac{\vec{p}_T^{CM}}{\sqrt{|\vec{p}_T^{CM}|^2 + \hat{s}}}$$



p_T corrections for M_{CT}

Attempts have been made to mitigate this problem:

(i) ‘Guess’ the lab $\rightarrow$ CM frame boost:

$$M_{CT(\text{corr})} = \begin{cases} M_{CT} & \text{after boosting by } \beta = p_b/E_{\text{cm}} & \text{if } A_{x(\text{lab})} \geq 0 \text{ or } A'_{x(\text{lo})} \geq 0 \\ M_{CT} & \text{after boosting by } \beta = p_b/\hat{E} & \text{if } A'_{x(\text{hi})} < 0 \\ M_{Cy} & & \text{if } A'_{x(\text{hi})} \geq 0 \end{cases}$$

x – parallel to boost

y – perp. to boost

with:

$$\begin{aligned} A_x &= p_x[q_1]E_y[q_2] + p_x[q_2]E_y[q_1] \\ M_{Cy}^2 &= (E_y[q_1] + E_y[q_2])^2 - (p_y[q_1] - p_y[q_2])^2 \end{aligned}$$

Giacomo Polesello and Daniel R. Tovey. Supersymmetric particle mass measurement with the boost-corrected contranverse mass. *JHEP*, 1003:030, 2010.

(ii) Only look at event along axis perpendicular to boost:

$$M_{CT\perp}$$

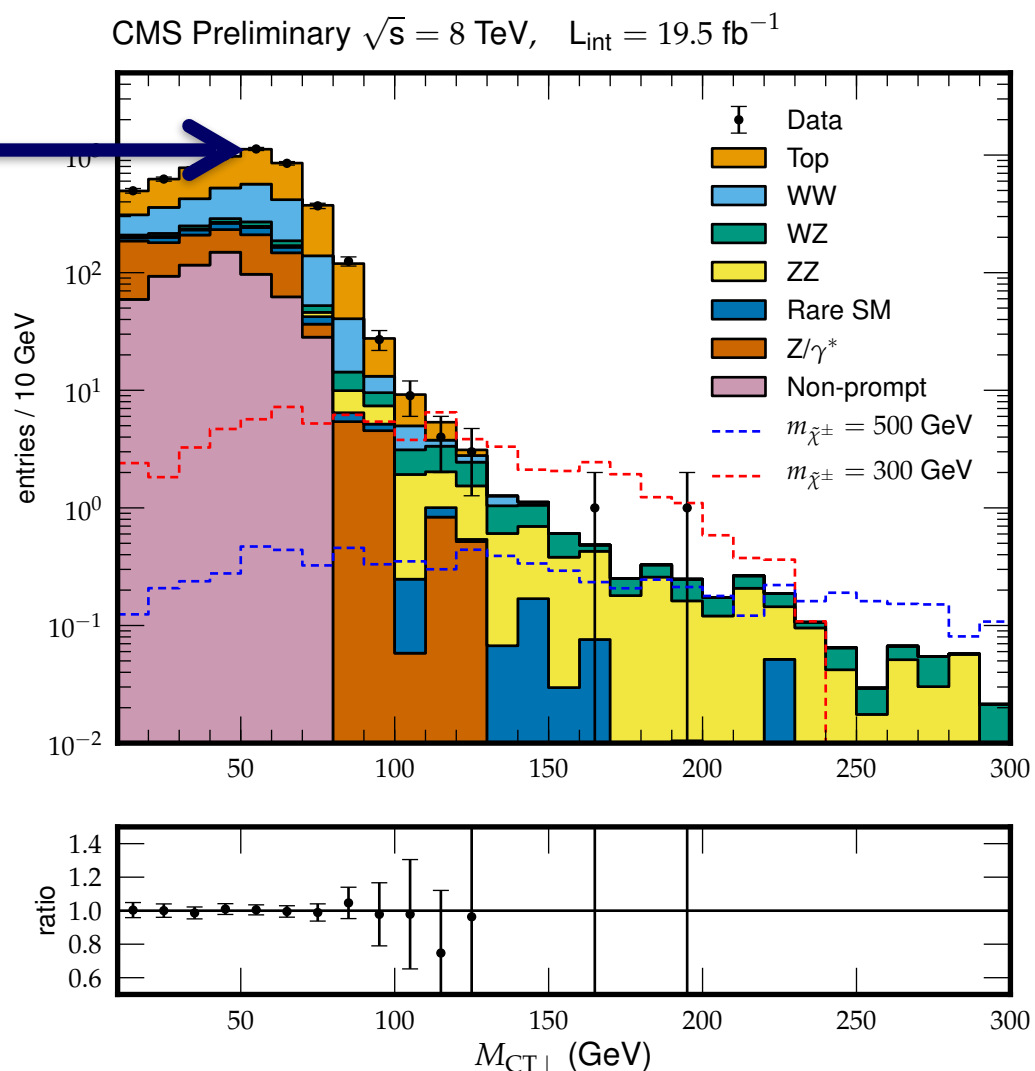
Konstantin T. Matchev and Myeonghun Park. A General method for determining the masses of semi-invisibly decaying particles at hadron colliders. *Phys.Rev.Lett.*, 107:061801, 2011.



$M_{CT\perp}$ in practice

‘peak position’ of signal and backgrounds due to other cuts (p_T , MET) and only weakly sensitive to sparticle masses

From:
CMS-SUS-PAS-13-006

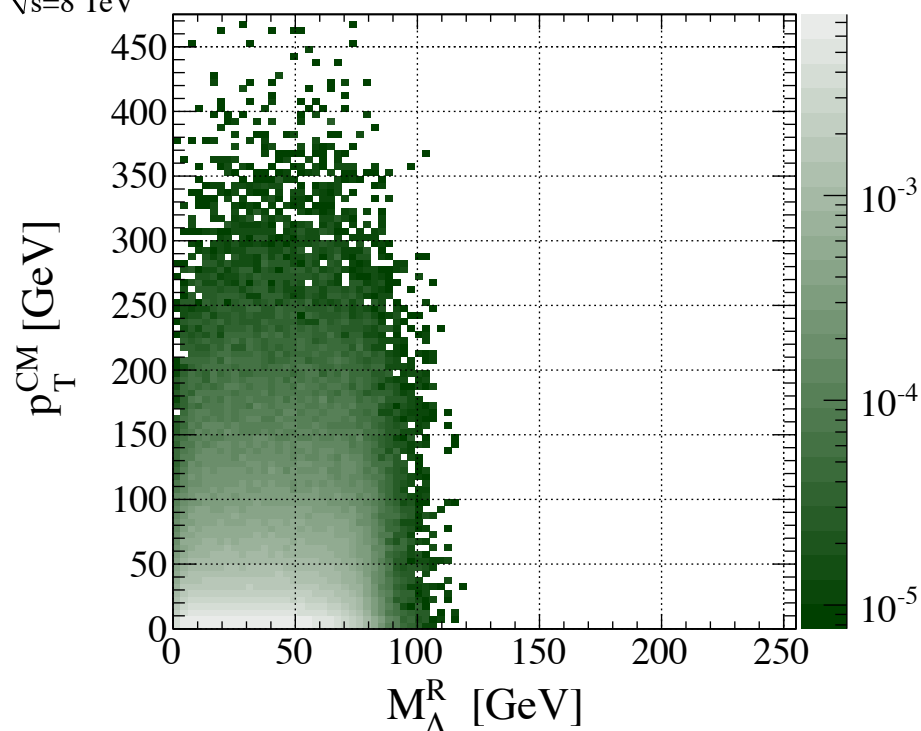




Recursive rest-frame reconstruction

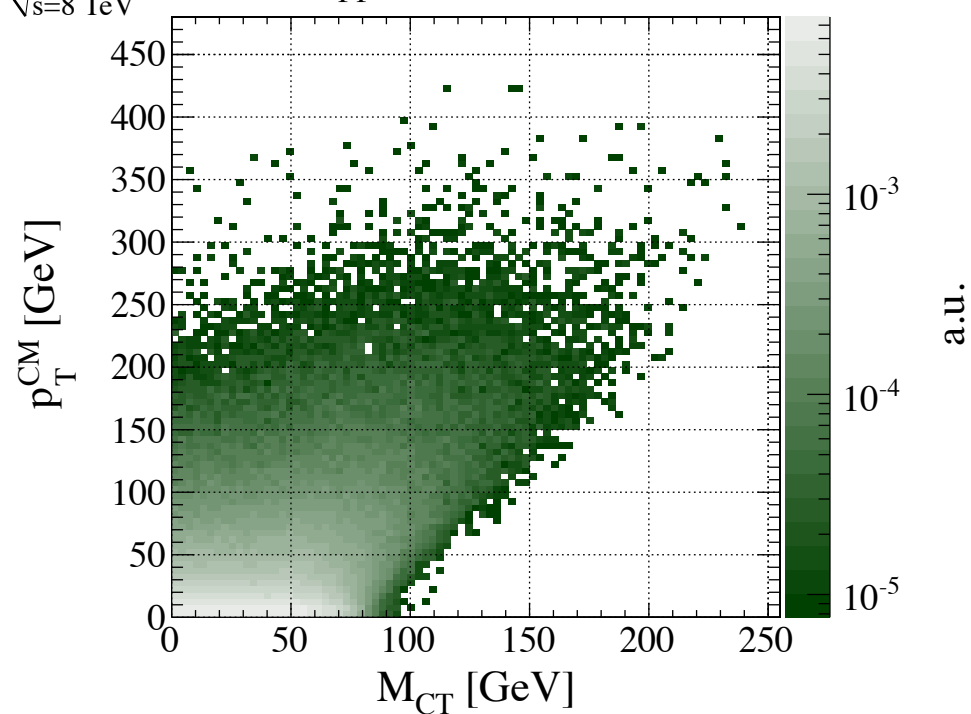
MadGraph+PGS
 $\sqrt{s}=8$ TeV

$pp \rightarrow W^+W^-; W^\pm \rightarrow l^\pm \nu$



MadGraph+PGS
 $\sqrt{s}=8$ TeV

$pp \rightarrow W^+W^-; W^\pm \rightarrow l^\pm \nu$

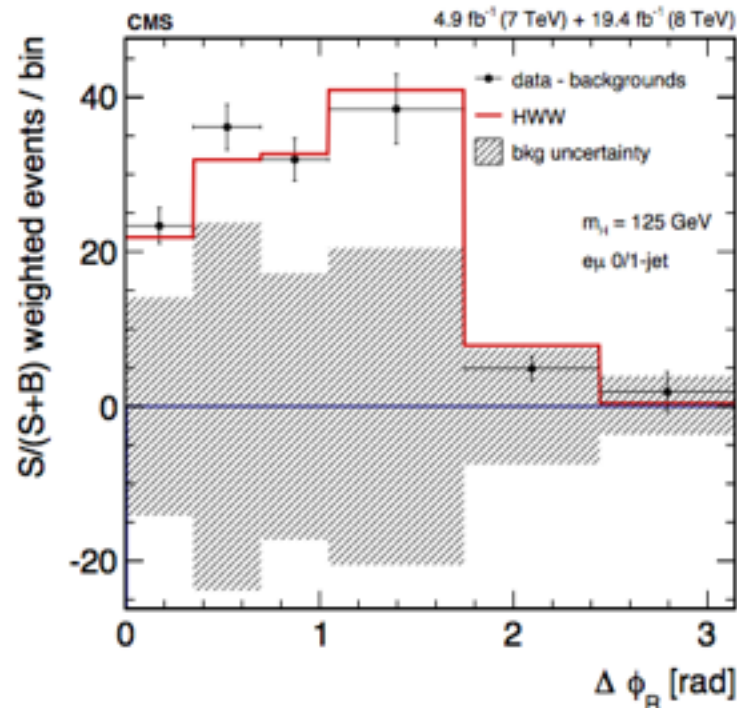


M_Δ^R is a singularity variable – in fact it is essentially identical to M_{CT} but evaluated *in a different reference frame*. Boost procedure ensures that new variable is invariant under the previous transformations



Resonant Higgs production

$$H \rightarrow WW^* \rightarrow 2\ell 2\nu$$



The $\Delta\phi$ between the leptons is evaluated in the R -frame, removing dependence on the p_T of the Higgs and correlation with $\sqrt{\hat{s}_R}$

CMS uses 2D fit of variables to measure Higgs mass in this channel

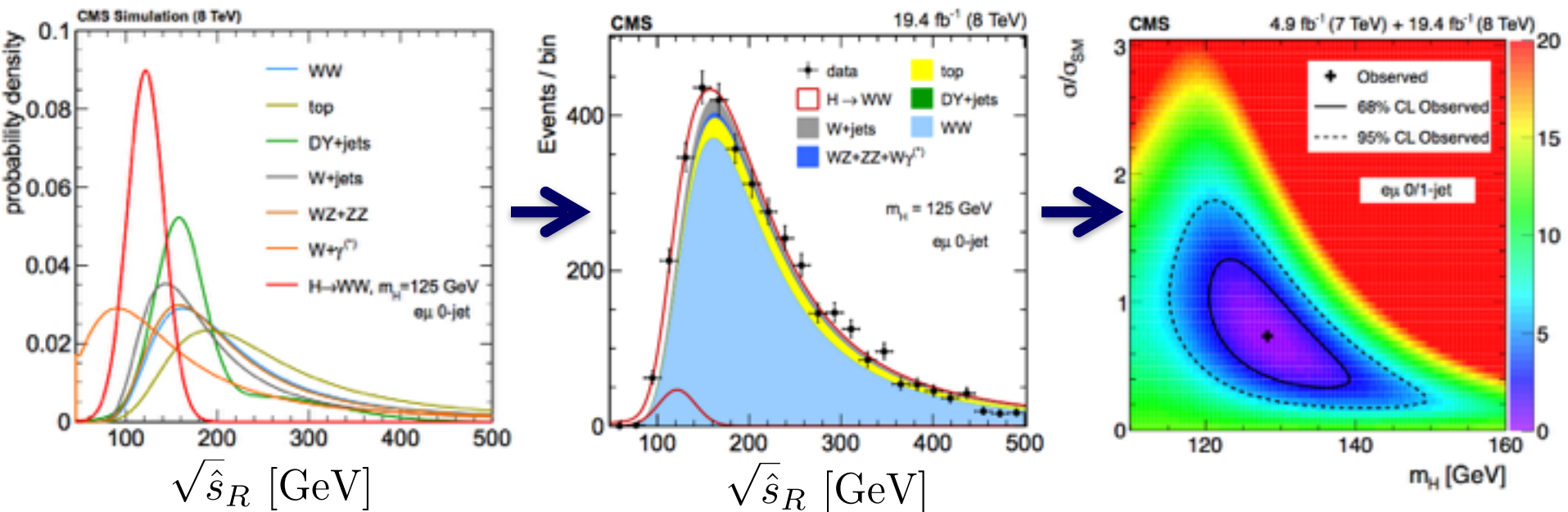
From:

CMS Collaboration, *Measurement of Higgs boson production and properties in the WW decay channel with leptonic final states*, arXiv:1312.1129v1 [hep-ex]



Resonant Higgs production

$$H \rightarrow WW^* \rightarrow 2\ell 2\nu$$



The shape of the $\sqrt{\hat{s}_R}$ distribution, for the Higgs signal and backgrounds, is used to extract both the Higgs mass and signal strength – even while information is lost with the two escaping neutrinos

From:

CMS Collaboration, *Measurement of Higgs boson production and properties in the WW decay channel with leptonic final states*, arXiv:1312.1129v1 [hep-ex]

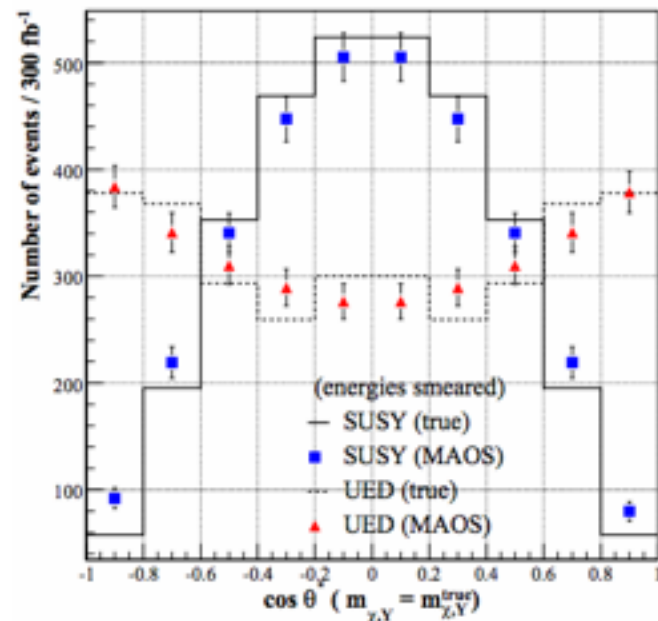


What other info can we extract?

Ex. M_{T2} extremization assigns values to missing degrees of freedom – if one takes these assignments literally, can we calculate other useful variables?

From:

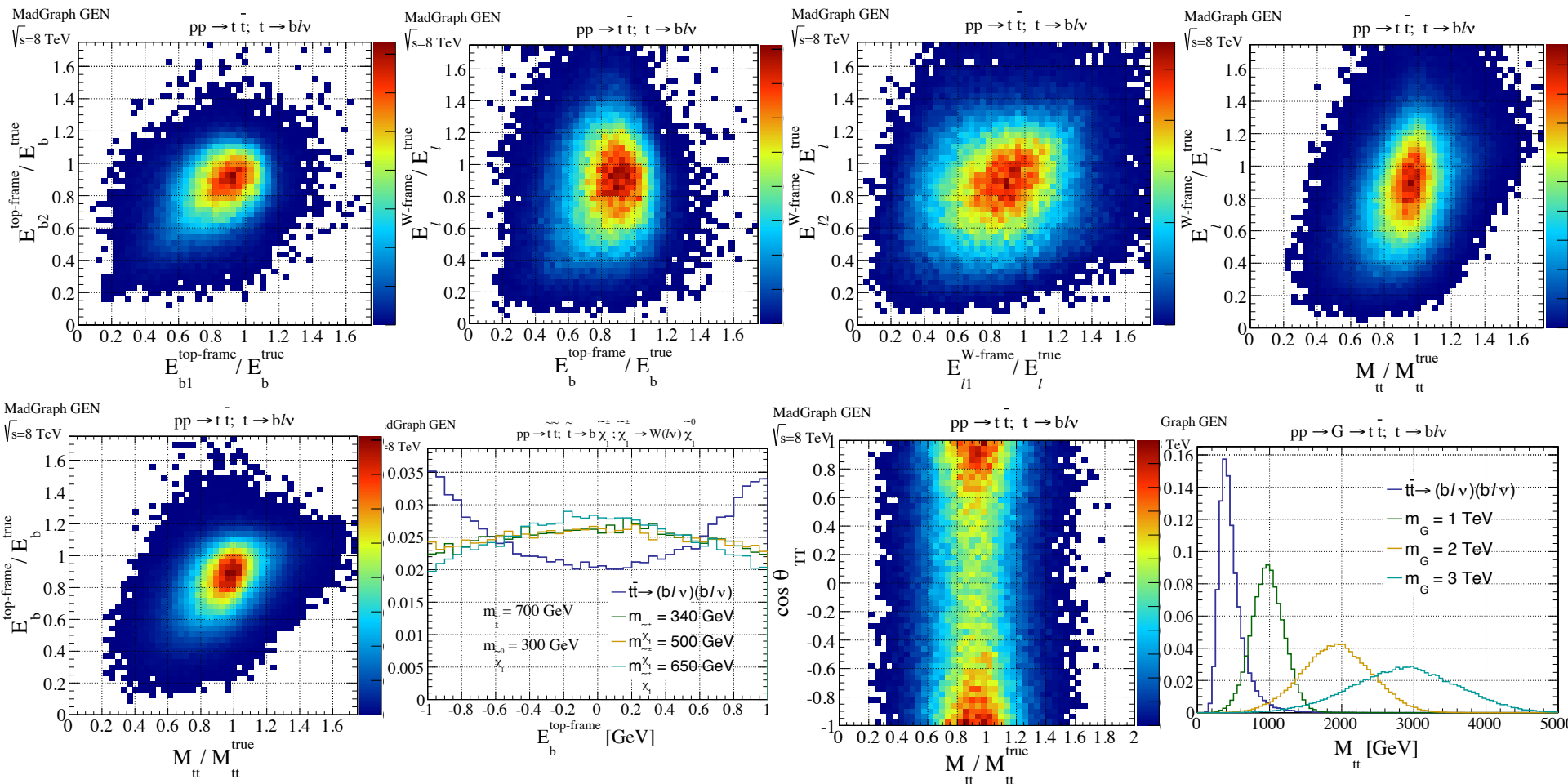
Mass and Spin Measurement with $M(T2)$ and MAOS Momentum - Cho, Won Sang et al.
Nucl.Phys.Proc.Suppl. 200-202 (2010) 103-112 arXiv:0909.4853 [hep-ph]



When we assign unconstrained d.o.f. by extremizing one quantity, what are the general properties of other variables we calculate? What are the correlations among them?



Example: the di-leptonic top basis



A rich basis of useful Recursive Jigsaw observables can be calculated, each with largely independent information

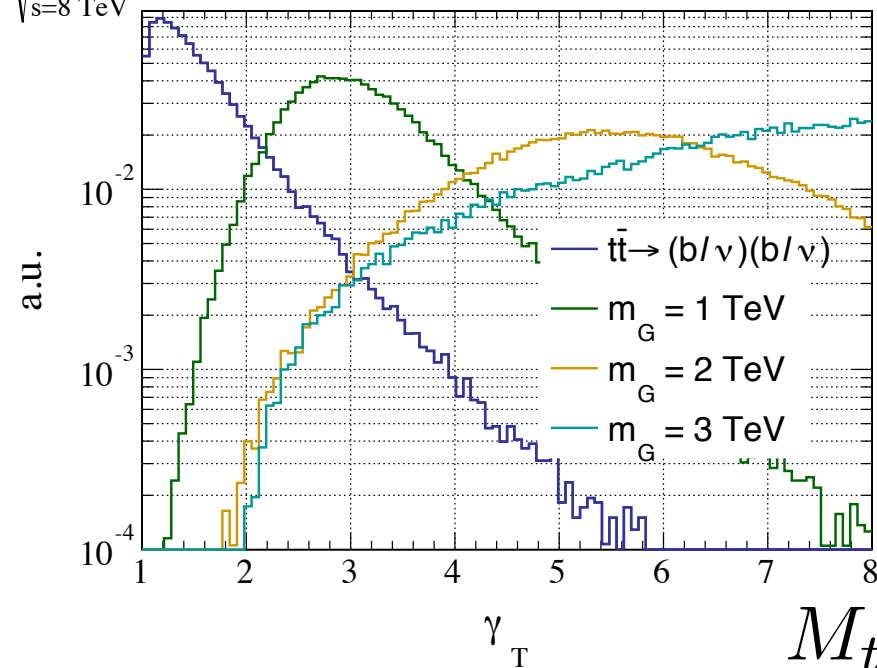


The di-leptonic top basis vs. stops

MadGraph GEN

$\sqrt{s}=8$ TeV

$pp \rightarrow G \rightarrow t \bar{t}; t \rightarrow b/\nu$

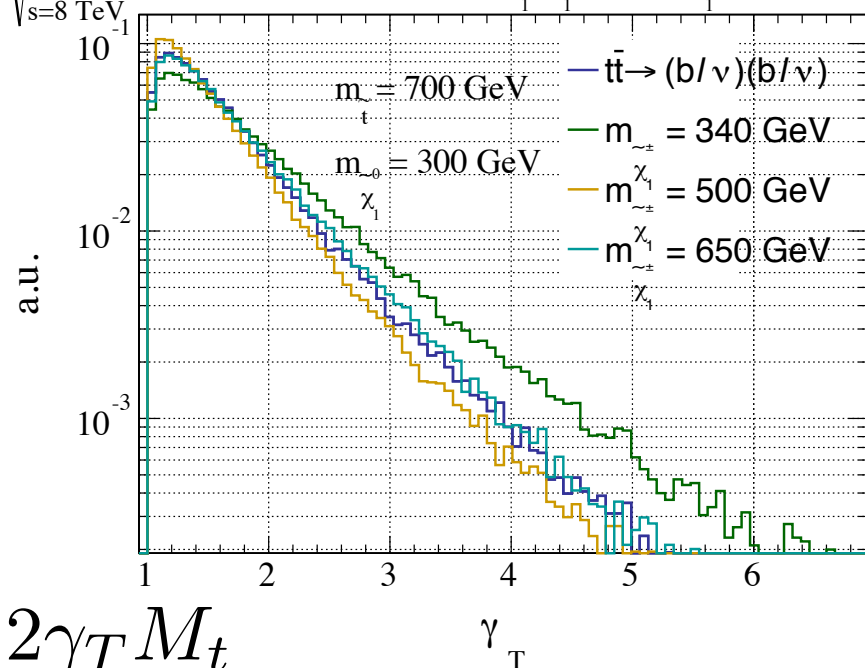


$$M_{tt} = 2\gamma_T M_t$$

MadGraph GEN

$\sqrt{s}=8$ TeV

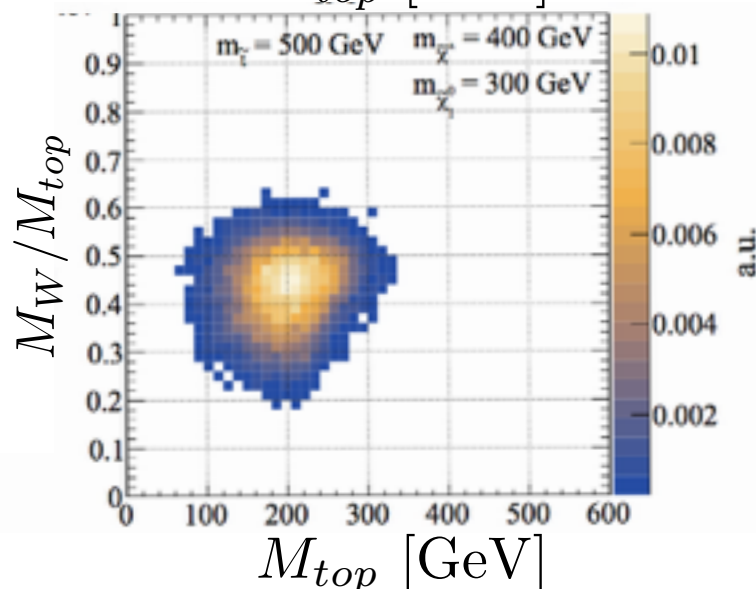
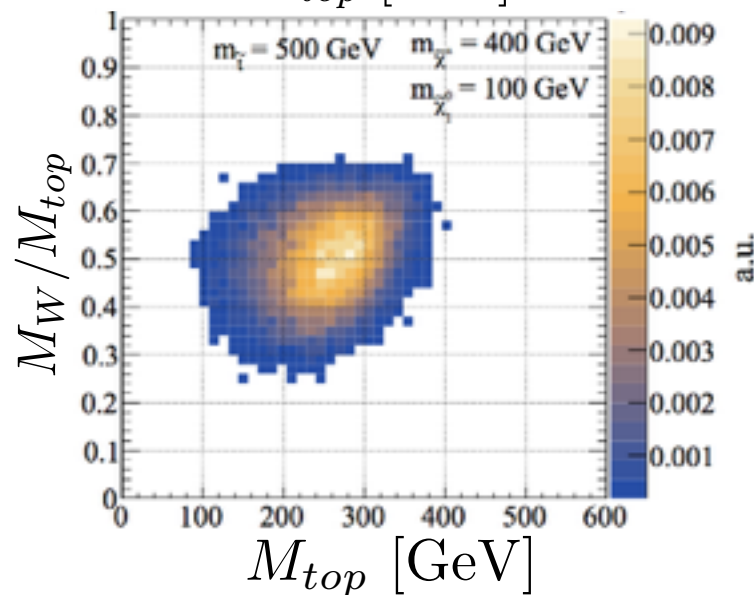
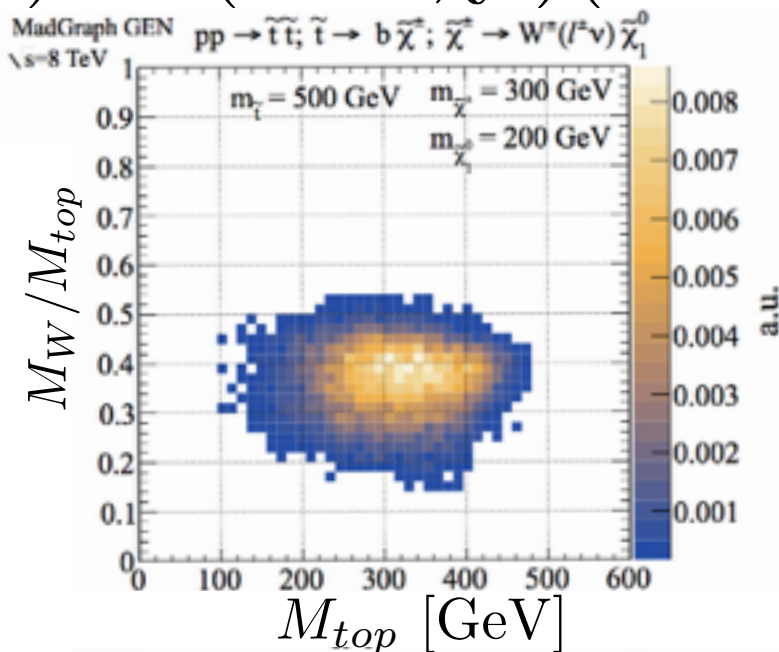
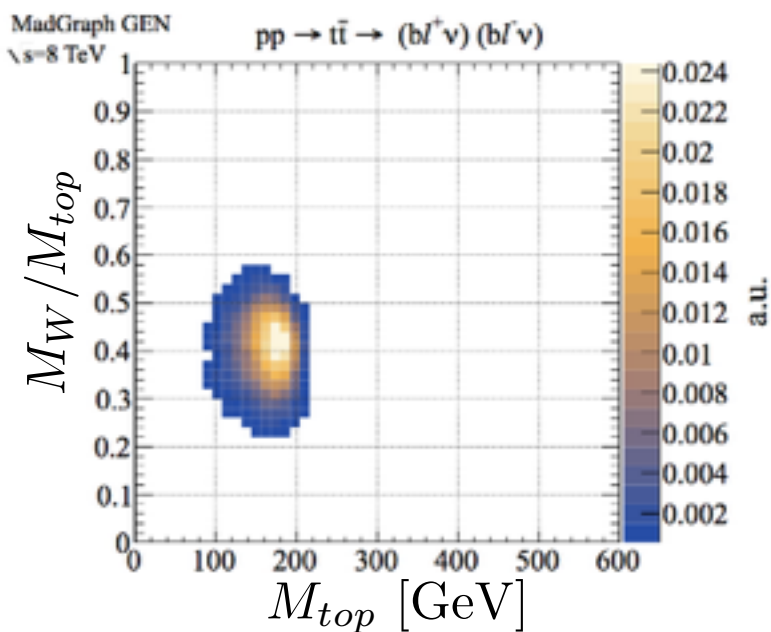
$pp \rightarrow t \bar{t}; t \rightarrow b \tilde{\chi}_1^\pm; \tilde{\chi}_1^\pm \rightarrow W(l\nu) \tilde{\chi}_1^0$



Observables sensitive to intermediate resonances cannot distinguish between non-resonant signals and background



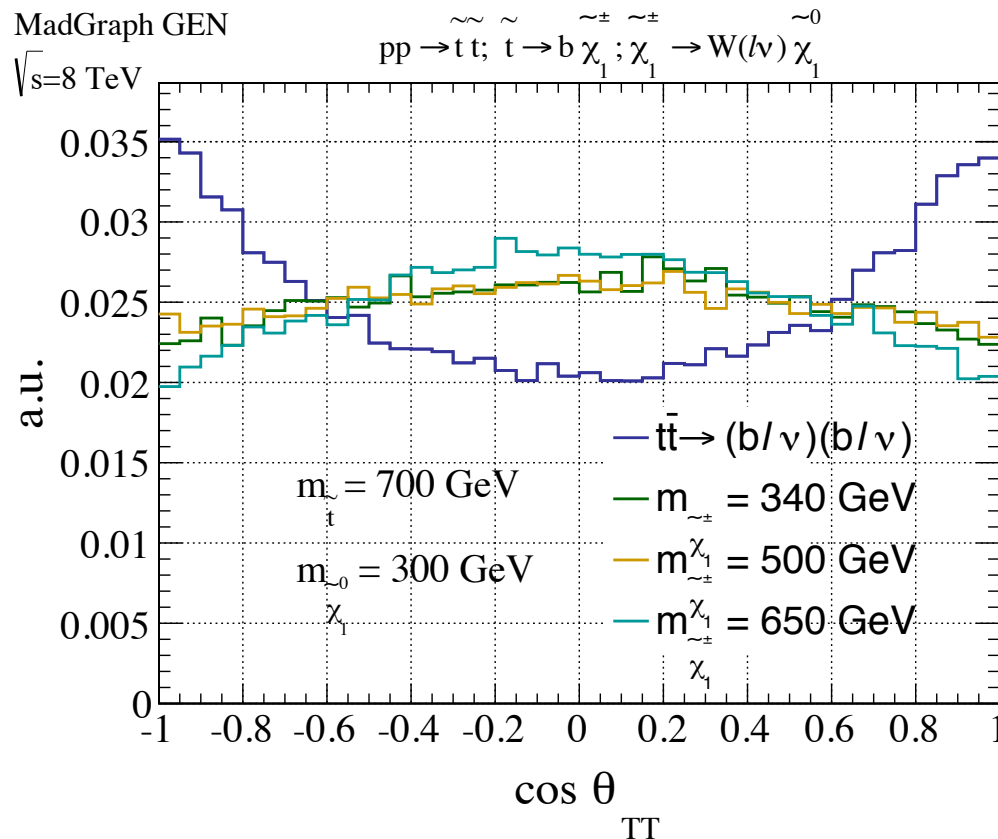
$$t\bar{t} \rightarrow (b\ell\nu)(b\ell\nu) \text{ vs. } t\bar{t} \rightarrow (b\tilde{\chi}^\pm)(b\tilde{\chi}^\pm) \rightarrow (b\ell^\pm\nu\tilde{\chi}^0)(b\ell^\pm\nu\tilde{\chi}^0)$$



Multi-dimensional bump-hunting



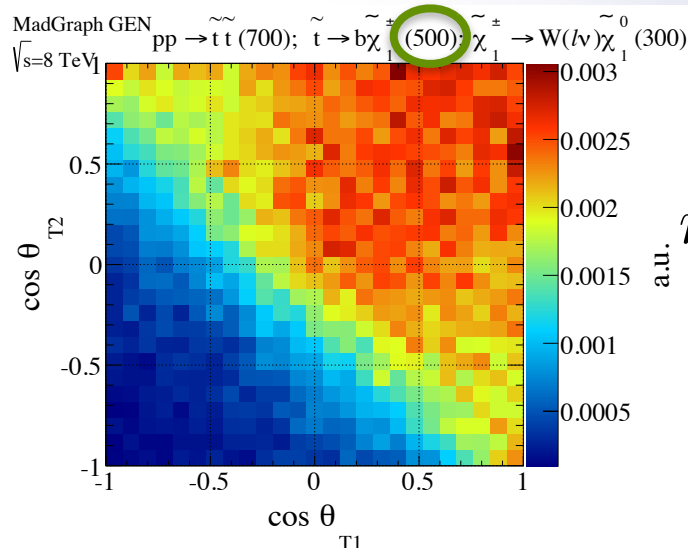
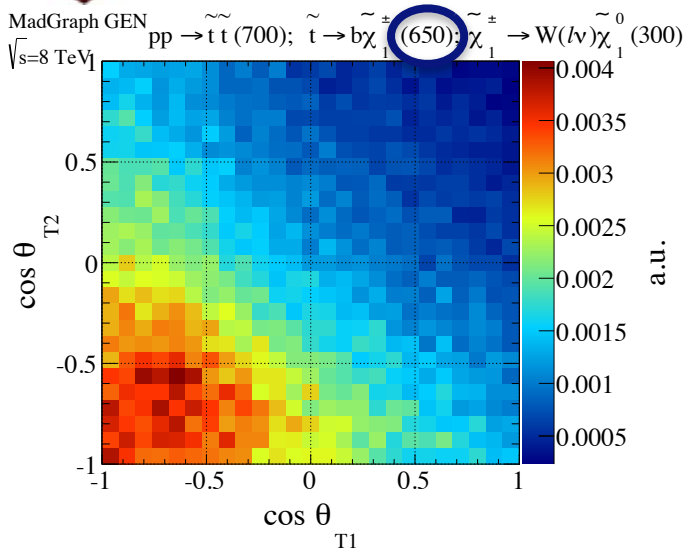
The di-leptonic top basis vs. stops



Decay angles are also sensitive to differences
 between stop signals and $t\bar{t}$ background



The di-leptonic top basis vs. stops



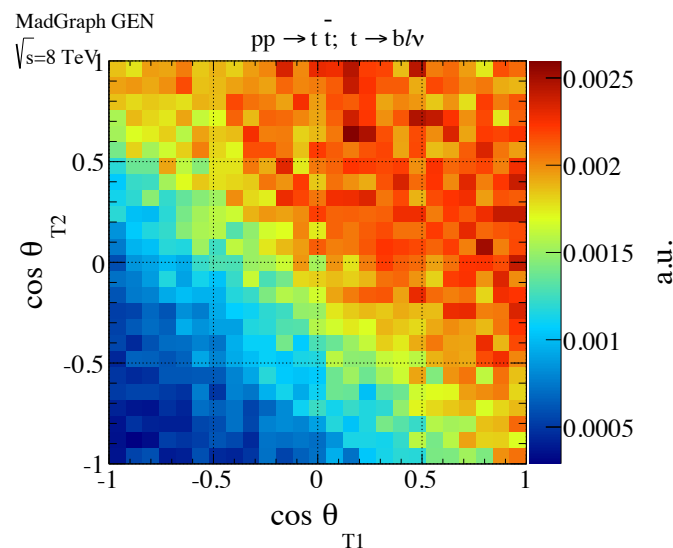
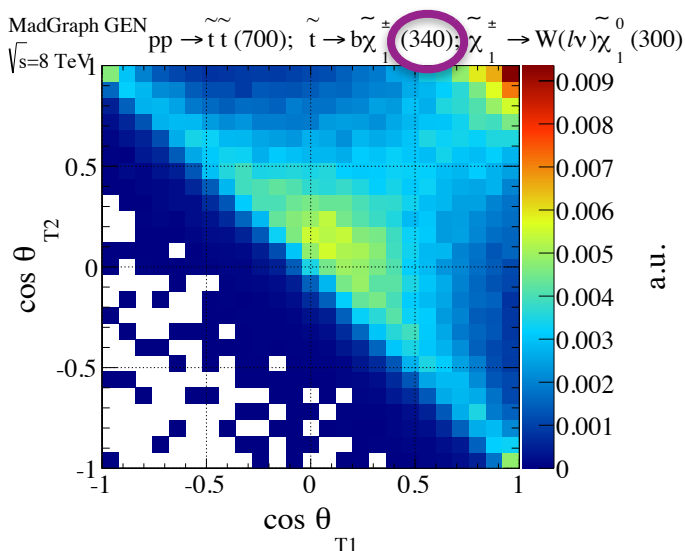
$$m_{\tilde{t}} = 700 \text{ GeV}$$

650

$$m_{\tilde{\chi}^\pm} = 500 \text{ GeV}$$

340

$$m_{\tilde{\chi}^0} = 300 \text{ GeV}$$

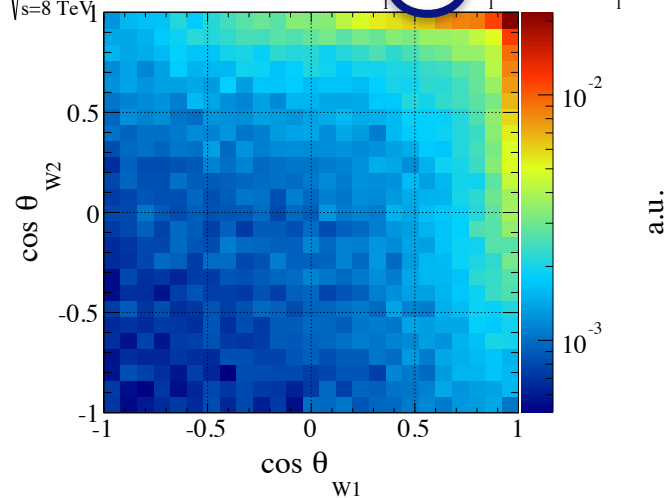


Decay angles are also sensitive to differences between stop signals and $t\bar{t}$ background

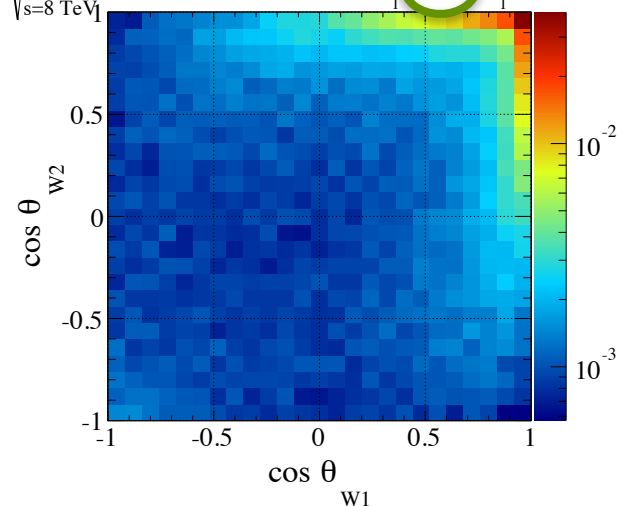


The di-leptonic top basis vs. stops

MadGraph GEN
 $\sqrt{s}=8 \text{ TeV}$
 $pp \rightarrow \tilde{t} \tilde{t}^* (700); \tilde{t} \rightarrow b \tilde{\chi}_1^\pm (650); \tilde{\chi}_1^\pm \rightarrow W(l\nu) \tilde{\chi}_1^0 (300)$



MadGraph GEN
 $\sqrt{s}=8 \text{ TeV}$
 $pp \rightarrow \tilde{t} \tilde{t}^* (700); \tilde{t} \rightarrow b \tilde{\chi}_1^\pm (500); \tilde{\chi}_1^\pm \rightarrow W(l\nu) \tilde{\chi}_1^0 (300)$



$$m_{\tilde{t}} = 700 \text{ GeV}$$

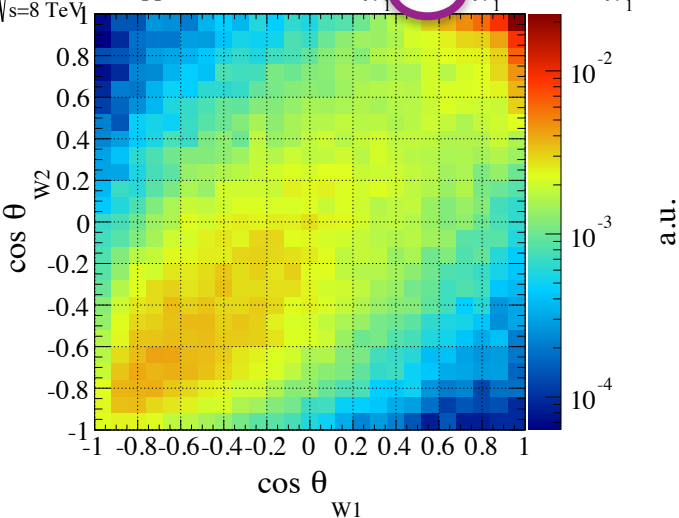
$$650$$

$$m_{\tilde{\chi}^\pm} = 500 \text{ GeV}$$

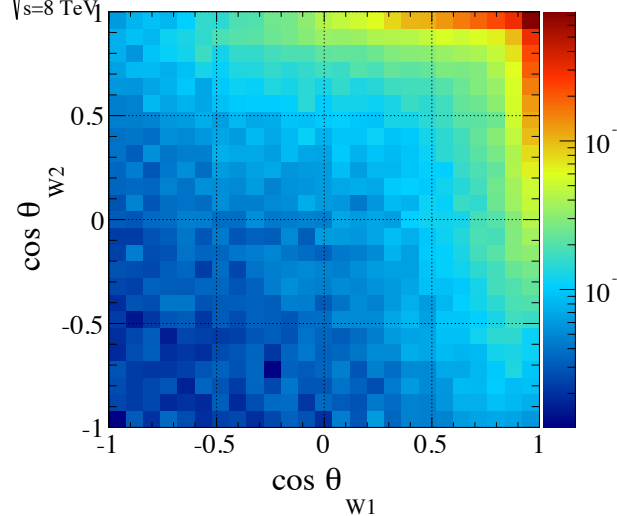
$$340$$

$$m_{\tilde{\chi}^0} = 300 \text{ GeV}$$

MadGraph GEN
 $\sqrt{s}=8 \text{ TeV}$
 $pp \rightarrow \tilde{t} \tilde{t}^* (700); \tilde{t} \rightarrow b \tilde{\chi}_1^\pm (340); \tilde{\chi}_1^\pm \rightarrow W(l\nu) \tilde{\chi}_1^0 (300)$



MadGraph GEN
 $\sqrt{s}=8 \text{ TeV}$
 $pp \rightarrow \tilde{t} \tilde{t}^*; \tilde{t} \rightarrow b/l\nu$

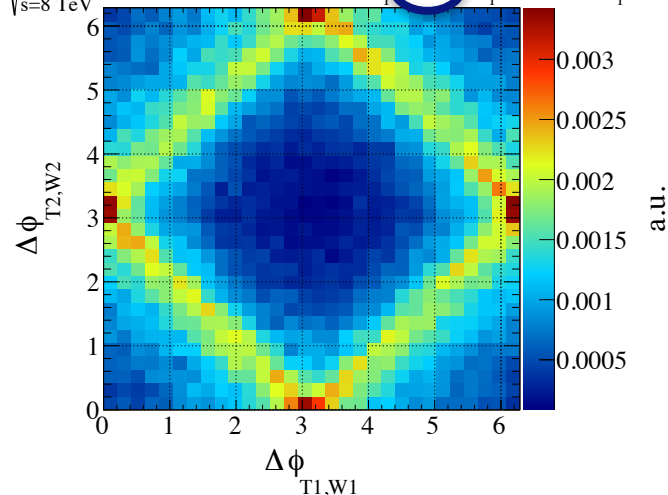


Decay angles are
also sensitive to
differences
between stop
signals and ttbar
background

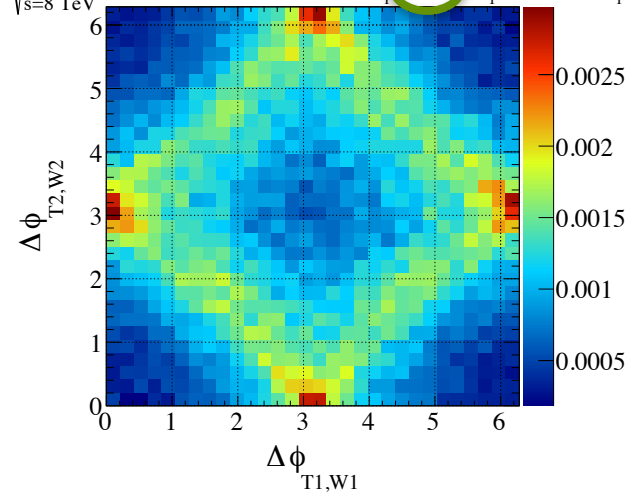


The di-leptonic top basis vs. stops

MadGraph GEN
 $\sqrt{s}=8$ TeV
 $pp \rightarrow \tilde{t}\tilde{t}^* (700); \tilde{t} \rightarrow b\tilde{\chi}_1^\pm; \tilde{\chi}_1^\pm \rightarrow W(l\nu)\tilde{\chi}_1^0 (300)$



MadGraph GEN
 $\sqrt{s}=8$ TeV
 $pp \rightarrow \tilde{t}\tilde{t}^* (700); \tilde{t} \rightarrow b\tilde{\chi}_1^\pm; \tilde{\chi}_1^\pm \rightarrow W(l\nu)\tilde{\chi}_1^0 (300)$



$$m_{\tilde{t}} = 700 \text{ GeV}$$

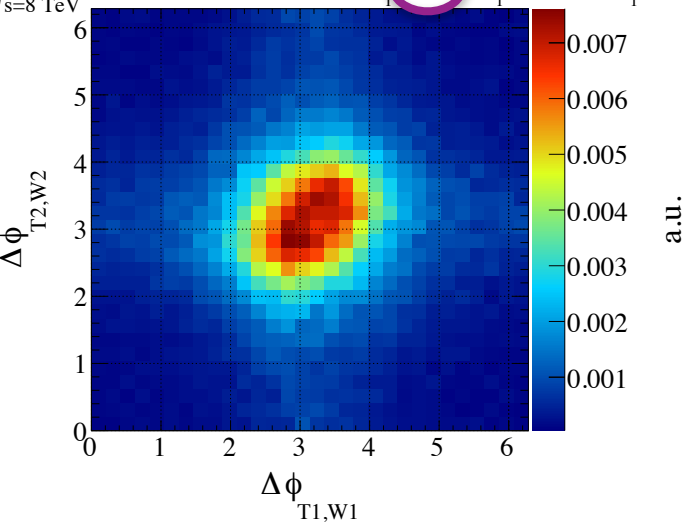
$$650$$

$$m_{\tilde{\chi}^\pm} = 500 \text{ GeV}$$

$$340$$

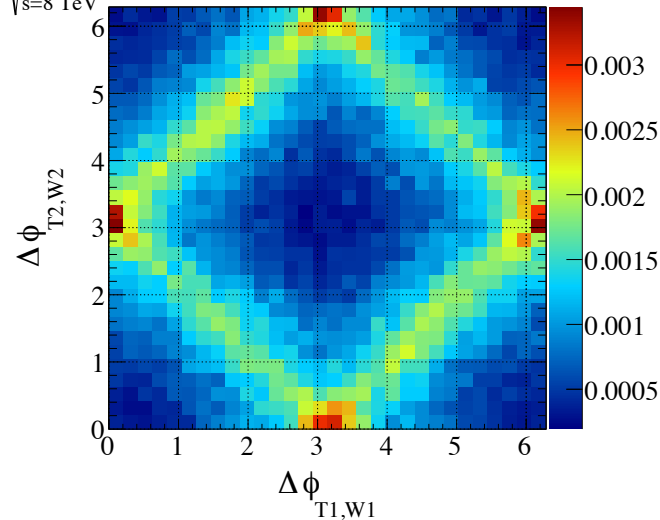
$$m_{\tilde{\chi}^0} = 300 \text{ GeV}$$

MadGraph GEN
 $\sqrt{s}=8$ TeV
 $pp \rightarrow \tilde{t}\tilde{t}^* (700); \tilde{t} \rightarrow b\tilde{\chi}_1^\pm; \tilde{\chi}_1^\pm \rightarrow W(l\nu)\tilde{\chi}_1^0 (300)$



MadGraph GEN

$\sqrt{s}=8$ TeV
 $pp \rightarrow \tilde{t}\tilde{t}^*; \tilde{t} \rightarrow b l \nu$

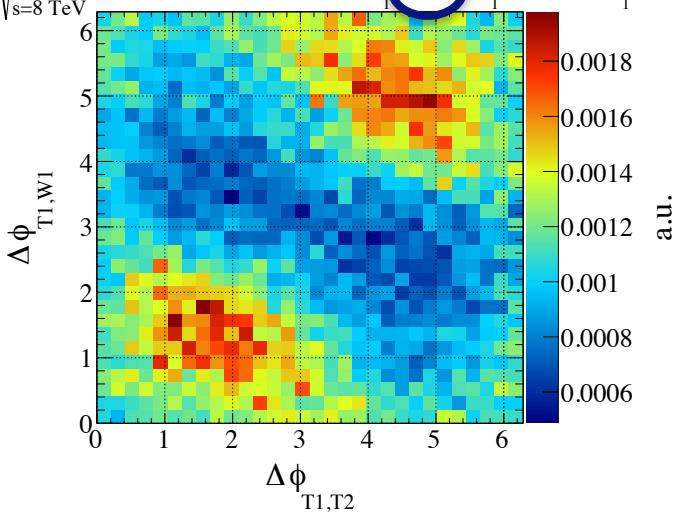


Here, the azimuthal angle between the the top and W decay planes $\Delta\phi_{T1,W1}$, for each of the two decay chains

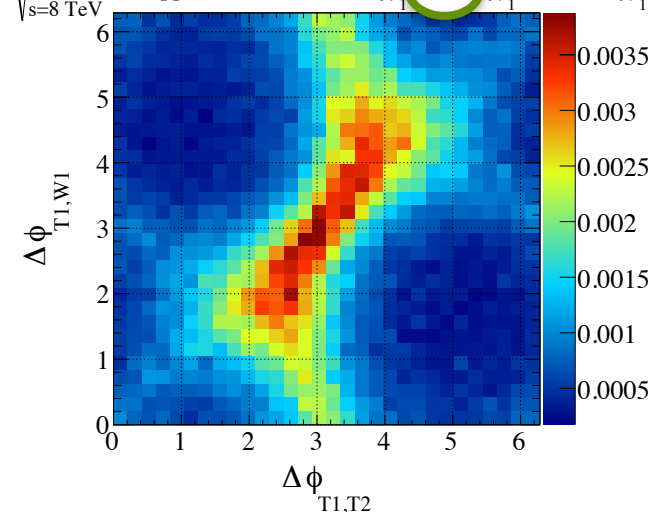


The di-leptonic top basis vs. stops

MadGraph GEN $\sqrt{s}=8$ TeV $pp \rightarrow \tilde{t}\tilde{t}^* (700); \tilde{t} \rightarrow b\tilde{\chi}_1^\pm (650); \tilde{\chi}_1^\pm \rightarrow W(l\nu)\tilde{\chi}_1^0 (300)$



MadGraph GEN $\sqrt{s}=8$ TeV $pp \rightarrow \tilde{t}\tilde{t}^* (700); \tilde{t} \rightarrow b\tilde{\chi}_1^\pm (500); \tilde{\chi}_1^\pm \rightarrow W(l\nu)\tilde{\chi}_1^0 (300)$



$$m_{\tilde{t}} = 700 \text{ GeV}$$

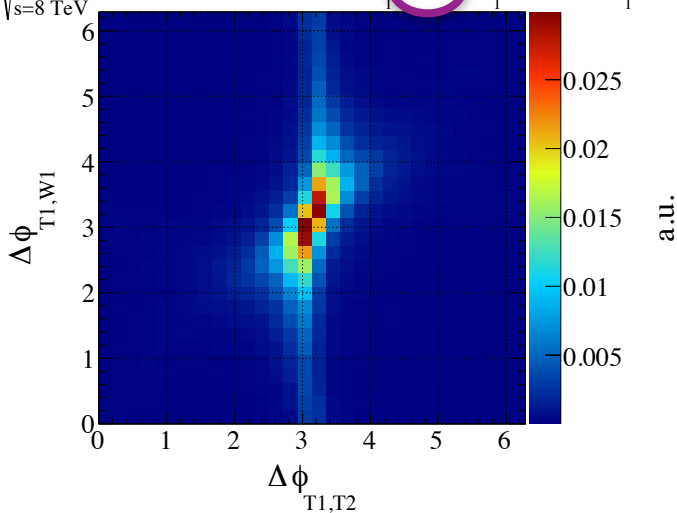
$$650$$

$$m_{\tilde{\chi}^\pm} = 500 \text{ GeV}$$

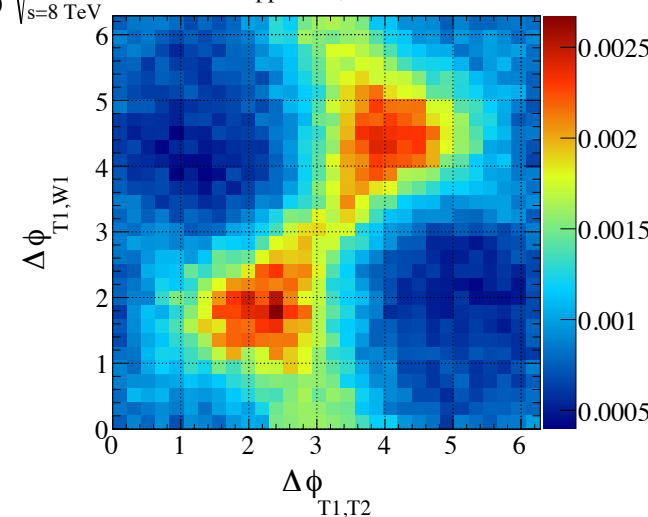
$$340$$

$$m_{\tilde{\chi}^0} = 300 \text{ GeV}$$

MadGraph GEN $\sqrt{s}=8$ TeV $pp \rightarrow \tilde{t}\tilde{t}^* (700); \tilde{t} \rightarrow b\tilde{\chi}_1^\pm (340); \tilde{\chi}_1^\pm \rightarrow W(l\nu)\tilde{\chi}_1^0 (300)$



MadGraph GEN $\sqrt{s}=8$ TeV

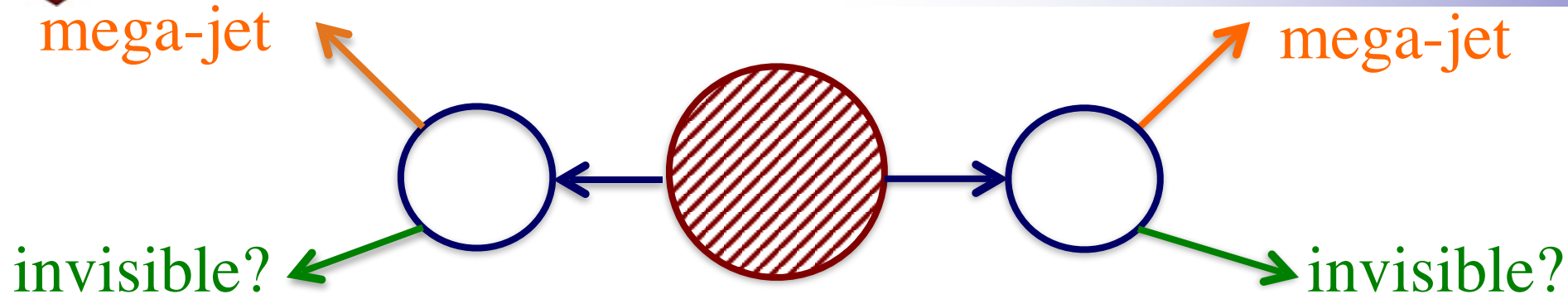


Here, the azimuthal angle between the the top and W decay planes $\Delta\phi_{T1,W1}$ and the angle between the two top decay planes

$$\Delta\phi_{T1,T2}$$



Razor kinematic variables



- Assign every reconstructed object to one of two **mega-jets**
- Analyze the event as a ‘canonical’ open final state:
 - two variables: M_R (mass scale) , R (scale-less event imbalance)
- An inclusive approach to searching for a large class of new physics possibilities with open final states

Razor variables

arXiv:1006.2727v1 [hep-ph]

CMS+ATLAS
analyses

PRD 85, 012004 (2012)

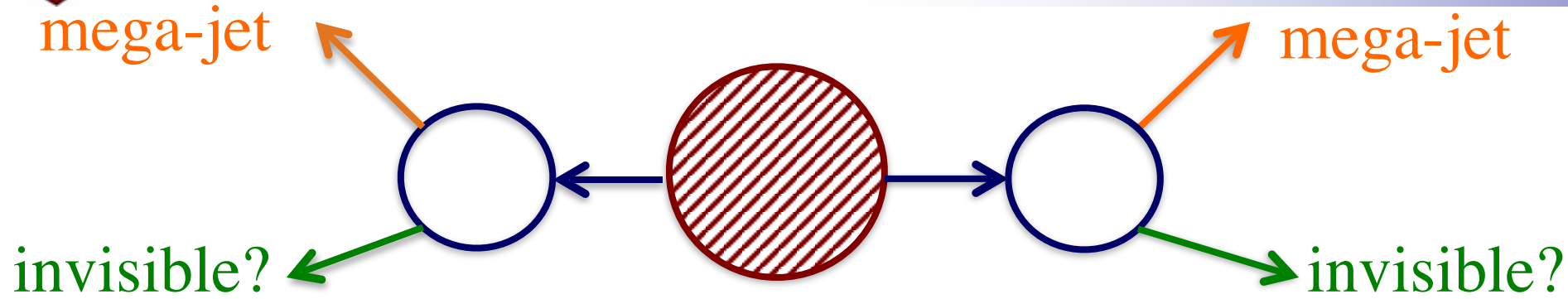
EPJC 73, 2362 (2013)

PRL 111, 081802 (2013)

CMS-PAS-SUS-13-004



Razor kinematic variables



- Assign every reconstructed object to one of two **mega-jets**
- Analyze the event as a ‘canonical’ open final state:
 - two variables: M_R (mass scale) , R (scale-less event imbalance)

$$M_R \sim \sqrt{\hat{s}} \qquad R = \frac{M_T^R}{M_R} \sim \frac{M_\Delta}{\sqrt{\hat{s}}}$$

Two distinct mass scales in event

Two pieces of complementary information



A Monte Carlo analysis to compare

■ Baseline Selection From PRD 89, 055020 (arXiv:1310.4827 [hep-ph])

- Exactly two opposite sign leptons with $p_T > 20 \text{ GeV}/c$ and $|\eta| < 2.5$
- If same flavor, $m(\ell\ell) > 15 \text{ GeV}/c^2$
- ΔR between leptons and any jet (see below) > 0.4
- veto event if b-tagged jet with $p_T > 25 \text{ GeV}/c$ and $|\eta| < 2.5$

■ Kinematic Selection

‘CMS selection’

$$|m(\ell\ell) - m_Z| > 15 \text{ GeV}$$

$$E_T^{\text{miss}} > 60 \text{ GeV}$$

CMS-PAS-SUS-12-022

‘ATLAS selection’

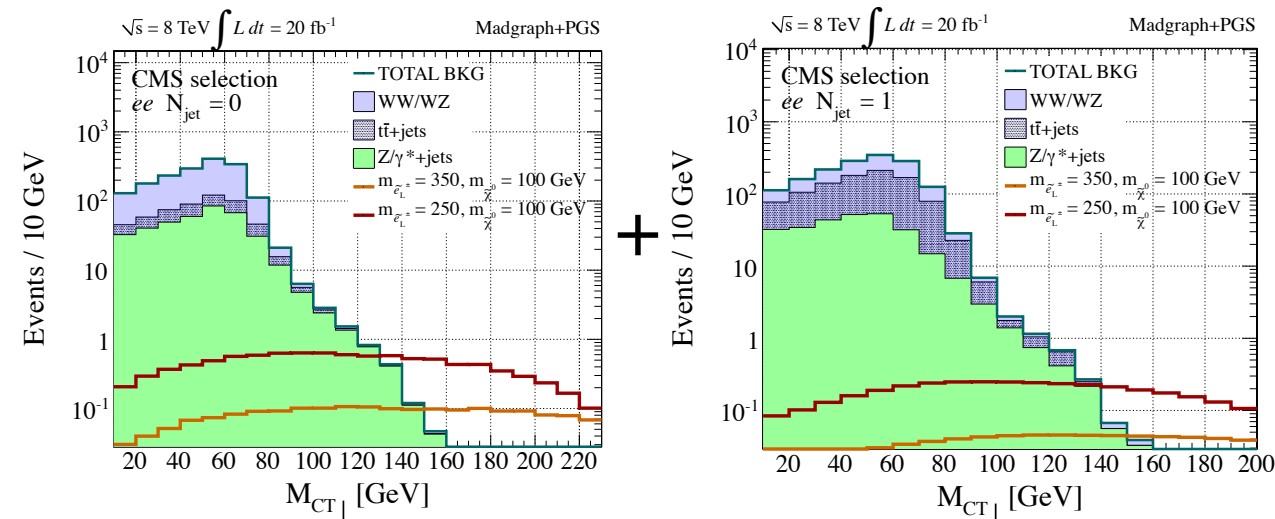
$$|m(\ell\ell) - m_Z| > 10 \text{ GeV}$$

$$E_T^{\text{miss,rel.}} = \begin{cases} E_T^{\text{miss}} & \text{if } \Delta\phi_{\ell,j} \geq \pi/2 \\ E_T^{\text{miss}} \times \sin \Delta\phi_{\ell,j} & \text{if } \Delta\phi_{\ell,j} < \pi/2 \end{cases} > 40 \text{ GeV}$$

ATLAS-CONF-2013-049



1D Shape Analysis



From PRD 89, 055020

Other jet
+ multiplicity and
lepton flavor
categories

■ Analysis Categories

- Consider final 9 different final states according to lepton flavor and jet multiplicity – simultaneous binned fit includes both high S/B and low S/B categories

$$(ee, \mu\mu, e\mu) \times (0, 1, \geq 2 \text{ jets}) \quad \text{with } p_T^{jet} > 30 \text{ GeV}/c, |\eta^{jet}| < 3$$

Fit to kinematic distributions (in this case, M_{Δ}^R , M_{T2} or M_{CTperp} in 10 GeV bins), over all categories for WW , $t\bar{t}$ and $Z/\gamma^* + \text{jets}$ yields



Systematic uncertainties

From PRD 89, 055020 (arXiv:1310.4827 [hep-ph])

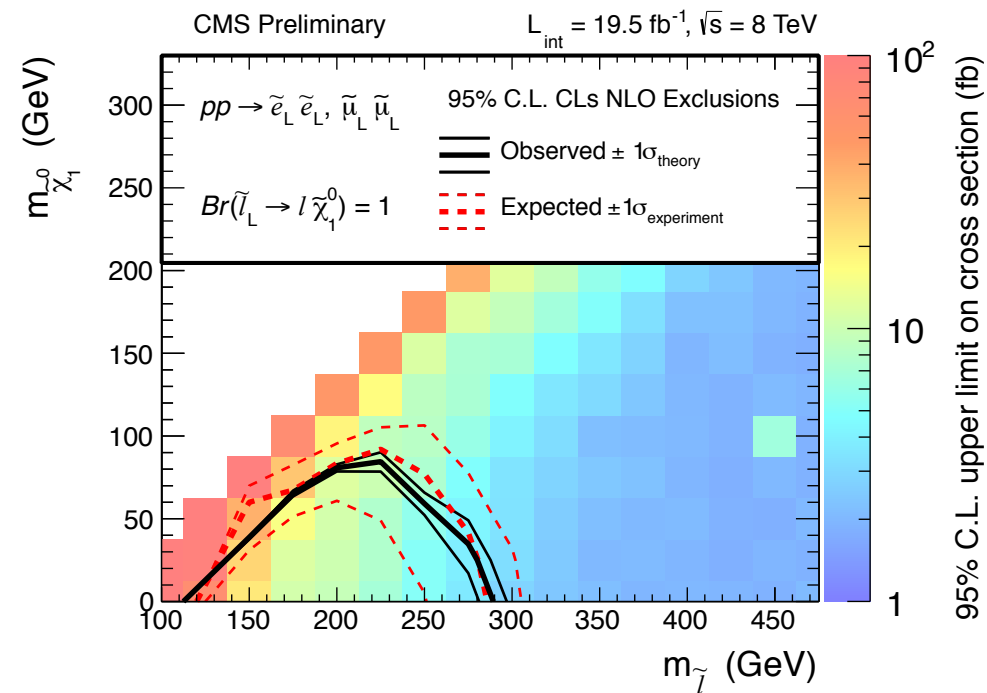
- 2% lepton ID (correlated btw bkg, uncorrelated between lepton categories)
- 10% jet counting (per jet) (uncorrelated between all processes)
- 10% x-section uncertainty for backgrounds (uncorrelated) + theoretical x-section uncertainty for signal (small)
- ‘shape’ uncertainty derived by propagating effect of 10% jet energy scale shift up/down to MET and recalculating shapes templates of kinematic variables
- Uncertainties are introduced into toy pseudo-experiments through marginalization (pdfs fixed in likelihood evaluation but systematically varied in shape and normalization in toy pseudo-experiment generation)



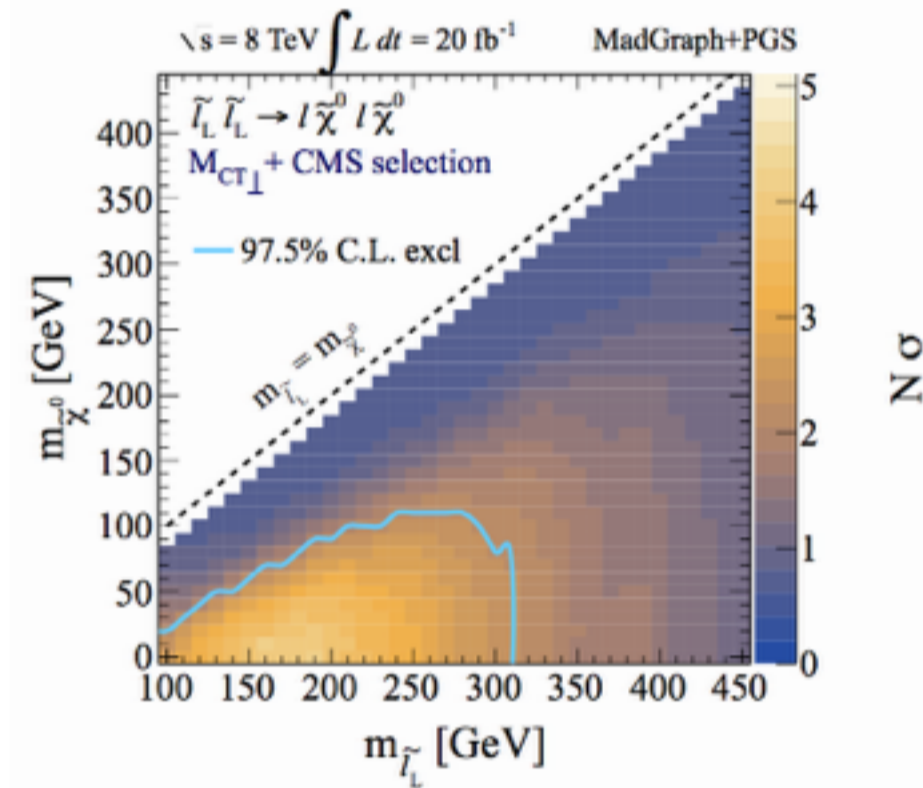
Compared to Reality

From PRD 89, 055020 (arXiv:1310.4827 [hep-ph])

$$pp \rightarrow \tilde{\ell}_L \tilde{\ell}_L; \quad \tilde{\ell}_L \rightarrow \tilde{\chi}_1^0 \ell$$

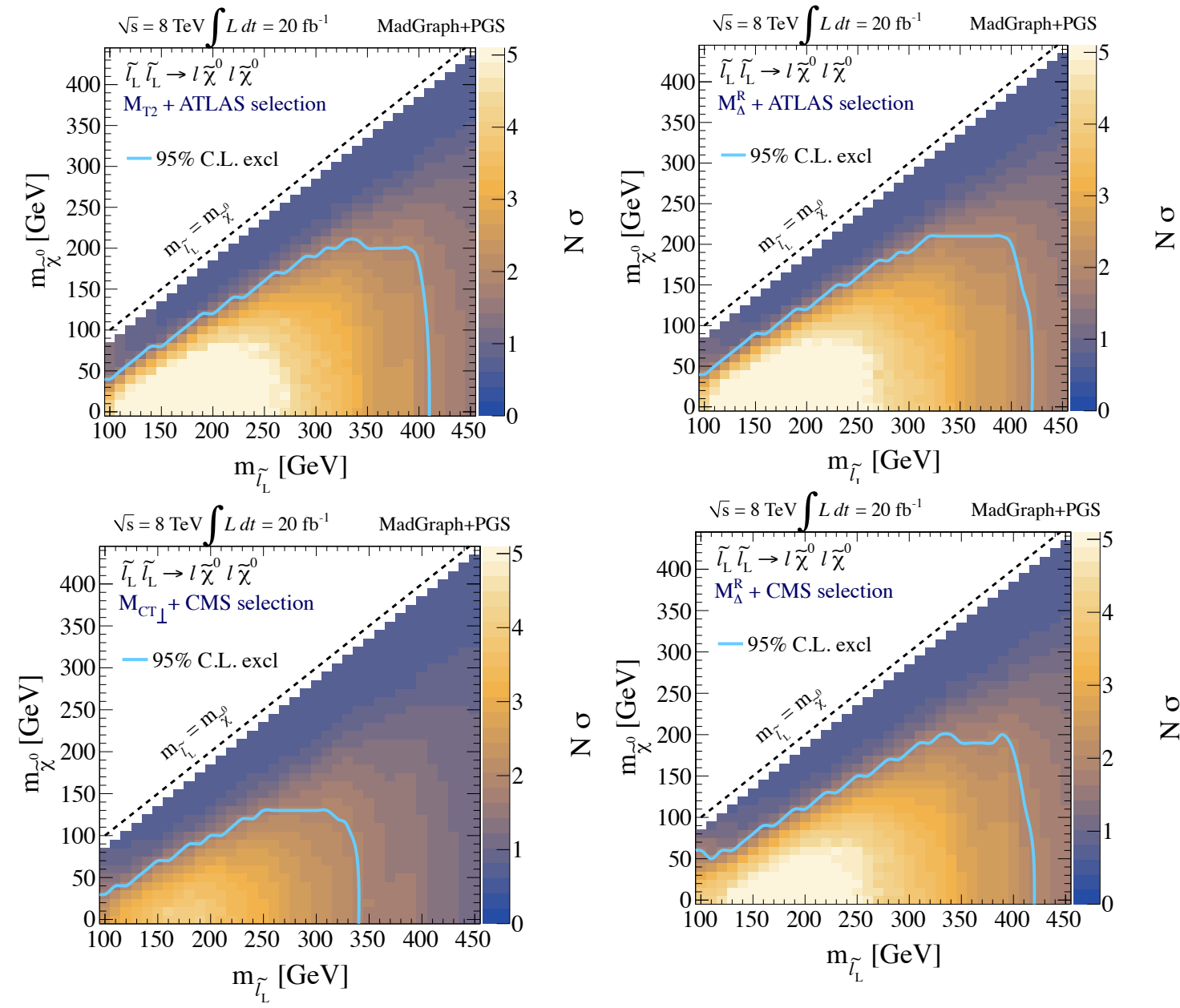


CMS-PAS-SUS-12-022





Expected Limit Comparison

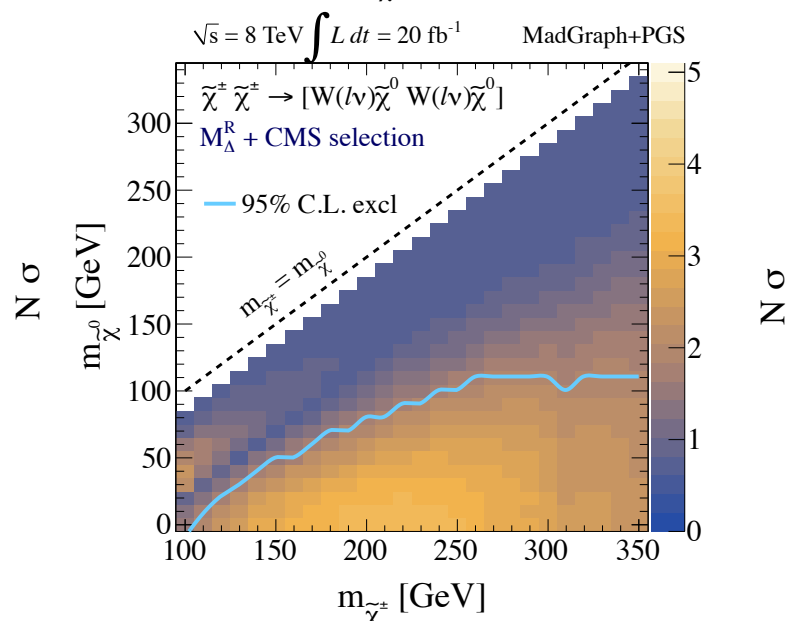
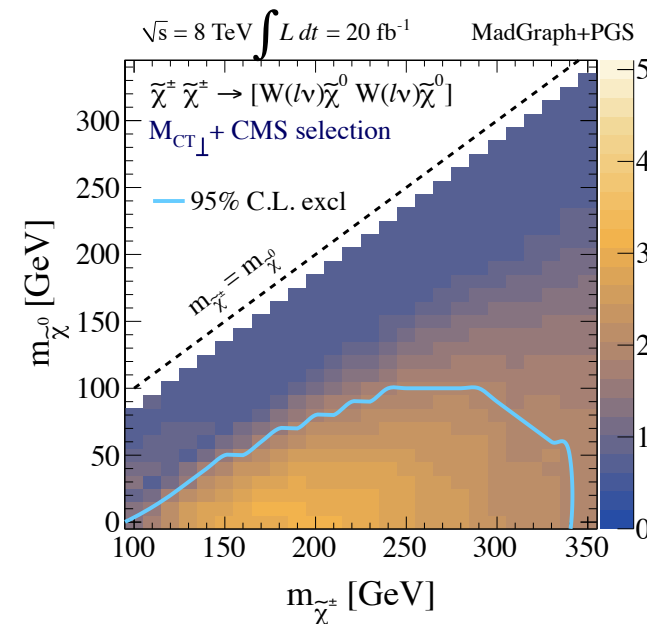
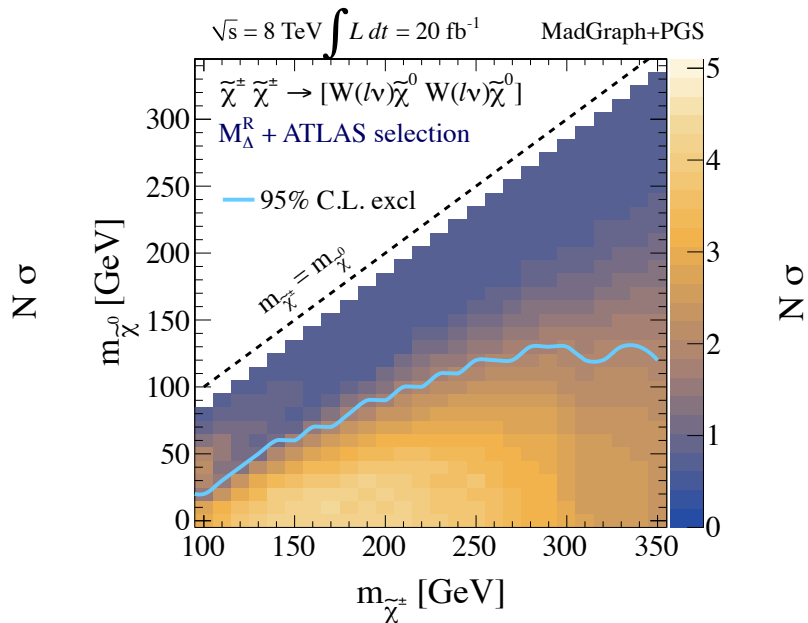
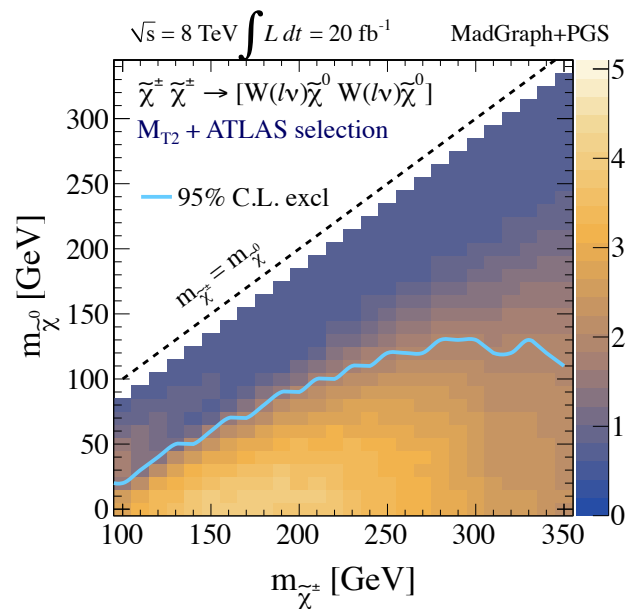


$$pp \rightarrow \tilde{\ell}_L \tilde{\ell}_L; \tilde{\ell}_L \rightarrow \tilde{\chi}_1^0 \ell$$

From PRD 89, 055020 (arXiv:1310.4827)



Charginos

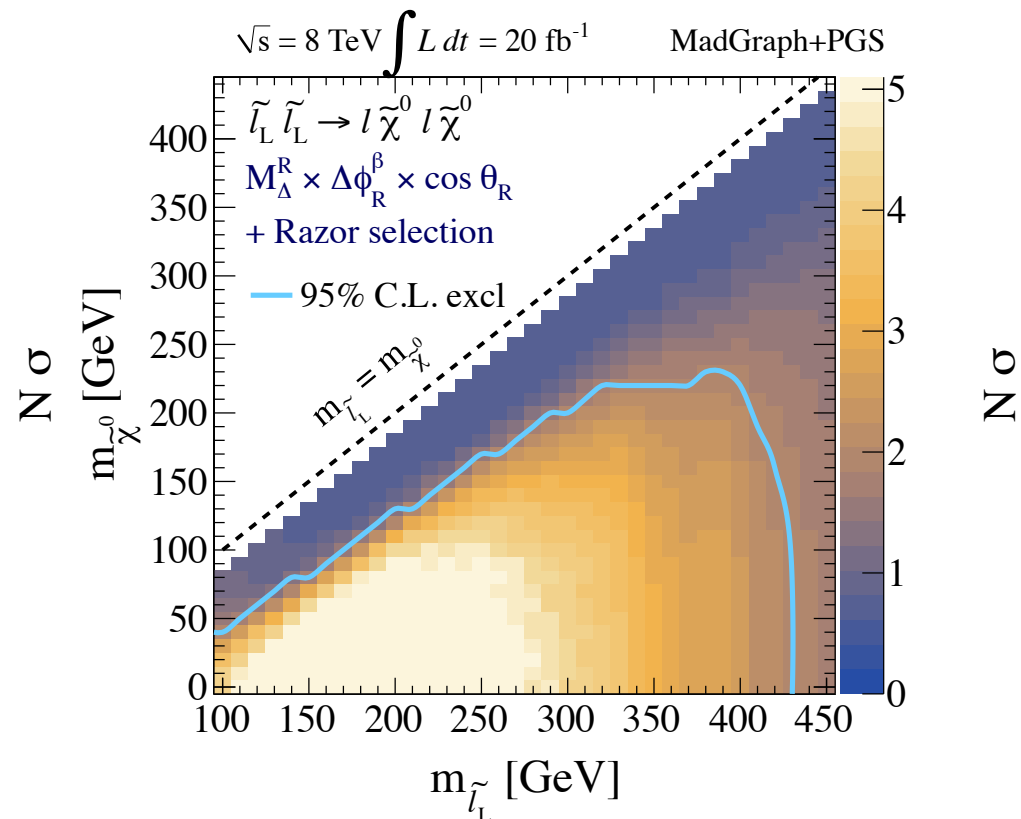
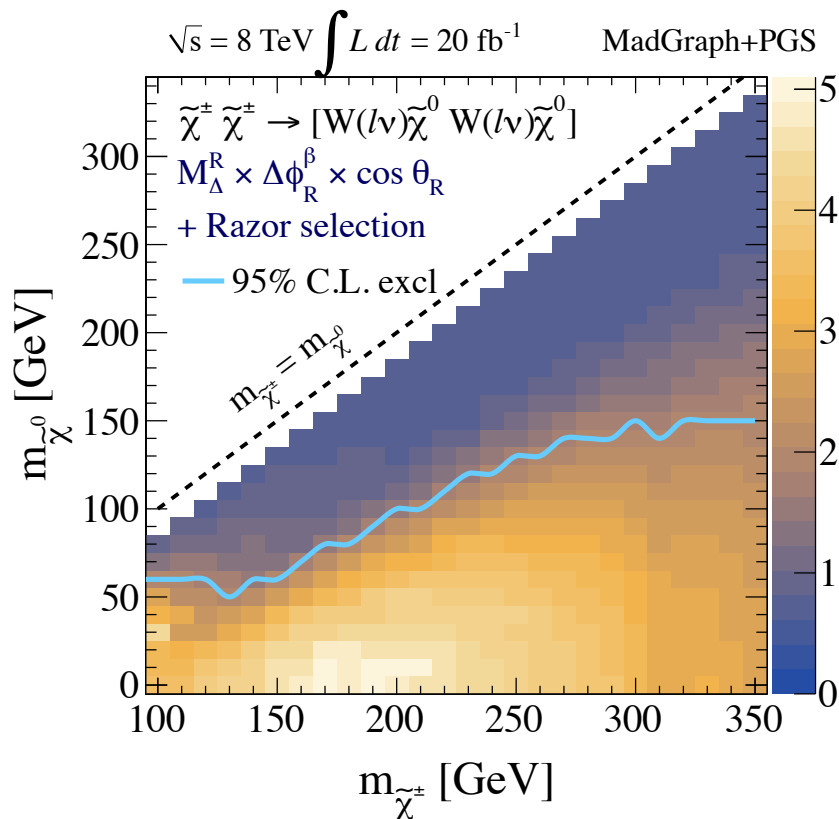


$$pp \rightarrow \tilde{\chi}_1^{\mp} \tilde{\chi}_1^{\pm}; \quad \tilde{\chi}_1^{\pm} \rightarrow \tilde{\chi}_1^0 W^{\pm} (\ell^{\pm} \nu)$$



Super-Razor Basis Selection

From PRD 89, 055020 (arXiv:1310.4827 [hep-ph])





Comparisons

