Consequences of Weyl Consistency Conditions

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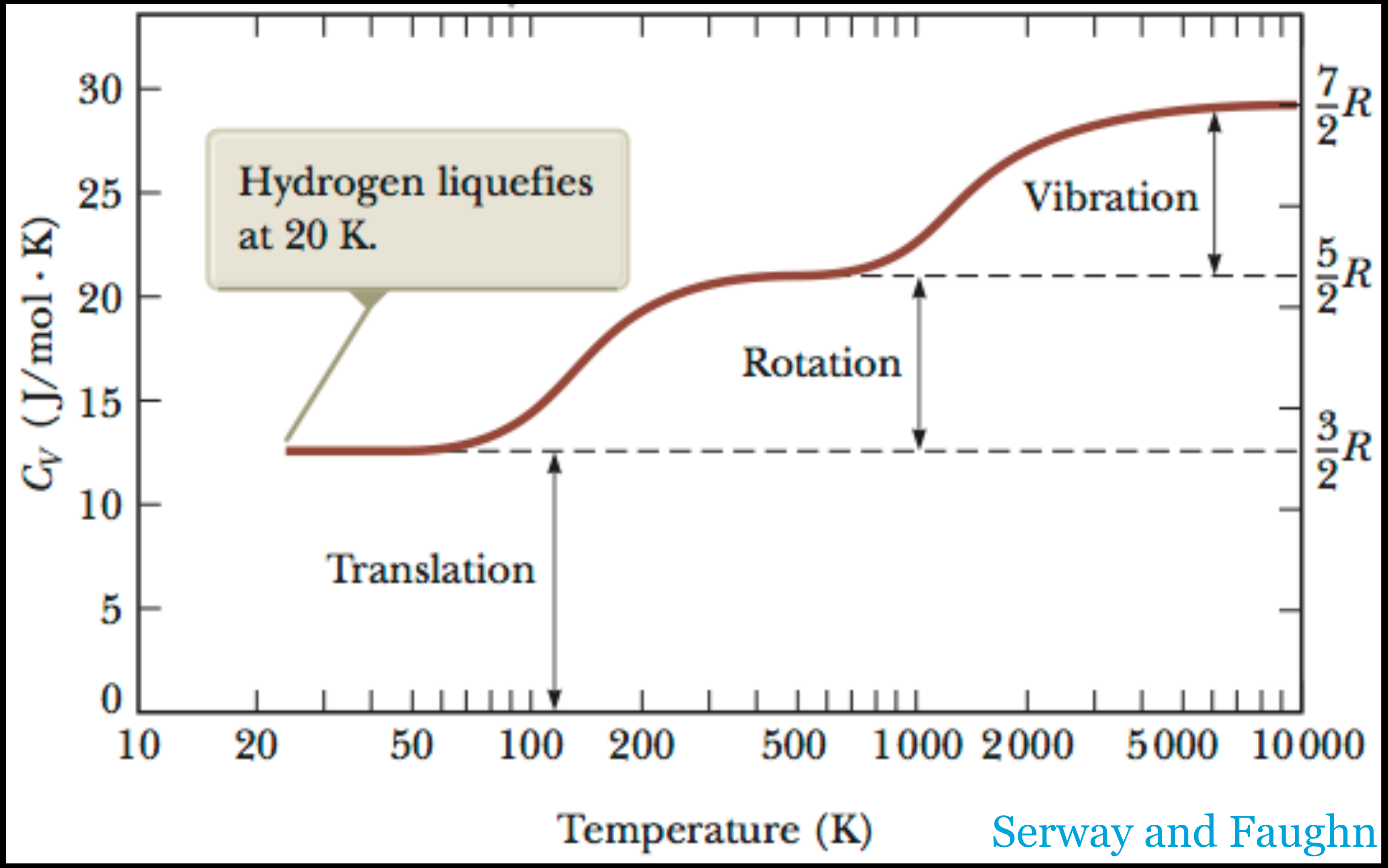
Invitation: DOFs in QFTs

- There is a clear interpretation in Physics 2C:



- The interpretation in QFTs is not so clear

DOFs in 2d QFTs

There is a quantity that is known to capture the number of degrees of freedom in a 2d CFT:
the central charge

In 2d QFTs it has been known since the 80s that there is a quantity that decreases monotonically as we slide our RG scale μ downward

An irreversibility of the renormalization group flux, captured by this so called *c*-function, which *is* the central charge at fixed points

The 2d c-function

Define:

$$\tilde{c} = 2z^4\langle T_{zz}(x)T_{zz}(0)\rangle + 4z^3\bar{z}\langle T_{zz}(x)\beta^i\Phi_i(0)\rangle - 6(z\bar{z})^2\langle\beta^i\Phi_i(x)\beta^j\Phi_j(0)\rangle$$

$$G_{ij}(g) = (x^2)^2\langle\Phi_i(x)\Phi_j(0)\rangle \geq 0$$

Then, in two dimensions:

$$\frac{d\tilde{c}}{d\log\mu} = \beta^i G_{ij}\beta^j$$

At fixed points:

The central charge–
our DOF counter

$$\beta^i(g^*) = 0 \qquad \Rightarrow \qquad \tilde{c}(g^*) = c(g^*)$$

Stationary at the fixed points

Zamolodchikov '86

Measuring the DOFs

We now have a “degrees of freedom” counter that is intuitive and has a fantastic property unique to quantum field theories

➡ But analogous to the time evolution of dissipative systems!

The existence of such a quantity in 4d would go a great distance in advancing our understanding of, *e.g.*, QCD

➡ Does such a quantity exist?

The Plan

- Local RGEs in QFTs
- Weyl consistency conditions
- A 2d example
- The 4d case
- The $2n$-d case ($n \geq 3$)
- The Future

Disclaimer I: we will only consider unitary, renormalizable, relativistic theories, *i.e.* we are being reasonable and unsuspecting physicists. The analysis changes completely without these requirements on the QFTs

The local RG flow

- Let the sliding scale μ be a function of spacetime (let the cutoff in a theory be spacetime dependent): $\mu \to \mu(x)$
- First consider making couplings and the (spacetime) metric a function of spacetime, and then relate these to $\mu(x)$

$$g^i \to g^i(x)$$

Acts as a source for composite operators:

$$\mathcal{O}_i(x) = \frac{\delta S}{\delta g^i(x)}$$

$$\gamma_{\mu\nu} \to \gamma_{\mu\nu}(x)$$

Acts as a source for the energy momentum tensor:

$$T^{\mu\nu}(x) = \frac{\delta S}{\delta \gamma_{\mu\nu}(x)}$$

These tools should allow us to compute something like *c*-function from 2d, but in any dimension

But what have the $g^i(x)$ and $\gamma^{\mu\nu}(x)$ got to do with the RG?

The “global” RG flow

- Take a step back- the quantity that concerns us is the generating functional W:

$$e^W = \int [d\phi] e^{-S}$$

- The Callan-Symanzik equation tells us how to compensate the sliding of μ by changing the couplings:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta^i \frac{\partial}{\partial g^i}\right) W = 0$$

The “global” RG flow

- Now let W be a function of a constant background metric $\gamma^{\mu\nu}$, so that rescalings of the length $|x| = \sqrt{\gamma_{\mu\nu}x^\mu x^\nu}$ are equivalent to rescalings of the metric:

$$\left(\mu\frac{\partial}{\partial\mu} + 2\gamma^{\mu\nu}\frac{\partial}{\partial\gamma^{\mu\nu}}\right) W = 0$$

- This relates scalings of the metric and couplings:

$$\left(2\gamma^{\mu\nu}\frac{\partial}{\partial\gamma^{\mu\nu}} - \beta^i\frac{\partial}{\partial g^i}\right) W = 0$$

By turning this into a local equation, we can capture the effects of Weyl transformations on our theory

(which, in turn, will tell us about (hopeful) c-function candidates)

The local RG flow

By sending $g^i \to g^i(x) \qquad \gamma_{\mu\nu} \to \gamma_{\mu\nu}(x)$

The appropriate transformations become

$$\left(\Delta_\sigma^W - \Delta_\sigma^\beta\right) W = \text{stuff from } x \text{ dependence of } \gamma^{\mu\nu} \text{ and } g^i$$

With

$$\Delta_\sigma^W = 2\int d^dx\sqrt{\gamma}\;\sigma\;\gamma^{\mu\nu}(x)\frac{\delta}{\delta\gamma^{\mu\nu}(x)}$$

$$\Delta_\sigma^\beta = \int d^dx\sqrt{\gamma}\;\sigma\;\beta^i(x)\frac{\delta}{\delta g^i(x)}$$

Here *σ(x)* is the Weyl transformation parameter

$$\delta\gamma^{\mu\nu}(x) \to 2\sigma(x)\gamma^{\mu\nu}(x)$$

$$\delta g^i(x) \to \sigma(x)\beta^i(x)$$

The local RG flow

But not so fast! Presuming we had properly renormalized our theory beforehand, W must now include new counterterms introduced by the spacetime dependence of g^i and $\gamma^{\mu\nu}$:

$$\left(\Delta_\sigma^W - \Delta_\sigma^\beta\right) W = \text{terms with derivatives of } \gamma_{\mu\nu},\ g^i, \text{ and } \sigma$$

This looks like a theory on a curved background- for a flat background and constant g^i and σ, we reacquire the Callan-Symanzik equation

This is our local RG equation

Note an equivalent form:

$$\gamma^{\mu\nu} T_{\mu\nu} = \beta^i \mathcal{O}_i + \text{ terms with derivatives on } \gamma_{\mu\nu},\ g^i$$

The general form of the Weyl anomaly

Weyl consistency conditions

The Weyl group is Abelian, so its generators obey

$$\left[\Delta^W_\sigma - \Delta^\beta_\sigma, \Delta^W_{\sigma'} - \Delta^\beta_{\sigma'}\right] W = 0$$

We can use this to constrain the form of the (Weyl) anomaly, much as Wess and Zumino did for SU(3) x SU(3)

In particular, might these consistency conditions pick out a c-function?

Osborn '91

Weyl CCs in 2d

Disclaimer II: we will only consider dimensionless couplings and flows driven by marginal operators.

Osborn '91
Jack and Osborn '13

In 2d, by power counting, diff-invariance, *etc.*

$$\left(\Delta_\sigma^W - \Delta_\sigma^\beta\right) W =$$

$$\int dv\, \sigma \left(\frac{1}{2} c R - \frac{1}{2}\chi_{ij}\partial_\mu g^i \partial^\mu g^j\right) - \int dv\, \partial_\mu \sigma w_i \partial^\mu g^i$$

There is one equation implied by the consistency conditions:

$$\partial_i (c + w_j \beta^j) = \chi_{ij}\beta^j + (\partial_i w_j - \partial_j w_i)\beta^j$$

Contracting with β^i:

$$\frac{d\tilde{c}}{d\log\mu} = \beta^i \chi_{ij} \beta^j \quad \text{with} \quad \tilde{c} = c + w_i \beta^i$$

It certainly looks like a c-theorem…

A c-theorem in 2d?

But will it blend?

Recall that the local RG was a statement of the Weyl anomaly- from there we can take functional derivatives to get correlation functions:

$$\left(\Delta^W_\sigma - \Delta^\beta_\sigma\right) W = \int dv\, \sigma \left(\frac{1}{2} c R - \frac{1}{2}\chi_{ij}\partial_\mu g^i \partial^\mu g^j\right) - \int dv\, \partial_\mu \sigma w_i \partial^\mu g^i$$

$$\downarrow$$

$$\langle T_{\rho\rho}(x) T_{\mu\nu}(0)\rangle - \langle \Theta(x) T_{\mu\nu}(0)\rangle = c(\partial^2 \delta_{\mu\nu} - \partial_\mu \partial_\nu)\delta^{(2)}(x)$$

$$\langle T_{\rho\rho}(x)\mathcal{O}_i(0)\rangle - \langle \Theta(x)\mathcal{O}_i(0)\rangle = w_i \partial^2 \delta^{(2)}(x)$$

$$\Theta = \beta^i \mathcal{O}_i$$

And...

$$\mathcal{D}\langle T_{\mu\nu}(x) T_{\rho\sigma}(0)\rangle = 0$$

$$\mathcal{D} = \mu \frac{\partial}{\partial \mu} + \beta^i \frac{\partial}{\partial g^i}$$

The consistency conditions are fundamentally a relation amongst correlation functions

$$\mathcal{D}\langle \mathcal{O}_i(x)\mathcal{O}_j(0)\rangle + \partial_i \beta^k \langle \mathcal{O}_k(x)\mathcal{O}_j(0)\rangle + \partial_j \beta^k \langle \mathcal{O}_i(x)\mathcal{O}_k(0)\rangle = -\chi_{ij}\partial^2 \delta^{(2)}(x)$$

Ambiguities in the CCs

Correlation functions containing contact terms are ambiguous

At the level of our consistency conditions, we can see this by adding to our generating functional (changing our scheme)

$$W' = -\int dv \left(\frac{1}{2}\alpha R - \alpha_{ij}\partial_\mu g^i \partial^\mu g^j \right)$$

This causes a shift in the parameters of the anomaly:

$$\delta c = \beta^i \partial_i \alpha, \qquad \delta w_i = -\partial_i \alpha + \alpha_{ij}\beta^j$$

$$\delta \chi_{ij} = \beta^k \partial_k \alpha_{ij} + \partial_i \beta^k \alpha_{kj} + \partial_j \beta^k \alpha_{ik}$$

A c-theorem in 2d

It blends!

Now the consistency condition

$$\partial_i(c + w_j\beta^j) = \chi_{ij}\beta^j + (\partial_i w_j - \partial_j w_i)\beta^j$$

is invariant under the ambiguities.

$$\delta c = \beta^i \partial_i \alpha, \qquad \delta w_i = -\partial_i \alpha + \alpha_{ij}\beta^j$$

$$\delta\chi_{ij} = \beta^k \partial_k \alpha_{ij} + \partial_i \beta^k \alpha_{kj} + \partial_j \beta^k \alpha_{ik}$$

A relation amongst the correlation functions allows χ_{ij} to be related to Zamolodchikov's G_{ij}, once the ambiguities are used to identify the two

Hence the (2d) c-theorem is derived by means of the Weyl consistency conditions!

Shall we try 4d?

The consistency conditions procedure can be extended to arbitrary number of even dimensions

But let's not be too hasty and try 4d first:

$$\left(\Delta^W_\sigma - \Delta^\beta_\sigma\right) W = \int dv\, \sigma\, \mathcal{T} - \int dv\, \partial_\mu \sigma\, \mathcal{Z}^\mu$$

This form chosen for convenience- we can always integrate by parts to put these terms in $\mathcal{T}$

$$\mathcal{T} = cW^2 + aE_4 + \frac{1}{9} bR^2$$

$$+ \frac{1}{3}\chi^e_i \partial_\mu g^i \partial^\mu R + \frac{1}{6}\chi^f_{ij}\partial_\mu g^i \partial^\mu g^j R + \frac{1}{2}\chi^g_{ij}\partial_\mu g^i \partial_\nu g^j G^{\mu\nu}$$

$$+ \frac{1}{2}\chi^a_{ij}\nabla^2 g^i \nabla^2 g^j + \frac{1}{2}\chi^b_{ijk}\partial_\mu g^i \partial^\mu g^j \nabla^2 g^k + \frac{1}{4}\chi^c_{ijkl}\partial_\mu g^i \partial^\mu g^j \partial_\nu g^k \partial^\nu g^l$$

$$\mathcal{Z}^\mu = G^{\mu\nu} w_i \partial_\nu g^i + \frac{1}{3}\partial^\mu (dR) + \frac{1}{3} R Y_i \partial^\mu g^i$$

$$+ \partial^\mu\left(U_i \nabla^2 g^i + \frac{1}{2} V_{ij}\partial_\nu g^i \partial^\nu g^j\right) + S_{ij}\partial^\mu g^i \nabla^2 g^j + \frac{1}{2} T_{ijk}\partial_\nu g^i \partial^\nu g^j \partial^\mu g^k$$

Weyl CCs in 4d

Rinse, repeat

In 4d there are six equations implied by the consistency conditions, one of which is:

$$\partial_i(a + \frac{1}{8}w_j\beta^j) = \frac{1}{8}\chi^g_{ij}\beta^j + \frac{1}{8}(\partial_i w_j - \partial_j w_i)\beta^j$$

Exactly analogous to the 2d case

However, it is not clear that $\chi^g{}_{ij}$ can be related to a positive definite "metric" like in 2d- the correlation functions are not as clean

The 4d version of the *c*-theorem has been proven perturbatively

A weak version of the *c*-theorem has perhaps been proven

Jack and Osborn '91

Komargodski and Schwimmer '11

On to 6d

The 6d case is motivated mostly by string theoretic constructions and misguided curiosity

6d:

$$\left(\Delta^W_\sigma - \Delta^\beta_\sigma\right) W = \int dv\, \sigma \sum_{i=1}^{65} \mathcal{T}_i - \int dv\, \partial_\mu \sigma \sum_{i=1}^{30} \mathcal{Z}_i^\mu$$

Problems: there are 95 independent dim. 6 diff-invariant terms contributing to the trace anomaly

Opportunities: many patterns in the CCs emerge that were not apparent in 2d or 4d

We've got 95 problems but a candidate *c*-function ain't one

On to 6d

First: the pure dim. 6 curvature terms (known for some time):

$$K_1 = R^3, \qquad K_2 = RR^{\kappa\lambda}R_{\kappa\lambda}, \qquad K_3 = RR^{\kappa\lambda\mu\nu}R_{\kappa\lambda\mu\nu}, \qquad K_4 = R^{\kappa\lambda}R_{\lambda\mu}R^{\mu}{}_{\kappa},$$
$$K_5 = R^{\kappa\lambda}R_{\kappa\mu\nu\lambda}R^{\mu\nu}, \qquad K_6 = R^{\kappa\lambda}R_{\kappa\mu\nu\rho}R_{\lambda}{}^{\mu\nu\rho}, \qquad K_7 = R^{\kappa\lambda\mu\nu}R_{\mu\nu\rho\sigma}R^{\rho\sigma}{}_{\kappa\lambda},$$
$$K_8 = R^{\kappa\lambda\mu\nu}R_{\rho\lambda\mu\sigma}R_{\kappa}{}^{\rho\sigma}{}_{\nu}, \qquad K_9 = R\Box R, \qquad K_{10} = R^{\kappa\lambda}\Box R_{\kappa\lambda}, \qquad K_{11} = R^{\kappa\lambda\mu\nu}\Box R_{\kappa\lambda\mu\nu},$$
$$K_{12} = R^{\kappa\lambda}\nabla_{\kappa}\partial_{\lambda}R, \qquad K_{13} = \nabla^{\kappa}R^{\lambda\mu}\nabla_{\kappa}R_{\lambda\mu}, \qquad K_{14} = \nabla^{\kappa}R^{\lambda\mu}\nabla_{\lambda}R_{\kappa\mu},$$
$$K_{15} = \nabla^{\kappa}R^{\lambda\mu\nu\rho}\nabla_{\kappa}R_{\lambda\mu\nu\rho}, \qquad K_{16} = \Box R^2, \qquad K_{17} = \Box^2 R.$$

A more useful basis:

- The Euler density (1) Whose coefficient is our *c* candidate
- Local Weyl invariants (3) Like W^2 in 4d
- Trivial anomalies (6) Like $\Box$ R in 4d- coefficients shifted by an addition to the action
- "Vanishing" anomalies (7) Like R^2 in 4d- do not satisfy the CCs at the fixed points

Bonora, Pasti, and Bregola '86
Bastianelli, Cuoghi, and Nocetti '00

In the 6d anomaly

Curvature terms with dimension less than 6

$$\frac{1}{d-1}R, \qquad G_{\mu\nu}, \qquad R_{\kappa\lambda\mu\nu}, \qquad \frac{1}{d-1}\partial_\mu R, \qquad \nabla_\kappa G_{\mu\nu}$$

$$E_4 = \tfrac{2}{(d-2)(d-3)}(R^{\kappa\lambda\mu\nu}R_{\kappa\lambda\mu\nu} - 4R^{\kappa\lambda}R_{\kappa\lambda} + R^2),$$

$$I = R^{\kappa\lambda\mu\nu}R_{\kappa\lambda\mu\nu} - \tfrac{4}{d-2}R^{\kappa\lambda}R_{\kappa\lambda} + \tfrac{2}{(d-1)(d-2)}R^2, \qquad \tfrac{1}{(d-1)^2}R^2, \qquad \tfrac{1}{d-1}\Box R,$$

$$H_{1\mu\nu} = \tfrac{(d-2)(d-3)}{2}E_4\gamma_{\mu\nu} - 4(d-1)H_{2\mu\nu} + 8H_{3\mu\nu} + 8H_{4\mu\nu} - 4R^{\kappa\lambda\rho}{}_{\mu}R_{\kappa\lambda\rho\nu},$$

$$H_{2\mu\nu} = \tfrac{1}{d-1}RR_{\mu\nu}, \qquad H_{3\mu\nu} = R_\mu{}^\kappa R_{\kappa\nu}, \qquad H_{4\mu\nu} = R^{\kappa\lambda}R_{\kappa\mu\lambda\nu},$$

$$H_{5\mu\nu} = \Box R_{\mu\nu}, \qquad H_{6\mu\nu} = \tfrac{1}{d-1}\nabla_\mu\partial_\nu R,$$

$$\partial_\mu E_4, \qquad \partial_\mu I, \qquad \tfrac{1}{(d-1)^2}R\,\partial_\mu R, \qquad \tfrac{1}{d-1}\partial_\mu\Box R, \qquad \nabla^\nu H_{(2,3,4)\mu\nu}$$

In the 6d anomaly

Some terms from $\mathcal{T}$

$$\mathcal{T}_1 = -c_1 I_1, \qquad \mathcal{T}_2 = -c_2 I_2, \qquad \mathcal{T}_3 = -c_3 I_3, \qquad \mathcal{T}_4 = -a E_6, \qquad \mathcal{T}_{5,\dots,11} = -b_{1,\dots,7} L_{1,\dots,7},$$

$$\mathcal{T}_{12} = \mathcal{I}_i^1\, \partial_\mu g^i\, \partial^\mu E_4, \qquad \mathcal{T}_{13} = \mathcal{I}_i^2\, \partial_\mu g^i\, \partial^\mu I, \qquad \mathcal{T}_{14} = \frac{1}{25}\mathcal{I}_i^3\, \partial_\mu g^i\, R\, \partial^\mu R,$$

$$\mathcal{T}_{15} = \frac{1}{5}\mathcal{I}_i^4\, \partial_\mu g^i\, \partial^\mu \Box R \qquad \mathcal{T}_{16,17,18} = \mathcal{I}_i^{5,6,7}\, \partial_\mu g^i\, \nabla_\nu H^{\mu\nu}_{2,3,4},$$

$$\mathcal{T}_{19} = \frac{1}{2}\mathcal{G}_{ij}^1\, \partial_\mu g^i \partial^\mu g^j\, E_4, \qquad \mathcal{T}_{20} = \frac{1}{2}\mathcal{G}_{ij}^2\, \partial_\mu g^i \partial^\mu g^j\, I, \qquad \mathcal{T}_{21} = \frac{1}{50}\mathcal{G}_{ij}^3\, \partial_\mu g^i \partial^\mu g^j\, R^2,$$

$$\mathcal{T}_{22} = \frac{1}{10}\mathcal{G}_{ij}^4\, \partial_\mu g^i \partial^\mu g^j\, \Box R, \qquad \mathcal{T}_{23,\dots,28} = \frac{1}{2}\mathcal{H}_{ij}^{1,\dots,6}\, \partial_\mu g^i \partial_\nu g^j\, H^{\mu\nu}_{1,\dots,6}$$

In the 6d anomaly

Some terms from $\mathcal{Z}^\mu$

$$\mathcal{Z}_1^\mu = -b_8\,\partial^\mu E_4, \qquad \mathcal{Z}_2^\mu = -b_9\,\partial^\mu I, \qquad \mathcal{Z}_3^\mu = -\frac{1}{25}b_{10}\,R\,\partial^\mu R,$$

$$\mathcal{Z}_4^\mu = -\frac{1}{5}b_{11}\,\partial^\mu \Box R, \qquad \mathcal{Z}_{5,6,7}^\mu = -b_{12,13,14}\,\nabla_\nu H_{2,3,4}^{\mu\nu}$$

$$\mathcal{Z}_8^\mu = \mathcal{G}_i^1\,\partial^\mu g^i\,E_4, \qquad \mathcal{Z}_9^\mu = \mathcal{G}_i^2\,\partial^\mu g^i\,I, \qquad \mathcal{Z}_{10}^\mu = \frac{1}{25}\mathcal{G}_i^3\,\partial^\mu g^i\,R^2,$$

$$\mathcal{Z}_{11}^\mu = \frac{1}{5}\mathcal{G}_i^4\,\partial^\mu g^i\,\Box R, \qquad \mathcal{Z}_{12,\ldots,17}^\mu = \mathcal{H}_i^{1,\ldots,6}\,\partial_\nu g^i\,H_{1,\ldots,6}^{\mu\nu}$$

Weyl CCs in 6d

In 6d there are thirty-six equations implied by the consistency conditions, one of which is *again*:

$$\partial_i(a + \frac{1}{6}b_1 - \frac{1}{90}b_3 + \frac{1}{6}\mathcal{H}^1_j\beta^j) = \frac{1}{6}\mathcal{H}^1_{ij}\beta^j + \frac{1}{6}(\partial_i\mathcal{H}^1_j - \partial_j\mathcal{H}^1_i)\beta^j$$

New terms- from vanishing anomalies

First point: we still don't know if we can relate $\mathcal{H}^1_{ij}$ to a positive definite metric

Second point: there must be some mechanism that guarantees such a CC will show up in any (even) dimension

Weyl CCs in 2n-d

Now notice the following pattern:

$$
\begin{array}{ll}
d=2 & 0=\delta_\sigma \int d^2x\,\sqrt{\gamma}\,R=\int d^2x\,\sqrt{\gamma}\,\gamma^{\mu\nu}\,\nabla_\mu\partial_\nu\sigma \\
d=4 & 0=\delta_\sigma \int d^4x\,\sqrt{\gamma}\,E_4=-8\int d^4x\,\sqrt{\gamma}\,G^{\mu\nu}\,\nabla_\mu\partial_\nu\sigma \\
d=2n & 0=\delta_\sigma \int d^{2n}x\,\sqrt{\gamma}\,E_{2n}=-8\int d^{2n}x\,\sqrt{\gamma}\,H^{\mu\nu}\,\nabla_\mu\partial_\nu\sigma
\end{array}
$$

Upon integrating by parts, in 2n-d

$$\nabla_\mu\nabla_\nu H^{\mu\nu}=0$$

In fact, it was shown in the 70s that $H^{\mu\nu}$ is the unique tensor with properties of the Einstein tensor:

$$\nabla_\nu H^{\mu\nu}=0 \text{ and } H^{\mu\nu}=H^{\nu\mu}$$

With *n-1* powers of curvature, etc.

The existence of this tensor is crucial to finding a *c* candidate in 2n-d!

Lovelock ’71

Weyl CCs in $2n$-d

The c-candidate terms are all proportional to

$$(\sigma\partial_\mu\sigma' - \sigma'\partial_\mu\sigma)H_1^{\mu\nu}$$

Example: A case in 6d where this does *not* occur:

This consistency condition is proportional to

$$(\sigma\partial_\mu\sigma' - \sigma'\partial_\mu\sigma)H_4^{\mu\nu}$$

$$\partial_i(-b_1 + \frac{2}{3}b_7 + \frac{1}{12}\mathcal{H}^4_j\beta^j) = \frac{1}{12}(\mathcal{H}^4_{ij} + \frac{1}{2}\mathcal{F}_{ij})\beta^j + \frac{1}{12}\partial_{[i}\mathcal{H}^4_{j]}\beta^j + \frac{1}{6}\mathcal{I}^7_i$$

This ruins everything!

But in fact, the appearance of this extra term $\sim\chi_i$ is generic, and ruins any hopes of c-function candidates other than that coming from the special $H^{\mu\nu}$ consistency condition

The coefficient of the Euler density and its associated CC are indeed quite exceptional, and is the only candidate CC that permits a c-function interpretation

Weyl CCs in $2n$-d

A consistency condition in any even-dimensional spacetime:

$$\partial_i \tilde{a} = \mathcal{H}_{ij}\beta^j + (\partial_i \mathcal{H}_j - \partial_j \mathcal{H}_i)\beta^j$$

Invariant under the arbitrariness of the $2n$-d Weyl anomaly

Coincides with the coefficient of E_{2n} at fixed points

Our metric in coupling space

The monotonicity of a function analogous to the *c*-function in 2d can be established if $\mathcal{H}_{ij}$ is shown to be positive definite

This has been done in 2d (χ_{ij}) and perturbatively in 4d ($\chi^g{}_{ij}$)

"Consequences of Weyl consistency conditions"

The Future

- Perturbative proof that $\mathcal{H}_{ij}$ is positive definite about a UV fixed point in 6d (in progress)
- Effects of (ir)relevant operators on montonicity results
- Weyl CCs in QFTs with a boundary
- Postdoc applications

Thank you!