Little Flavor

Spacetime as a topological insulator
Phys. Rev. Lett. 108 (2012) 181807
DBK, Sichun Sun

Little Flavor: A model of weak-scale flavor physics
Phys.Rev. D87 (2013) 125036
Sichun Sun, DBK, A. E. Nelson

Effective field theory for Little Flavor
in preparation
Dorota Grabowska, DBK

The three major flavor puzzles:

1. The big question: why 3 generations of quark and leptons?
    - new symmetries?
    - new dimensions?
    - new dynamics?

2. Why so much hierarchical structure in flavor parameters?
    - couplings: gauge ~ Higgs ~ top Yukawa ~ O(1)
      CP violating phase~O(1)
    - angles: $V_{us} \sim 2\text{x}10^{-1}$, $V_{cb} \sim 4\text{x}10^{-2}$, $V_{ub} \sim 2\text{x}10^{-3}$
    - masses: $b/t \sim 5\text{x}10^{-2}$, $c/t \sim 10^{-2}$, $s/t \sim 10^{-3}$, $u/t \sim d/t \sim 10^{-5}$

3. What is the scale of flavor physics?
    - EW higgs sector, dark matter suggest new TeV physics
    - Absence of FCNC *seems* to suggest this new physics contains no new flavor structure.

- **Hors d'oeuvres**:

spacetime as a topological insulator and the origin of families... an interesting (?) failure to explain the number of families

- **Main course**:

A new framework for flavor physics in 4d

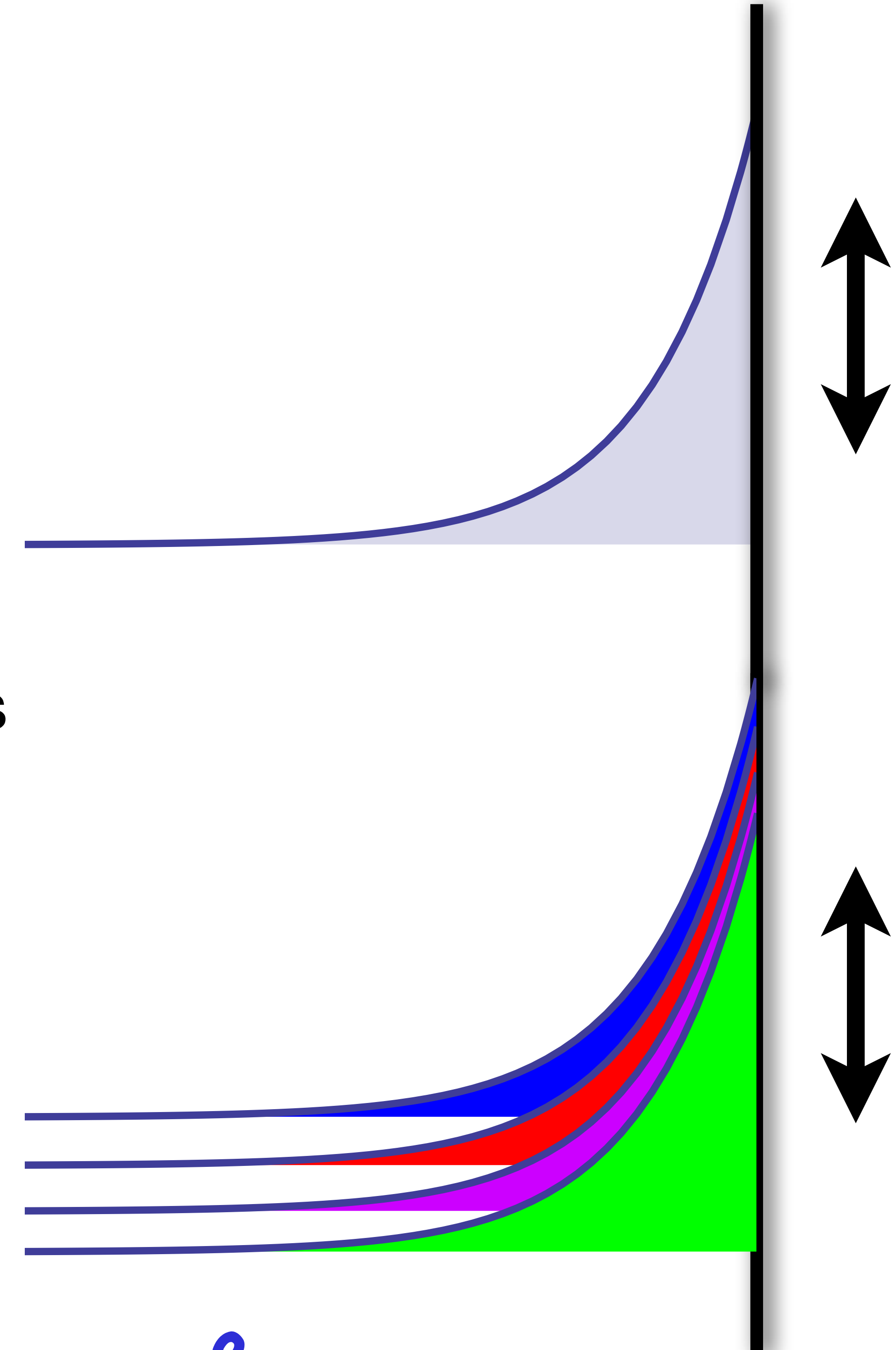
- new low scale flavor physics at a few TeV)
- small Flavor Changing Neutral Currents (FCNC) in a phenomenological model with realistic quark masses, CKM matrix
- unusual flavor/Higgs structure

- **Hors d'oeuvres:**

1. Why 3 generations of quark and leptons?

Spacetime as a topological insulator

Jackiw & Rebbi (1976):
Odd spacetime dimensions: Dirac fermion has a massless chiral surface mode



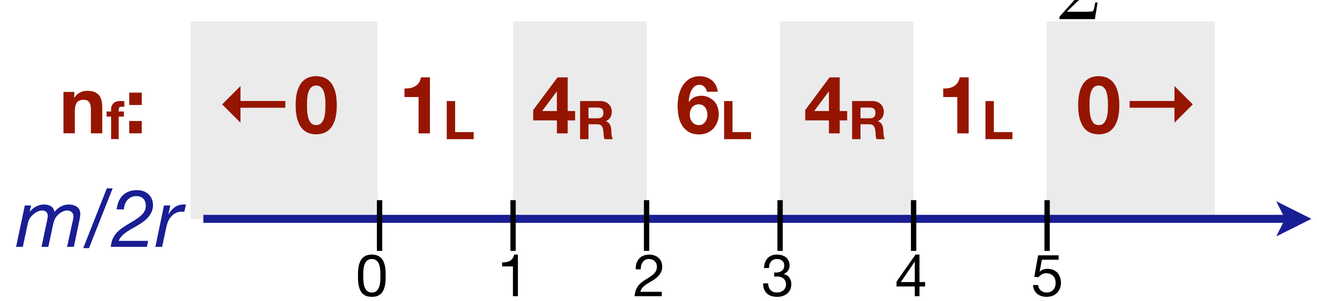
DBK (1992), Jansen & Schmaltz (1992):
Lattice version has $\mathbf{n_f}$ copies of chiral surface modes

- n_f changes discontinuously when Lagrangian parameters are varied continuously

E.g, d=5 lattice:

lattice derivatives

$$\mathcal{L} = \bar\psi i\boxed{\partial\!\!\!/}\psi - m\bar\psi\psi + \frac{r}{2}\bar\psi\boxed{\partial^2}\psi$$



topology?
on a lattice??

lattice derivatives

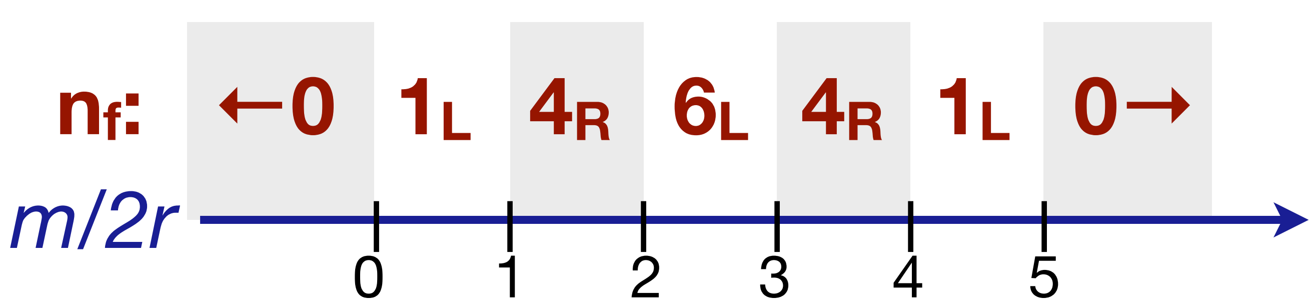
$$\mathcal{L} = \bar\psi i \not\partial \psi - m\bar\psi\psi + \frac{r}{2}\bar\psi \partial^2 \psi$$

$$S(p)^{-1} = m + \sum_{i=1}^{5}\left[i\gamma_i \frac{\sin p_i a}{a} + r\frac{(\cos a p_i - 1)}{a^2}\right]$$

Momenta lie on a d-torus (Brouillion zone): $-\pi/a < p_i \leq \pi/a$

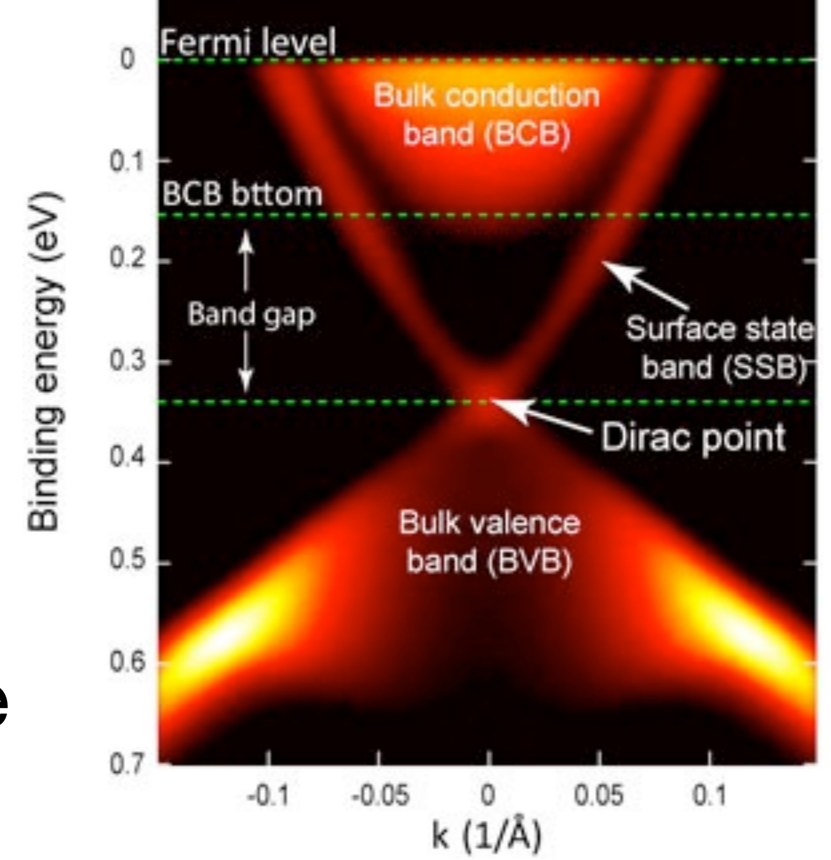
Fermion propagator S(p) maps $T_d \Rightarrow S_d$, integer winding number = n_f

Number of zero modes changes when S(p) can have a pole for some Euclidian momentum: m/r=0,2,4...

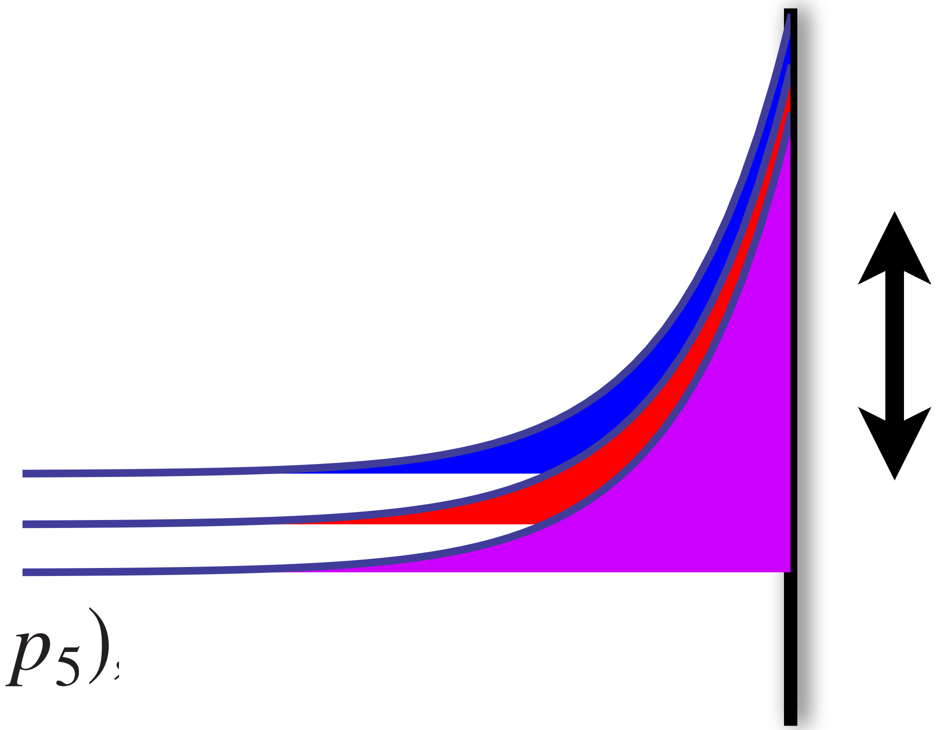


Goltermann, Jansen, DBK, PLB301, 219, (1992)

- Number of chiral surface modes result of topology of bulk fermion dispersion relation in <u>momentum</u> space
- Exactly the same physics subsequently discovered in CMT, called "topological insulators"
- Can 3 families of 4d fermions arise from a single family of 5d fermion through this mechanism? *# families determined by coupling constant values?*



Possible to engineer model in semi-infinite 5th dimension with 3 families of zeromodes:



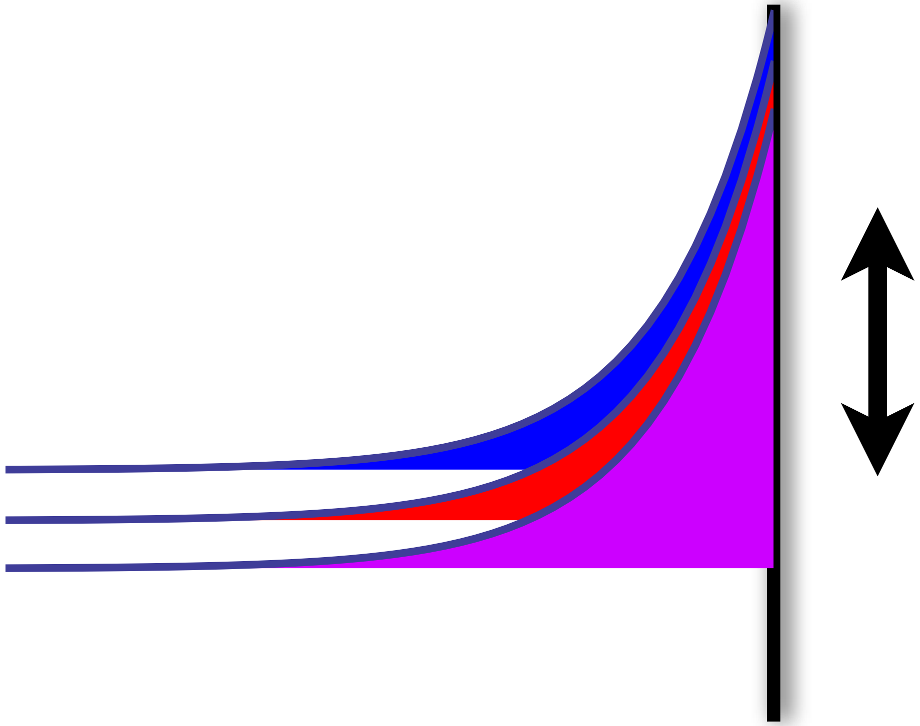
Bulk dispersion relation:

$$iG^{-1}(p_\mu, p_5) = iZ_\mu(p)\gamma^\mu + iZ_5(p_5)\gamma^5 - \Sigma(p, p_5),$$

Z, Σ can be chosen so that there are three 4d chiral families on surface of 5d

- Possible to construct model in semi-infinite 5th dimension with 3 families of zeromodes; **but**...

  1. can't have SM gauge fields live in noncompact extra dim
     - ...so compactify
  2. on compact manifold, find <u>vector-like</u> fermions instead of chiral
     - can be made chiral with chiral orbifold projection
  3. relies on UV physics:
     - topology in x depends on large-x behavior of fields
     - $\Rightarrow$ topology in p depends on large p behavior of G^{-1}
     - Need UV completion to make sense..eg, deconstruction



Outcome of deconstructing the 5d model: obtain a type of moose (quiver) diagram that has 3 families built into it. ☹

- **Main course**:

  Outcome of deconstruction lost the ability to explain 3 families, but it motivated looking at models that can tolerate a low scale for new flavor physics (Little Flavor)

Some allowed dim 6 FCNC operators:

$$\frac{c_{sd}}{\Lambda^2}\left(\bar{s}\gamma_\mu d\right)^2 \qquad \frac{c_{uc}}{\Lambda^2}\left(\bar{u}\gamma_\mu c\right)^2 \qquad \frac{c_{bd}}{\Lambda^2}\left(\bar{b}\gamma_\mu d\right)^2$$

experimental constraints

- Im[c_{sd}] = O(1) ⇒ Λ > O(10^4) TeV... 10^5 x M_Z!
- Re[c_{sd}] = O(1) ⇒ Λ > O(10^3) TeV
- c_{uc} = O(1) ⇒ Λ > O(10^3) TeV
- c_{bd} = O(1) ⇒ Λ > O(10^2) TeV

Reasonable conclusion: new flavor physics arises from very high energy scale physics. 

Necessary conclusion? No: eg, Minimal Flavor Violation

Minimal Flavor Violation (Chivukula & Georgi, 1987)

- Yukawa couplings *Y* in SM explicitly break $U(3)^5$ chiral symmetry: {Q, L, U*, D*, E*} x 3 families
- Assume that in the UV theory that *Y* are the *only* "spurions" that break $U(3)^5$
- Then the $U(3)^5$ transformation which diagonalizes *Y* to go to mass eigenstate basis will diagonalize *all* dim 6 operators as well...no FCNC

Other approaches to flavor: other chiral symmetries

- fermion mass matrices arise as products of various spurions that break some chiral flavor symmetry (eg, Froggatt Nielsen invoke a U(1) chiral symmetry which forbids fermion coupling to the Higgs)
- FCNC are not zero, but suppressed by small parameters related to small Yukawa couplings

Little Flavor (restricted to quark sector):

- **No** approximate chiral flavor symmetry
- Sizes of masses controlled by an approximate SU(4) x U(3) symmetry
    - ✦ U(3) is a vector-like flavor symmetry
    - ✦ SU(4) is a nonlinearly realized symmetry related to pseudo-Goldstone Boson nature of the Higgs
- FCNC is nonzero but can be acceptable
- Combines features of conventional flavor models (such as Froggatt-Nielsen) with Little Higgs

How flavor models typically work (e.g. Froggatt-Nielsen):

- Start with a large chiral flavor symmetry G that forbids fermion Yukawa couplings
- Include "sparse" spurions ε which break G ⇒ G' at 1st order in ε; G' ⇒ G" at 2nd order in ε, ...
- Fermion Yukawa matrices are built up in a hierarchical way with multiple insertions of spurions

Problems:

- SM provides little clue to RH fermion flavor structure, not enough about LH...have to guess at textures, symmetries
- models tend to be rather complicated, not extremely predictive.

Pluses:

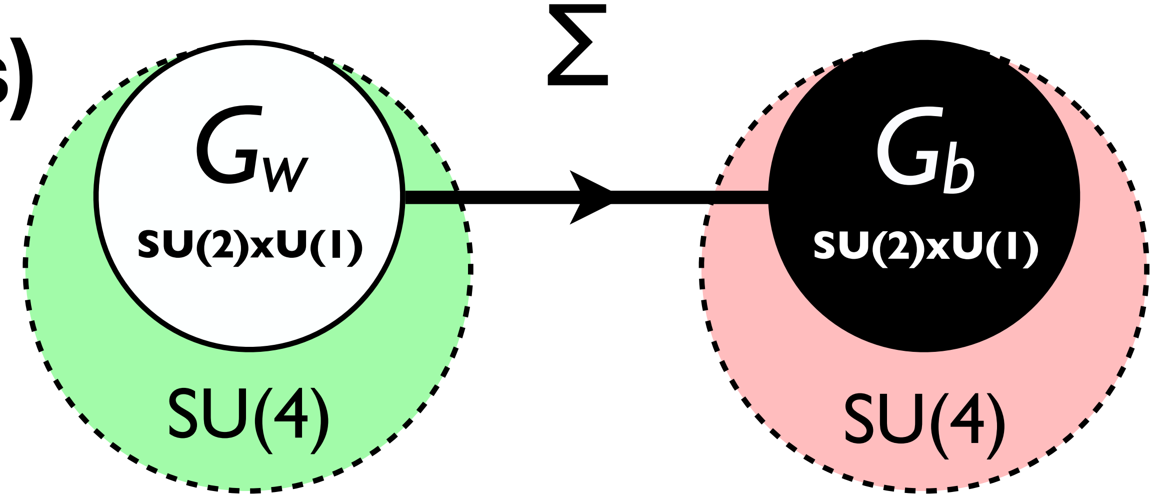
- same spurions can suppress FCNC
- flavor structure related to symmetry

How Little Higgs models work:

(Arkani-Hamed, Cohen, Georgi (2001); Arkani-Hamed, Cohen, Katz, Nelson (2002))

- Start with Higgs as a Goldstone boson of G/H, with scale f; $h \to h+f$ forbids Higgs potential (Kaplan, Georgi, 1984)
- Include "sparse" spurions $\varepsilon_{1,2}$ which break $G \Rightarrow G_{1,2}$, two different subgroups of G
- Both $G_{1,2}$ individually retain an exact shift symmetry for the Higgs, $h \to h+f$, but the $\varepsilon_{1,2}$ spurions break it when both are combined
- Higgs potential starts at order $m^2 \propto \varepsilon_1 x\ \varepsilon_2 f^2$, typically at 2-loops for extra $1/(4\pi)^4$...so Higgs is much lighter ("littler") compared to scale of new physics f than naive naturalness estimates
- New physics can start at the few TeV scale
- New top partner at ~ 1 TeV to cancels quadratic contribution to Higgs mass^2

The Little Flavor model (for quarks)



Gauge symmetry:

- Nonlinear *SU(4) x SU(4)/SU(4)* Σ field lives on the link (scale f ~ *1.5 TeV*)

- Gauge group $G_w \times G_b = [SU(2) \times U(1)]^2 \subset SU(4) \times SU(4)$

  ★ *$[SU(2) \times U(1)]^2$ broken to $[SU(2) \times U(1)]_{SM}$ by $\langle\Sigma\rangle = 1$*

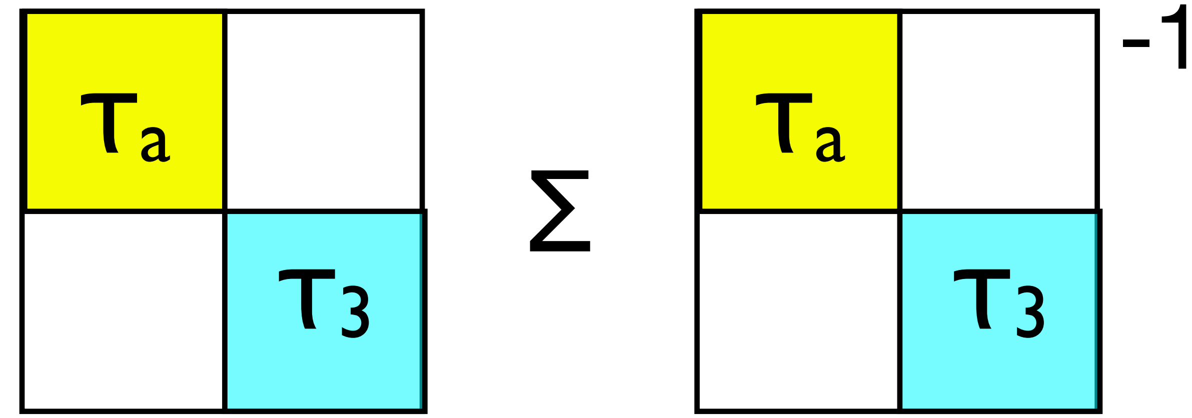
- Σ contains two composite Higgs doublets H_u, H_d

  ★ *$[SU(2) \times U(1)]_{SM}$ broken to $U(1)_{EM}$ by $\langle\Sigma\rangle = 1 + O(v/f)$, $v \sim 10^2$ GeV*

gauge couplings $g_{1,w}$, $g_{2,w}$, $g_{1,b}$, $g_{2,b}$ related to SM couplings g, g' via two angles $\gamma_{1,2}$:

$$g_{1,w} = \frac{g'}{\cos\gamma_1}\,, \qquad g_{1,b} = \frac{g'}{\sin\gamma_1}\,, \qquad g_{2,w} = \frac{g}{\cos\gamma_2}\,, \qquad g_{2,b} = \frac{g}{\sin\gamma_2}$$

Embedding of
$[SU(2) \times U(1)]^2 \supset SU(4)^2$:



$G_w = SU(2) \times U(1)$ $G_b = SU(2) \times U(1)$

Parametrization of the Σ field:

$$\Sigma_H = \exp\left[\left(\frac{i\sqrt{2}}{f}\right)\begin{pmatrix} 0 & \Phi^\dagger \\ \Phi & 0 \end{pmatrix}\right] \qquad \Phi = \begin{pmatrix} H_u^T \\ H_d^T \end{pmatrix}$$

$$\Sigma = \xi \Sigma_H \xi \qquad \xi = \exp\left[(i/2f)\begin{pmatrix} \vec{\pi}' \cdot \vec{\sigma} + \eta/\sqrt{2} & 0 \\ 0 & \vec{\pi} \cdot \vec{\sigma} - \eta/\sqrt{2} \end{pmatrix}\right]$$

- $\pi'^{\pm}$, π'^{0} = *SU(2)* triplet; eaten by heavy W', Z'
- $\pi^{\pm}$, π^{0}, η = *SU(2)* singlets; π^0 is eaten by heavy Z"
- H_u, H_d = SM Higgs doublets

Gauge boson masses:

Ignoring Higgs vev v:

$$M_\gamma = M_W = M_Z = 0$$

$$M_{W'} = M_{Z'} = \frac{gf}{\sin 2\gamma_2}\ , \qquad M_{Z''} = \frac{g' f}{\sin 2\gamma_1}$$

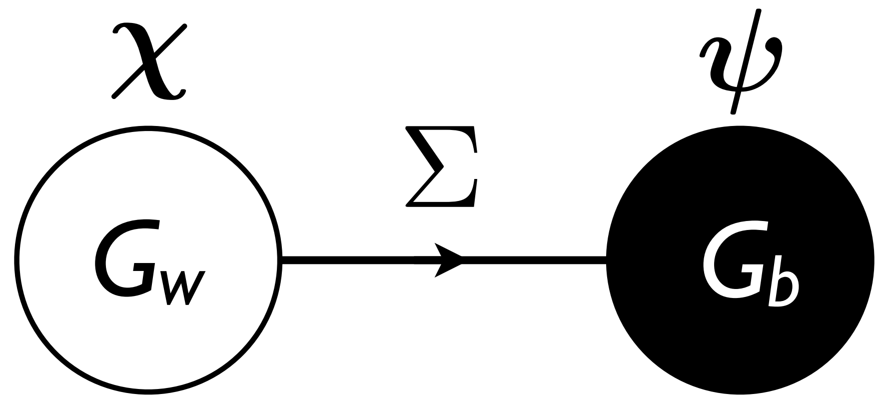
For f=1.5 TeV, $\gamma_1=\gamma_2=\pi/8$: $M_{W'} = M_{Z'}$ = 1.4 TeV, $M_{Z''}$ = 750 GeV

Z', Z" will have to be leptophobic to not be ruled out

Including Higgs vev v:

- *SM gauge bosons have conventional masses*
- *Exotic gauge boson masses receive $O(v^2/f^2)$ corrections*

Fermions:



on the **black** site:

- gauge group G_b = $SU(2) \times U(1)$
- 3 copies of 4 **Dirac** fermions
- transform as a 4 of $SU(4)_b$

$$\Psi = \begin{pmatrix} u \\ d \\ \hdashline U \\ D \end{pmatrix}_{i=1,2,3}$$

SU(2) doublet

SU(2) singlets

on the **white** site:

- gauge group G_w = $SU(2) \times U(1)$
- 3 copies of 4 **Chiral** fermions
- incomplete multiplets of $SU(4)_w$

$$\chi_L = \begin{pmatrix} u \\ d \\ \hdashline 0 \\ 0 \end{pmatrix}_{i,L} \qquad \chi_R = \begin{pmatrix} 0 \\ 0 \\ \hdashline U \\ D \end{pmatrix}_{i,R}$$

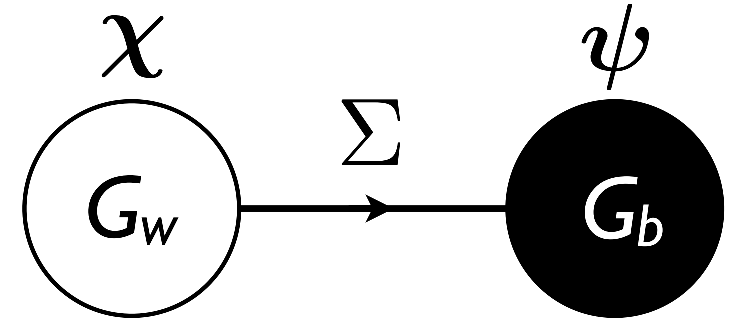
SU(2) doublet

SU(2) singlets

$i=1,2,3$

Fermion mass and Yukawa interactions:

U(3) x SU(4) symmetric terms



$$\Psi = \begin{pmatrix} u \\ d \\ U \\ D \end{pmatrix} \qquad \chi_L = \begin{pmatrix} u \\ d \\ 0 \\ 0 \end{pmatrix}_{i,L} \qquad \chi_R = \begin{pmatrix} 0 \\ 0 \\ U \\ D \end{pmatrix}_{i,R}$$

SU(2) doublet

SU(2) singlets

$$\mathcal{L}_{\rm sym} = \overline{\psi}\left(i\not{D} - M\right)\psi + \overline{\chi} i\not{D}\chi + \lambda f\left(\overline{\chi}\Sigma\gamma_5\psi + h.c.\right)$$

- Gives a common mass M~5 TeV to black Dirac fermions
- Σ (including Higgs) couples black Dirac fermions to white chiral fermions; f~ 1.5 TeV, λ=O(1).
- exact U(3) symmetry (acts on family index, not a chiral symmetry!)
- exact nonlinearly realized SU(4) symmetry (acts on black Dirac fermions and Σ)

$$\mathcal{L}_{\rm sym} = \sum_{n=1}^{3} \left[M\bar{\Psi}_n\Psi_n + \lambda f\left(\bar{\psi}_{L,n}\Sigma\Psi_{R,n} - \bar{\Psi}_{L,n}\Sigma^\dagger\psi_{R,n}\right)\right]$$

Expand to give Higgs couplings:

$$i\sqrt{2}\,\lambda\left[\left((\bar{u}_{w,n},\bar{d}_{w,n})_L\Phi^\dagger\begin{pmatrix}U_{b,n}\\ D_{b,n}\end{pmatrix}_R - (\bar{u}_{b,n},\bar{d}_{b,n})_L\Phi\begin{pmatrix}U_{w,n}\\ D_{w,n}\end{pmatrix}_R\right)\right]$$

$$\Phi^\dagger = (H_u^*, H_d^*)$$

- *Looks* like a Higgs vev would give all fermions a mass...
- ...but not true: can rotate <Σ> ⇒ 1 with SU(4) symmetry: then Higgs only has derivative couplings to fermions ⇒ at tree level Higgs looks like exact GB in $L_{\rm sym}$ and so no Yukawa term
- So: still have 3 massless chiral quark families after $[SU(2)xU(1)]_{SM}$ ⇒ $U(1)_{em}$.

Fermion mass and Yukawa interactions:

Introduce U(3) x SU(4) symmetry breaking terms in Dirac fermion masses

$$\mathcal{L}_{\rm asym} = \sum_{m,n=1}^{3} \bar\Psi_{m,L}\left(M^u X_u + M^d X_d\right)_{mn}\Psi_{n,R} + h.c.$$

- Acts only on black-site Dirac fermions
- M^u, M^d break the U(3) symmetry ⇒ $U(1)_B$

$$M^u = \begin{pmatrix} \mathcal{M}^u_{11} & \mathcal{M}^u_{12} & 0 \\ 0 & \mathcal{M}^u_{22} & 0 \\ \mathcal{M}^u_{31} & 0 & \mathcal{M}^u_{33}\end{pmatrix} \quad , \qquad M^d = \begin{pmatrix} \mathcal{M}^d_{11} & 0 & 0 \\ \mathcal{M}^d_{21} & \mathcal{M}^d_{22} & 0 \\ 0 & \mathcal{M}^d_{32} & \mathcal{M}^d_{33}\end{pmatrix}$$

- X_u, X_d break the SU(4) symmetry ⇒ different $SU(3)$ subgroups

$$X_u = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -3 & \\ & & & 1\end{pmatrix} \quad , \qquad X_d = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -3\end{pmatrix}$$

• Allows masses for ordinary families
• Little Higgs mechanism

Peculiar symmetry structure ensures Little Higgs mechanism in the fermion sector:

If M is the full fermion mass matrix, then

- $\mathrm{Tr}\, M^\dagger M$ is independent of H vevs
- $\mathrm{Tr}\, (M^\dagger M)^2$ is independent of H vevs

So there are neither <u>quadratic</u> nor <u>log</u> divergent contributions to the Higgs potential from fermions at one loop

There will be a finite Coleman-Weinberg contribution, $\mathrm{Tr}\, (M^\dagger M)^2 \ln(M^\dagger M)$. To avoid fine tuning of the Higgs potential, there needs to be a Dirac top-partner at ~ 1 TeV (will see it in this model)

At this level there is a Peccei-Quinn symmetry protecting against flavor violating Higgs couplings...to be softly broken in Higgs potential

Phenomenological fit to quark masses (RG scaled to 1 TeV) and CKM angles...not predictive, but useful for investigating FCNC

$$M = 5000 \text{ GeV} \,, \qquad f = 1500 \text{ GeV} \,, \qquad \tan\beta = \frac{v_u}{v_d} = 1$$

$$\lambda = 1.49794$$

$$M^u = \begin{pmatrix} 1189.54 & 15.4904 & 0 \\ 0 & 6.96490 & 0 \\ 3.50799 e^{-i1.224428} & 0 & 0.01441071 \end{pmatrix} \,, \qquad M^d = \begin{pmatrix} 45.7769 & 0 & 0 \\ -1.60269 & 0.600984 & 0 \\ 0 & 0.137582 & 0.0336607 \end{pmatrix} \quad \text{(GeV)}$$

Yields quark masses (GeV)

$$m_t = 153.2 \qquad m_c = 5.32 \times 10^{-1} \qquad m_u = 1.10 \times 10^{-3}$$
$$m_b = 2.45 \qquad m_s = 4.69 \times 10^{-2} \qquad m_d = 2.50 \times 10^{-3}$$

and angles:

$$|V_{\text{CKM}}| = \begin{pmatrix} 0.974 & 0.226 & 0.00385 \\ 0.226 & 0.973 & 0.0423 \\ 0.00892 & 0.0415 & 0.998 \end{pmatrix} \qquad \sin(2\alpha) = 0.052 \,, \qquad \sin(2\beta) = 0.72 \,, \qquad \sin(2\gamma) = 0.68$$

The model also has heavy SM fermion partners:

bottom-like masses (TeV):

6.628, 6.456, 5.489, 5.486, 5.482, 5.482

top-like masses (TeV):

6.628, 5.489, 5.482, 5.482, 5.463, 2.684

the top partner required
by Little Higgs

But what about FCNC?? First, look at *Z*, *Z'*, *Z"* couplings

Flavor dependence of neutral gauge boson couplings (Z, Z', Z")

$$M_{Z'} = 750 \text{ GeV} \;, \qquad M_{Z''} = 1400 \text{ GeV}$$

$$|\mathcal{L}^u_Z| = \begin{pmatrix} 2.6\times10^{-1} & 0 & 1.9\times10^{-6} \\ 0 & 2.6\times10^{-1} & 9.7\times10^{-6} \\ 1.9\times10^{-6} & 9.7\times10^{-6} & 2.6\times10^{-1} \end{pmatrix}, \qquad |\mathcal{R}^u_Z| = \begin{pmatrix} 1.1\times10^{-1} & 0 & 2.3\times10^{-6} \\ 0 & 1.1\times10^{-1} & 1.0\times10^{-5} \\ 2.3\times10^{-6} & 1.0\times10^{-5} & 1.1\times10^{-1} \end{pmatrix},$$

$$|\mathcal{L}^d_Z| = \begin{pmatrix} 3.2\times10^{-1} & 1.0\times10^{-6} & 5.0\times10^{-6} \\ 1.0\times10^{-6} & 3.2\times10^{-1} & 2.3\times10^{-5} \\ 5.0\times10^{-6} & 2.3\times10^{-5} & 3.2\times10^{-1} \end{pmatrix}, \qquad |\mathcal{R}^d_Z| = \begin{pmatrix} 5.5\times10^{-2} & 0 & 0 \\ 0 & 5.5\times10^{-2} & 3.6\times10^{-6} \\ 0 & 3.6\times10^{-6} & 5.5\times10^{-2} \end{pmatrix},$$

$$|\mathcal{L}^u_{Z'}| = \begin{pmatrix} 2.6\times10^{-3} & 0 & 0 \\ 0 & 2.6\times10^{-3} & 3.4\times10^{-5} \\ 0 & 3.4\times10^{-5} & 3.8\times10^{-3} \end{pmatrix}, \qquad |\mathcal{R}^u_{Z'}| = \begin{pmatrix} 1.4\times10^{-2} & 0 & 4.0\times10^{-4} \\ 0 & 1.5\times10^{-2} & 1.7\times10^{-3} \\ 4.0\times10^{-4} & 1.7\times10^{-3} & 3.7\times10^{-1} \end{pmatrix}$$

$$|\mathcal{L}^d_{Z'}| = \begin{pmatrix} 5.\times10^{-3} & 1.9\times10^{-5} & 8.9\times10^{-5} \\ 1.9\times10^{-5} & 4.9\times10^{-3} & 4.1\times10^{-4} \\ 8.9\times10^{-5} & 4.1\times10^{-4} & 3.7\times10^{-3} \end{pmatrix}, \qquad |\mathcal{R}^d_{Z'}| = \begin{pmatrix} 6.7\times10^{-3} & 0 & 2.6\times10^{-5} \\ 0 & 6.6\times10^{-3} & 2.0\times10^{-4} \\ 2.6\times10^{-5} & 2.0\times10^{-4} & 8.8\times10^{-3} \end{pmatrix}$$

$$|\mathcal{L}^u_{Z''}| = \begin{pmatrix} 1.9\times10^{-2} & 0 & 7.9\times10^{-5} \\ 0 & 1.9\times10^{-2} & 2.8\times10^{-4} \\ 7.9\times10^{-5} & 2.8\times10^{-4} & 2.9\times10^{-2} \end{pmatrix}, \qquad |\mathcal{R}^u_{Z''}| = \begin{pmatrix} 1.4\times10^{-3} & 0 & 0 \\ 0 & 1.4\times10^{-3} & 0 \\ 0 & 0 & 1.3\times10^{-3} \end{pmatrix}$$

$$|\mathcal{L}^d_{Z''}| = \begin{pmatrix} 2.0\times10^{-2} & 1.0\times10^{-4} & 5.0\times10^{-4} \\ 1.0\times10^{-4} & 1.9\times10^{-2} & 2.3\times10^{-3} \\ 5.0\times10^{-4} & 2.3\times10^{-3} & 2.9\times10^{-2} \end{pmatrix}, \qquad |\mathcal{R}^d_{Z''}| = \begin{pmatrix} 1.6\times10^{-3} & 0 & 0 \\ 0 & 1.6\times10^{-3} & 0 \\ 0 & 0 & 9.7\times10^{-4} \end{pmatrix}$$

Can read off $\Delta S = 2$ dim 6 operators from Z, Z', Z" exchange:

$$\frac{1\times 10^{-12}}{M_Z^2} \simeq \frac{1}{(10^5\ \mathrm{TeV})^2}\,, \qquad \frac{4\times 10^{-10}}{M_{Z'}^2} \simeq \frac{1}{(4\times 10^4\ \mathrm{TeV})^2}\,, \qquad \frac{1\times 10^{-8}}{M_{Z''}^2} \simeq \frac{1}{(1.3\times 10^4\ \mathrm{TeV})^2}$$

...all safe, even though:

- flavor physics is at the few TeV scale
- full theory does not have any approximate chiral flavor symmetry

Easy to show that dim 6 contact operators from above the cutoff Λ~4πf give tiny FCNC contributions (suppressed by spurions)

Numerical fit is not very informative... what does FCNC look like parametrically?

EFT analysis of Little Flavor (in preparation with Dorota Grabowska)

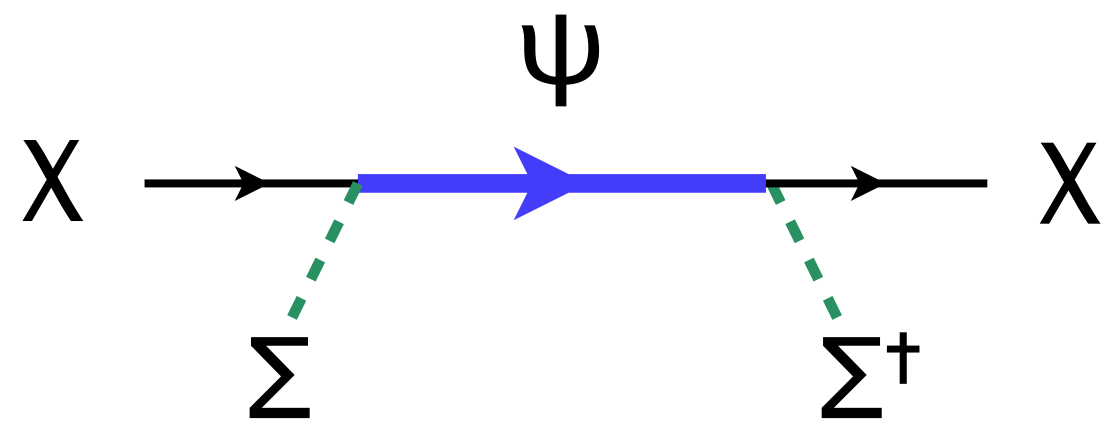
$$\mathcal{L} = \bar{\chi} i \not{D} \chi + \bar{\psi}(\not{D} - \mathbf{M})\psi + \lambda f \bar{\chi} \Sigma \gamma_5 \psi$$

$$\mathbf{M} = M + \mathcal{M}_u X_u + \mathcal{M}_d X_d$$

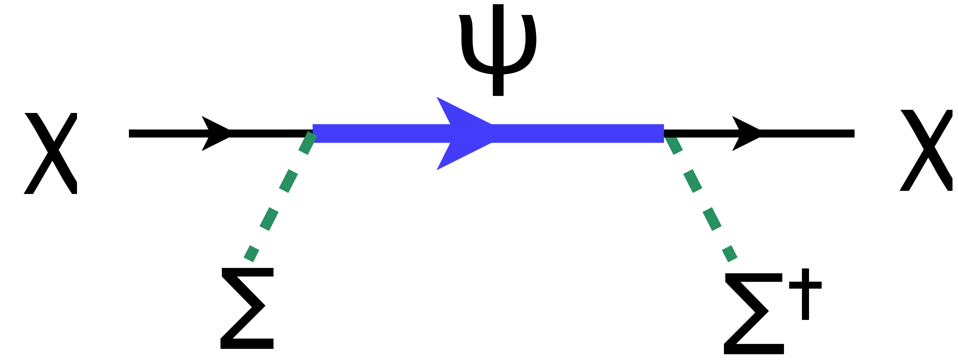
5 TeV

expand in 1/M

- Integrate out Dirac fermions ψ at tree level
  - ✦ *Not mass eigenstates...but that's OK!*



- Left with EFT for the chiral fermions χ, 1:1 with SM quarks



$$\mathcal{L}_\chi = \bar\chi i \not{D} \chi - (\lambda f)^2\, \bar\chi\, \Sigma \left(\frac{1}{i\not{D} + \mathbf{M} P_+ + \mathbf{M}^\dagger P_-} \right) \Sigma^\dagger\, \chi$$

Expand in powers of D, Φ:

$$\mathcal{L}_{\rm eff} = i\bar\chi \left[\not{D} + (\lambda f)^2\, \not{D} \left(\frac{P_-}{\mathbf{M}^\dagger \mathbf{M}} + \frac{P_+}{\mathbf{M}\mathbf{M}^\dagger} \right) \right] \chi$$

- w.f. renormalization
- F.C. gauge couplings

$$-2i\lambda^2 f\, \bar\chi \left[\Phi, \left(\frac{P_+}{\mathbf{M}} + \frac{P_-}{\mathbf{M}^\dagger} \right) \right] \chi$$

- quark Yukawa coupling to Higgs
- commutator vanishes w/o SU(4) breaking in M

$$+\, O(D^2, \langle\Phi\rangle D, \langle\Phi\rangle^2)$$

Restricting to the 2 light families, we find

$$M^{\rm SM}_{\rm up} \simeq -\frac{4(\lambda f)^2}{M^2+(\lambda f)^2}\,\frac{v_u}{f}\,\mathcal{M}_u\ , \qquad M^{\rm SM}_{\rm down} \simeq -\frac{4(\lambda f)^2}{M^2+(\lambda f)^2}\,\frac{v_d}{f}\,\mathcal{M}_d\ ;$$

Assuming the texture: $\mathcal{M}_u = \begin{pmatrix} \mathcal{M}_{u,11} & 0 \\ 0 & \mathcal{M}_{u,22} \end{pmatrix} \qquad \mathcal{M}_d = \begin{pmatrix} \mathcal{M}_{d,11} & \mathcal{M}_{d,12} \\ 0 & \mathcal{M}_{d,22} \end{pmatrix}$

we can fit these five real parameters to m_u, m_d, m_c, m_s, $\sin\theta_c$:

$$\mathcal{M}_{d,11} = \rho_d\, m_d \sec\theta_c \left[1-\left(\frac{m_d}{m_s}\right)^2 \frac{\tan^2\theta_c}{2}+\ldots\right]\ , \qquad \mathcal{M}_{u,11} = \rho_u\, m_u\ ,$$

$$\mathcal{M}_{d,12} = \rho_d\, m_s \sin\theta_c \left[1-\left(\frac{m_d}{m_s}\right)^2 \frac{1+\sec^2\theta_c}{2}+\ldots\right]\ , \qquad \mathcal{M}_{u,22} = \rho_u\, m_c\ ,$$

$$\mathcal{M}_{d,22} = \rho_d\, m_s \cos\theta_c \left[1+\left(\frac{m_d}{m_s}\right)^2 \frac{\tan^2\theta_c}{2}+\ldots\right]\ ,$$

$\rho_{u,d} \equiv \frac{M^2+(\lambda f)^2}{4(\lambda f)^2}\,\frac{f}{v_{u,d}}$ ~13 for our parameters, $v_u=v_d$

With this fit, mass diagonalization requires rotations of *d* quarks:

$$L_d = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}, \qquad R_d = \begin{pmatrix} \cos\theta_r & \sin\theta_r \\ -\sin\theta_r & \cos\theta_r \end{pmatrix}$$

$$\sin\theta_r = \underline{\tan\theta_c \frac{m_d}{m_s}} \left[1 - \left(\frac{m_d}{m_s}\right)^2 \frac{\tan\theta_c^2}{2} + \ldots\right]$$

Note: small angle

Using the w.f. correction term & these rotations, can compute all of the flavor dependence in gauge boson couplings to linear order in the light Yukawa couplings...

For example: flavor couplings of the lighter Z" (~ 750 GeV):

$$Z''_{L,u} = \frac{g'}{6}\beta_{\gamma_1}\left[1+\frac{\zeta_{\gamma_1}}{v_u}\begin{pmatrix} 2\left(m_u + m_d\sec\theta_c\right) & m_s\sin\theta_c \\ m_s\sin\theta_c & 2\left(m_c + m_s\cos\theta_c\right)\end{pmatrix}\right]$$

$$Z''_{R,u} = \frac{2g'}{3}\beta_{\gamma_1}\left[1+\frac{\zeta_{\gamma_1}}{v_u}\begin{pmatrix} 2\left(m_d\sec\theta_c - 3m_u\right) & m_s\sin\theta_c \\ m_s\sin\theta_c & 2\left(m_s\cos\theta_c - 3m_c\right)\end{pmatrix}\right]$$

$$Z''_{L,d} = \frac{g'}{6}\beta_{\gamma_1}\left[1+\frac{\zeta_{\gamma_1}}{v_d}\begin{pmatrix} 2\cos\theta_c\left(m_u + m_d\cos\theta_c + m_c\tan\theta_c\sin\theta_c\right) & -\sin\theta_c\left(2m_c\cos\theta_c + m_s\right) \\ -\sin\theta_c\left(2m_c\cos\theta_c + m_s\right) & 2\cos\theta_c\left(m_s + m_c\cos\theta_c\right)\end{pmatrix}\right]$$

$$Z''_{R,d} = -\frac{g'}{3}\beta_{\gamma_1}\left[1+\frac{\zeta_{\gamma_1}}{v_d}\begin{pmatrix} 2\left(m_u - 3m_d\cos\theta_c\right) & 3\sin\theta_c\left(2m_d - m_s\right) - 2\frac{m_d}{m_s}m_c\tan\theta_c \\ 3\sin\theta_c\left(2m_d - m_s\right) - 2\frac{m_d}{m_s}m_c\tan\theta_c & 2\left(m_c - 3m_s\cos\theta_c\right)\end{pmatrix}\right]$$

$$\beta_\gamma = \tan\gamma\frac{M^2 - f^2\lambda^2\cot^2\gamma}{M^2 + f^2\lambda^2} \simeq -0.06$$

$$\zeta_\gamma = \frac{f}{4M}\frac{1+\cot^2\gamma}{1-\frac{f^2\lambda^2}{M^2}\cot^2\gamma} \simeq 2.8$$

for:
M=5 TeV
f=1.5 TeV
λ=1.5
γ=π/8

$$
Z''_{L,d} = \frac{g'}{6}\beta_{\gamma_1}\left[1 + \frac{\zeta_{\gamma_1}}{v_d}\begin{pmatrix} 2\cos\theta_c\,(m_u + m_d\cos\theta_c + m_c\tan\theta_c\sin\theta_c) & -\sin\theta_c\,(2m_c\cos\theta_c + m_s) \\ -\sin\theta_c\,(2m_c\cos\theta_c + m_s) & 2\cos\theta_c\,(m_s + m_c\cos\theta_c) \end{pmatrix}\right]
$$

$$
Z''_{R,d} = -\frac{g'}{3}\beta_{\gamma_1}\left[1 + \frac{\zeta_{\gamma_1}}{v_d}\begin{pmatrix} 2\,(m_u - 3m_d\cos\theta_c) & 3\sin\theta_c\,(2m_d - m_s) - 2\frac{m_d}{m_s}m_c\tan\theta_c \\ 3\sin\theta_c\,(2m_d - m_s) - 2\frac{m_d}{m_s}m_c\tan\theta_c & 2\,(m_c - 3m_s\cos\theta_c) \end{pmatrix}\right]
$$

$$
\beta_\gamma = \tan\gamma\,\frac{M^2 - f^2\lambda^2\cot^2\gamma}{M^2 + f^2\lambda^2} \simeq -0.06
$$

small of FCNC due to:

- small Yukawas, θ_c
- small β
- ...but not MFV

So what have you just seen?

phenomenological

A ~~predictive~~ ~~theory~~ of ~~flavor~~

~~model~~ quarks

scenario

- A model to explain how 3 families can arise from the dispersion relation of 5D fermions...but no satisfactory renormalizable formulation.
- A model with a novel set of flavor symmetries in a 4D theory
    - U(3) flavor symmetry explains hierarchies,
    - SU(4) symmetry on Dirac quarks + PGB nature of Higgs explains why quarks are light
    - flavor symmetries interplay with EW symmetry breaking
    - Natural flavor @ few TeV scale with very small FCNC

Little Flavor pros:

- New flavor symmetry to explore: nonlinear, not chiral, connected to the Higgs
- Can try to build flavor models at the few TeV scale
- FCNC are suppressed enough, but possibly visible
- Extra EW gauge bosons to discover

Little Flavor cons:

- Radiative corrections to Yukawa couplings in simplest model raise light quark masses to ~ 100 MeV
- Little Higgs potential needs work -- sort of ugly, need to get rid of the η
- Needs leptons with Z', Z'' being leptophobic
- Would like a more predictive framework for flavor