

# Improving Vector Boson Fusion (VBF) LHC Higgs Analyses with Fox- Wolfram Moments

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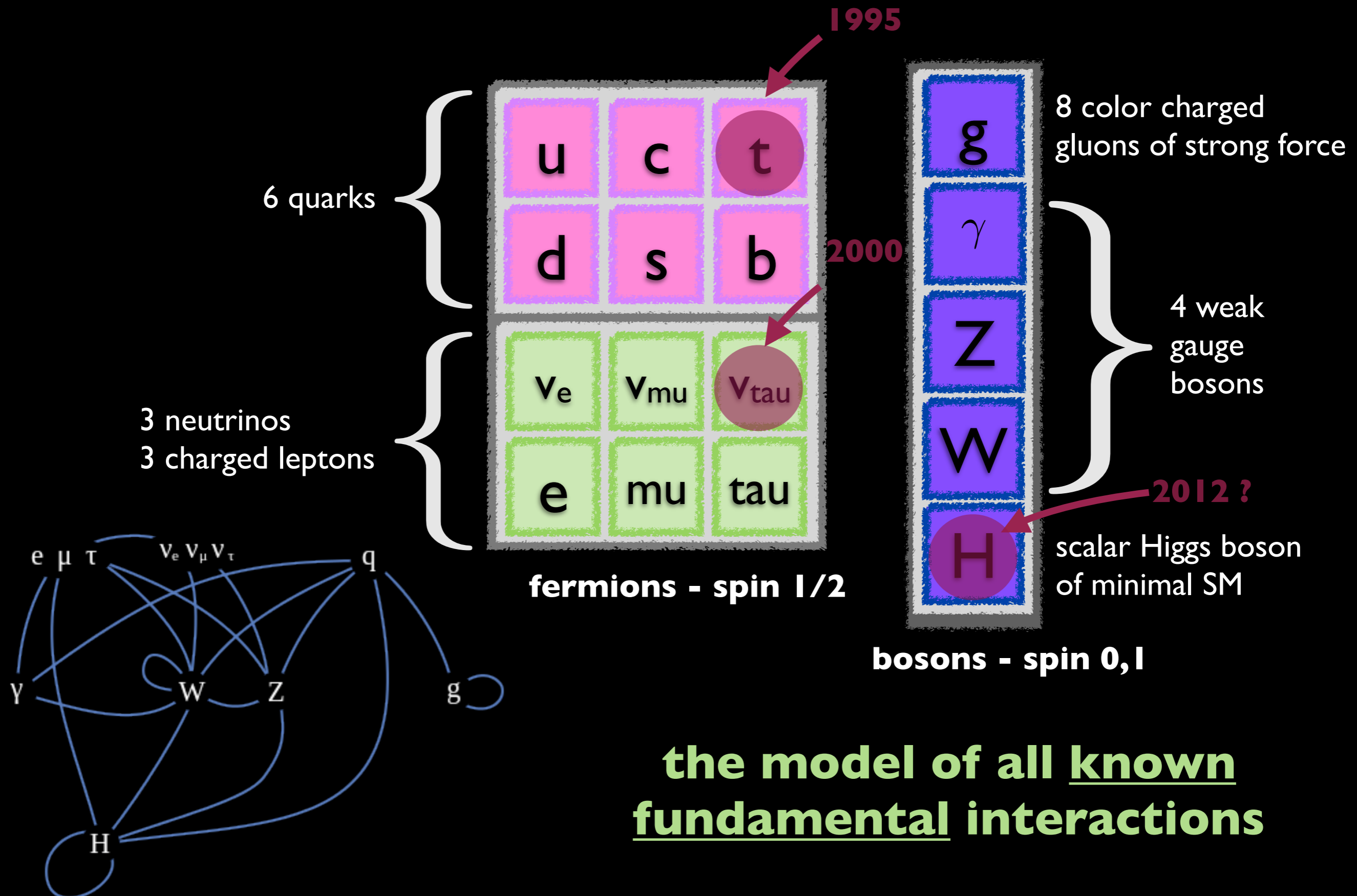
this work supported by BMBF



# Outline

- Review of Standard Model Higgs Mechanism (4)
- Phenomenology of Standard Model Higgs (6)
- Fox - Wolfram Moments (8)
- Results of Cut-Based and Boosted Decision Tree Analyses (16)

# The Standard Model Of Particle Physics



# Electroweak Symmetry Breaking in the Standard Model - QED as a toy model

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_{\mu}\phi|^2 - V(\phi)$$

covariant derivative:  $D_{\mu} = \partial_{\mu} - ieA_{\mu}$

simplest, renormalizable,  
U(1) invariant potential

$$\phi \rightarrow e^{-ie\eta(x)}\phi(x)$$

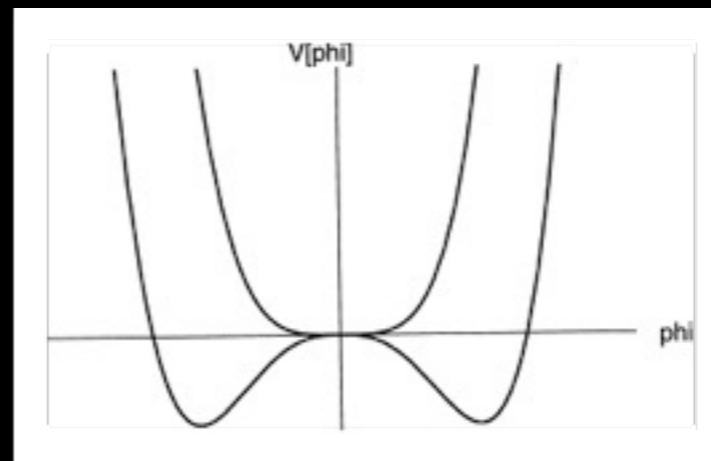
$$V(\phi) = \mu^2|\phi|^2 + \lambda(|\phi|^2)^2$$

$$\mu^2 < 0$$

$$\mu^2 > 0$$

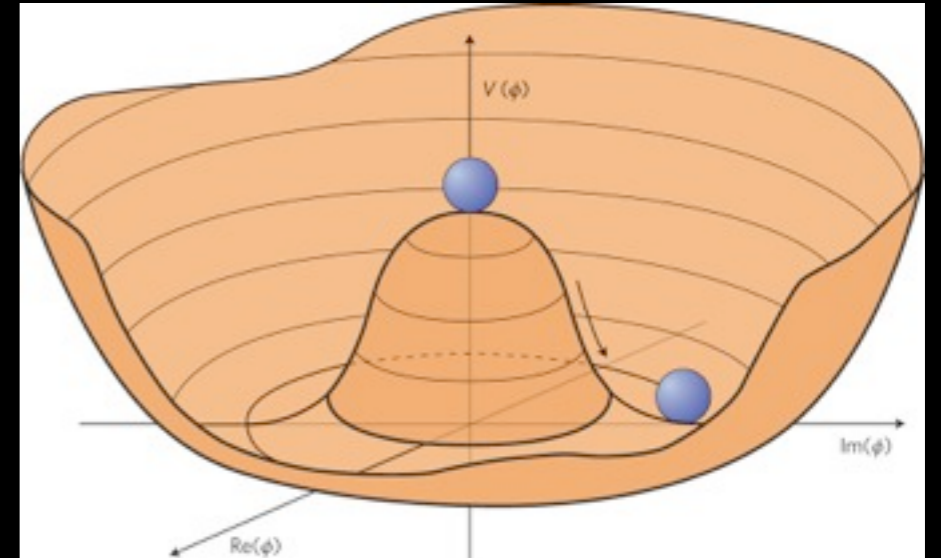
$$\langle\phi\rangle = 0$$

$$\langle\phi\rangle = \sqrt{-\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$



$$\langle \phi \rangle = \sqrt{-\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$$

U(1) symmetry  
is broken with  
nonzero vev



expand ito of non-vev  
fields

$$\phi \equiv \frac{1}{\sqrt{2}} e^{i\frac{x}{v}} (v + h)$$

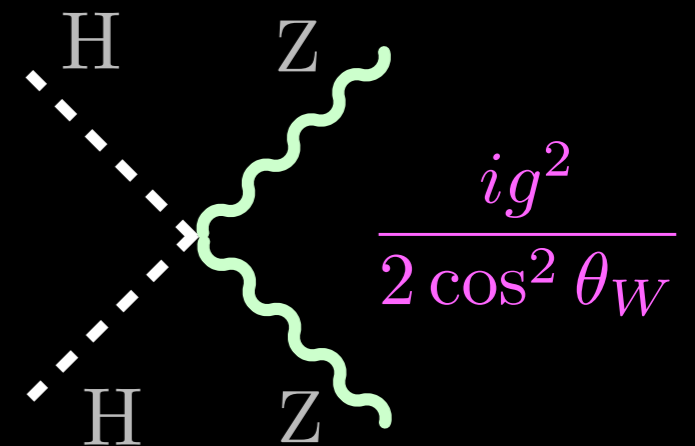
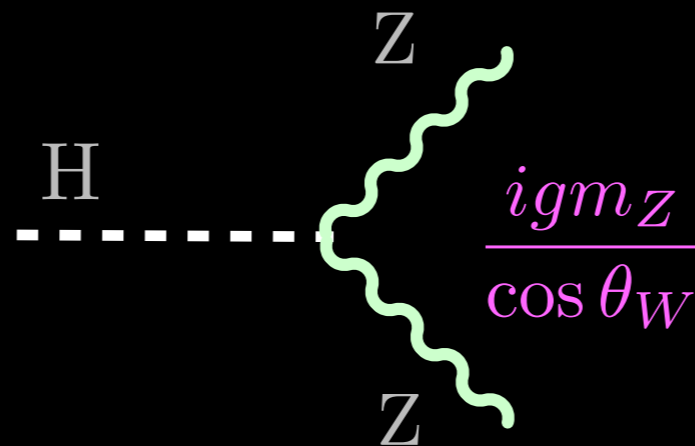
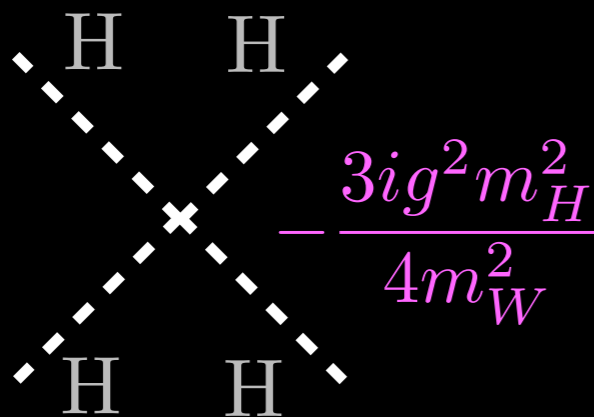
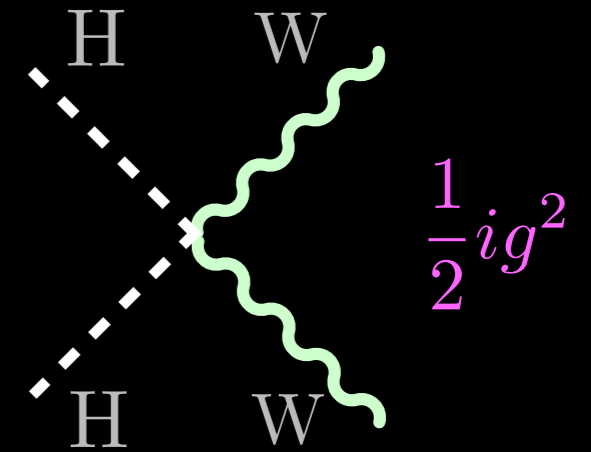
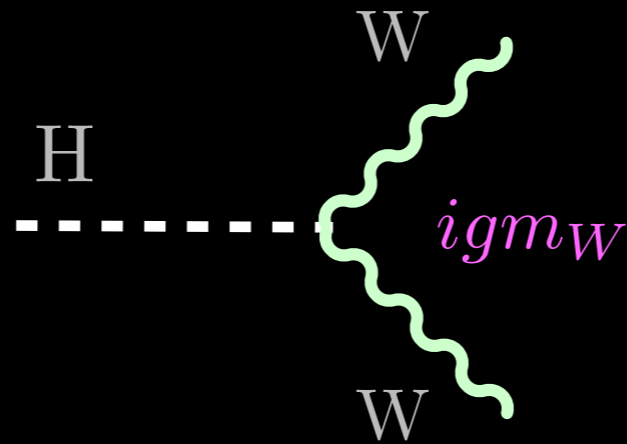
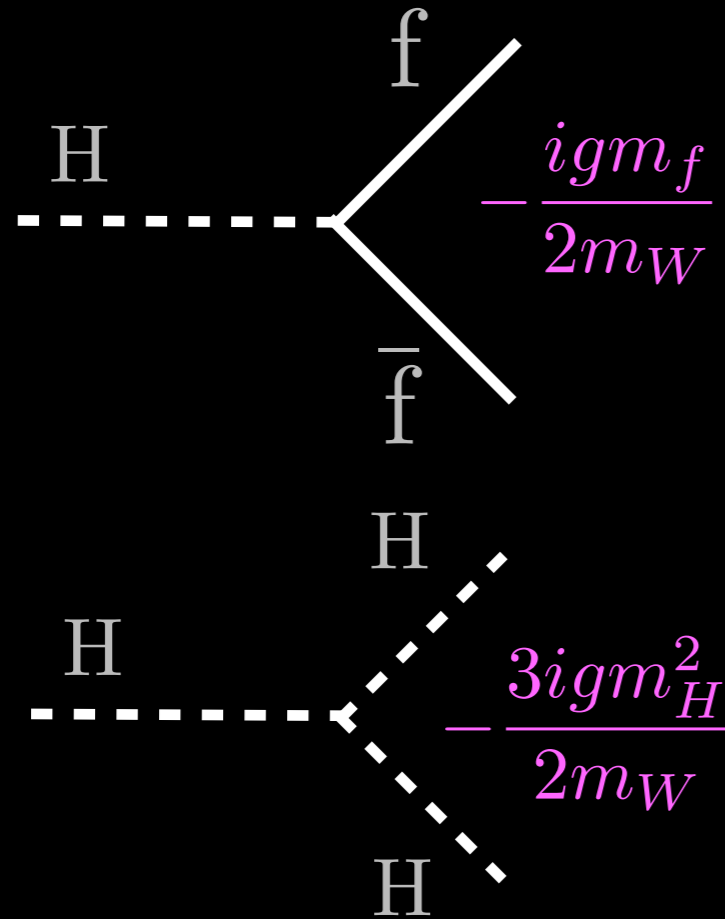
and mass is acquired:

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - ev A_{\mu} \partial^{\mu} \chi + \frac{e^2 v^2}{2} A_{\mu} A^{\mu} + \dots$$

photon field with  $M_A = ev$

same principle, when applied to electroweak theory causes  
weak bosons to acquire mass - Higgs field emerges as  
physical particle....

# SM Higgs Interactions

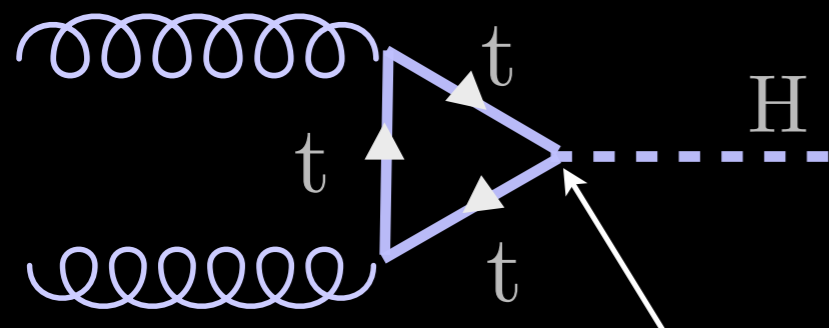


**coupling strengths proportional to masses**

- once  $m_H$  is known, couplings can be measured and compared to SM prediction

# SM Higgs Production at the LHC

the gluon fusion channel - **main** LHC production mechanism



“gluon fusion” ggF

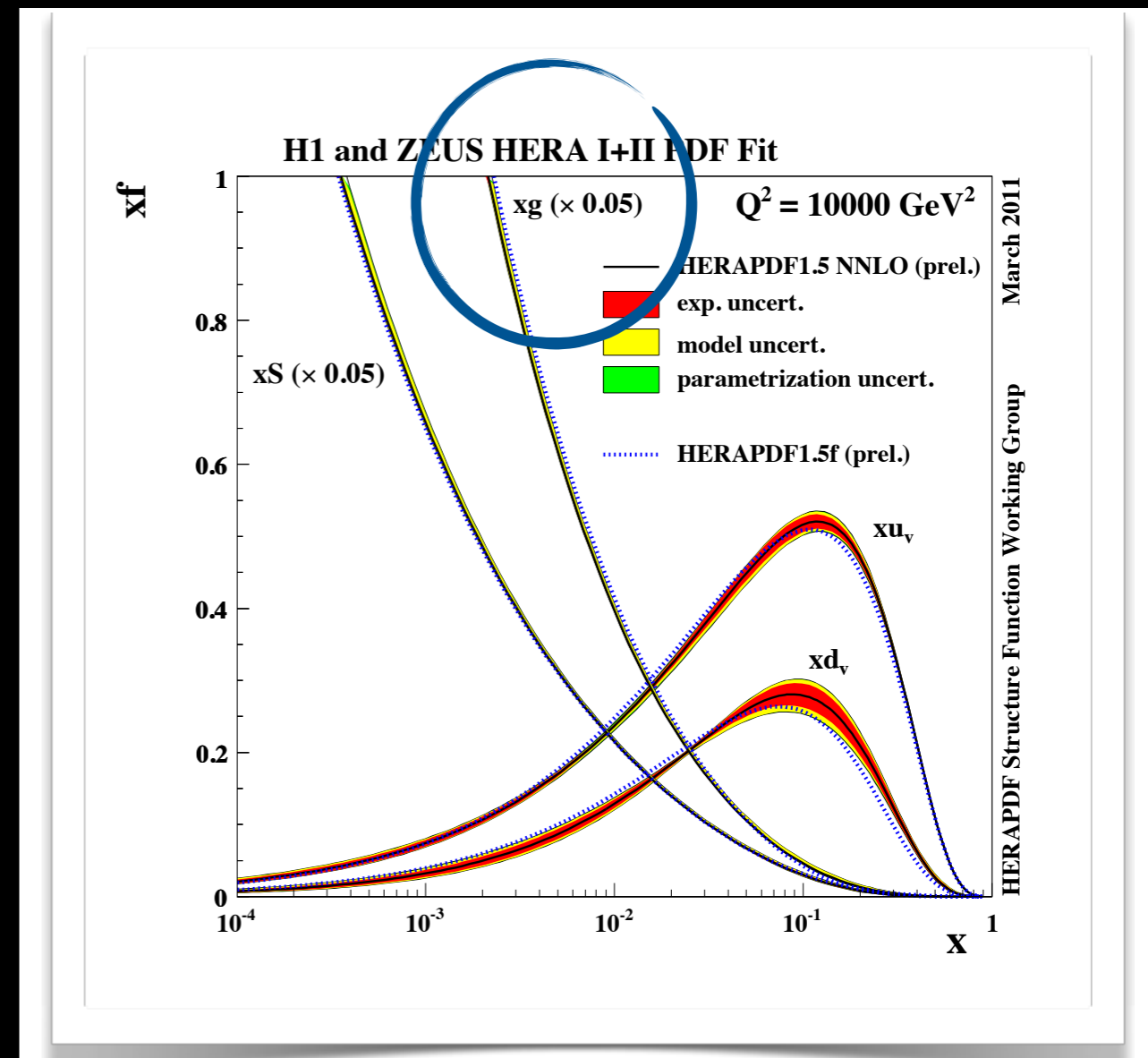
bottom loop suppressed  
by  $\sim 0.1\%$  - lighter  
quark loops even less  
likely

$$\sigma(gg \rightarrow H) \approx 15 \text{ pb at } 7 \text{ TeV}$$

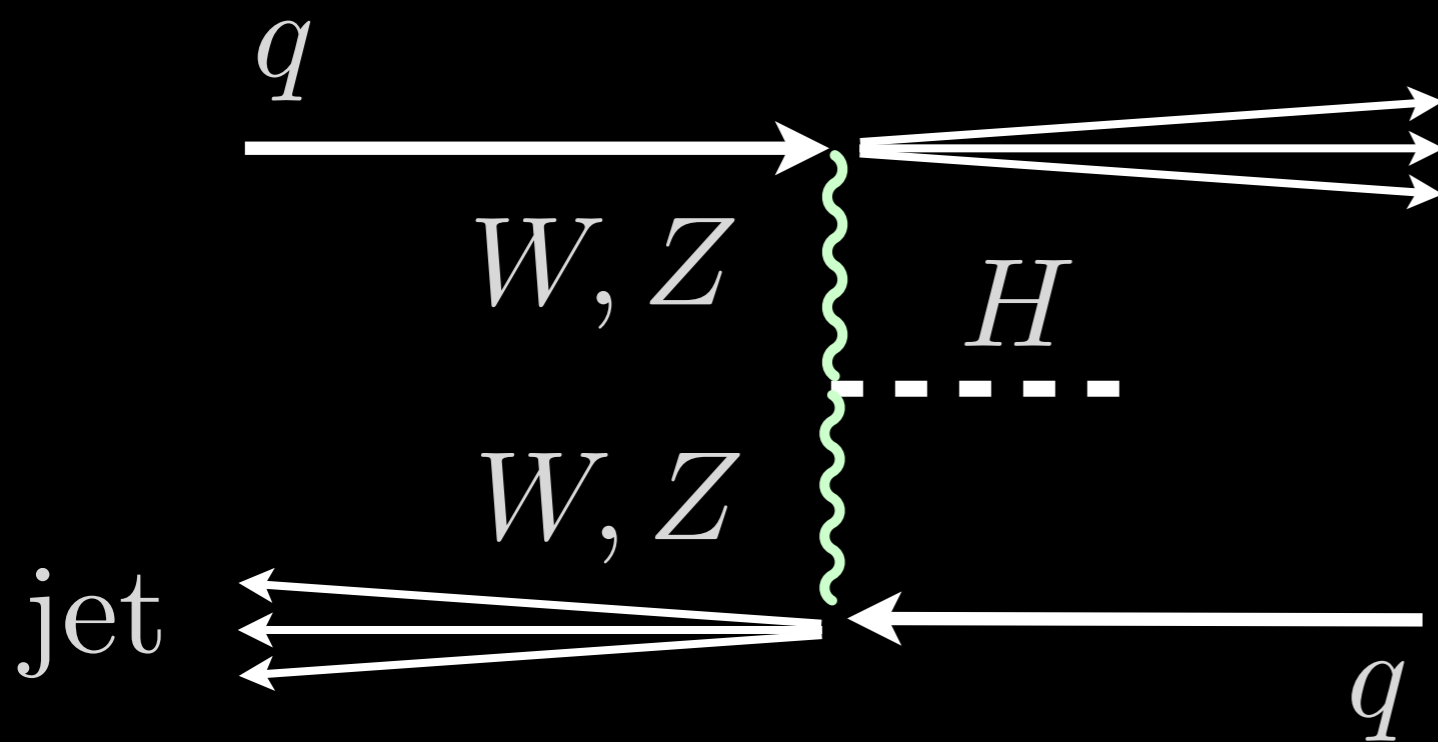
$$\sigma(gg \rightarrow H) \approx 50 \text{ pb at } 14 \text{ TeV}$$

for  $M_H = 125 \text{ GeV}$

why?! more likely to find a gluon in the proton



# SM Higgs Production at the LHC



## Vector Boson Fusion

essential probe of EW  
higgs couplings - deviations from  
predicted rates could indicate BSM  
higgs physics

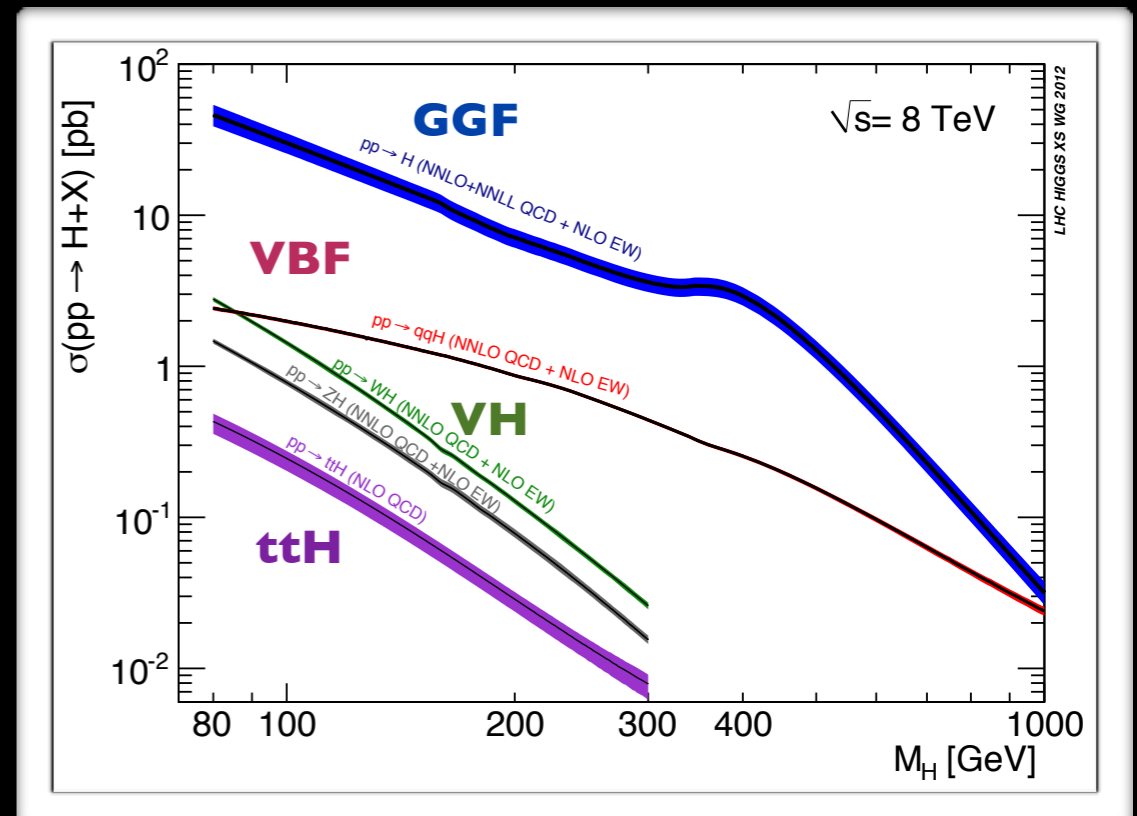
$$\sigma(qqH) \approx 1.3 \text{ pb at } 7 \text{ TeV}$$

$$\sigma(qqH) \approx 4 \text{ pb at } 14 \text{ TeV}$$

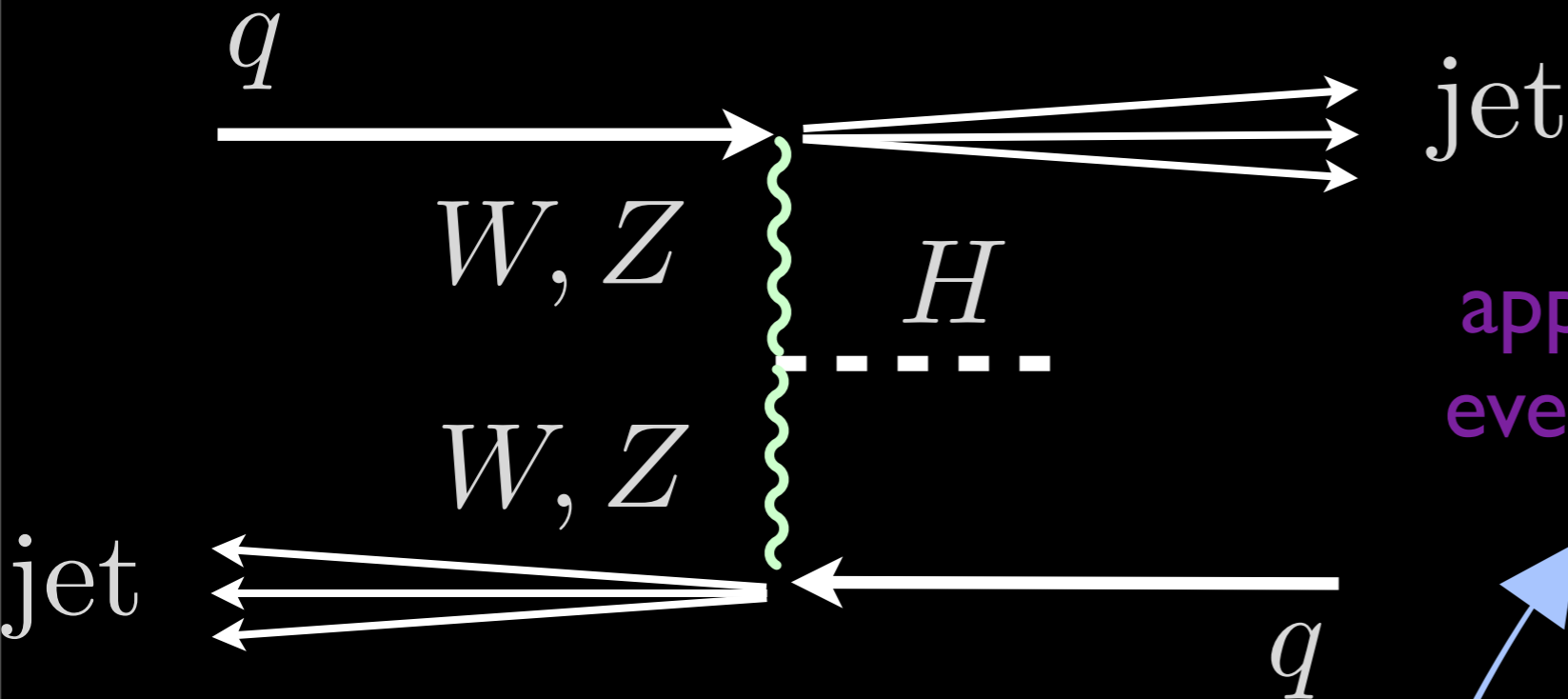
for  $M_H = 125 \text{ GeV}$

distinctive “forward - backward” jet  
topology unlike any background  
processes

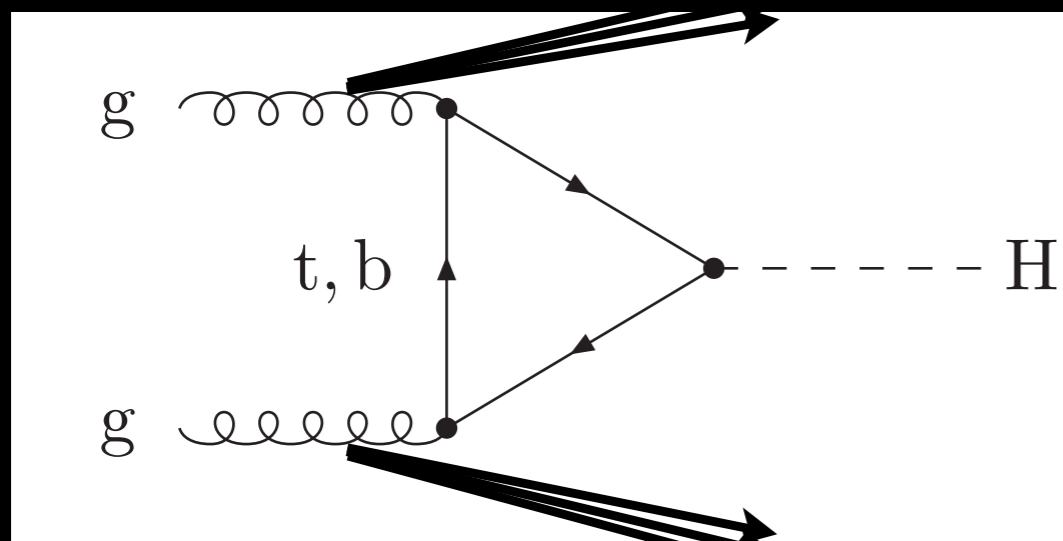
lack of central jet activity - handle for  
discerning from backgrounds



# SM Higgs Production at the LHC



**ggH + 2jet production could mimic VBF production**



**solution:**

apply acceptance criteria on events to disfavor ggH + 2jet kinematics

$$p_{Tj_1j_2} > 20 \text{ GeV}$$

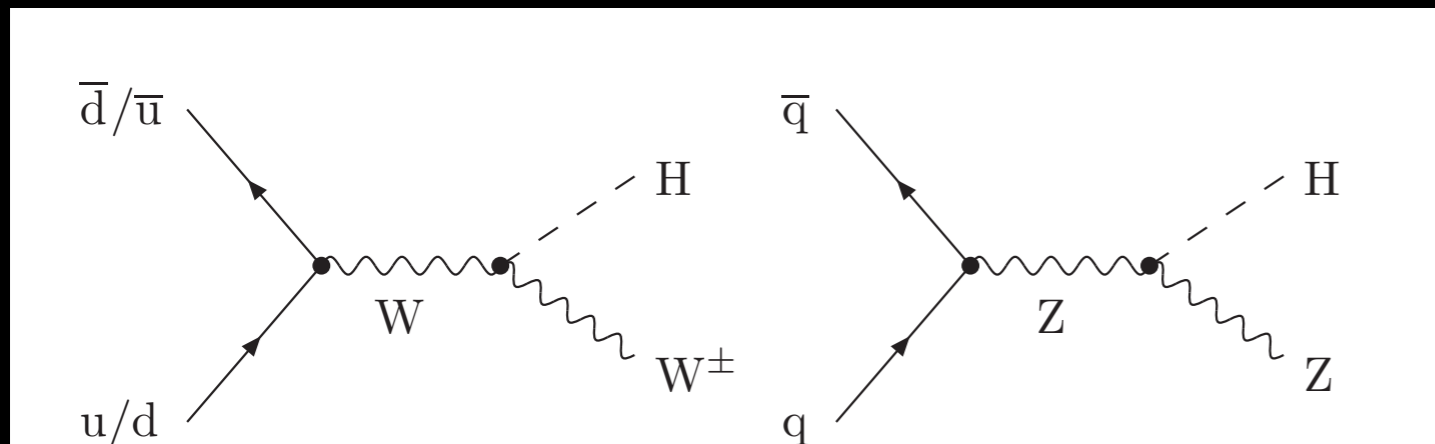
$$\eta_{j_1} \cdot \eta_{j_2} < 0$$

$$\Delta\eta_{j_1,j_2} > 4$$

after applying VBF selection, gg events contribute only 4% - 5% to Higgs production

# SM Higgs Production at the LHC

## the **Higgs-strahlung** channel

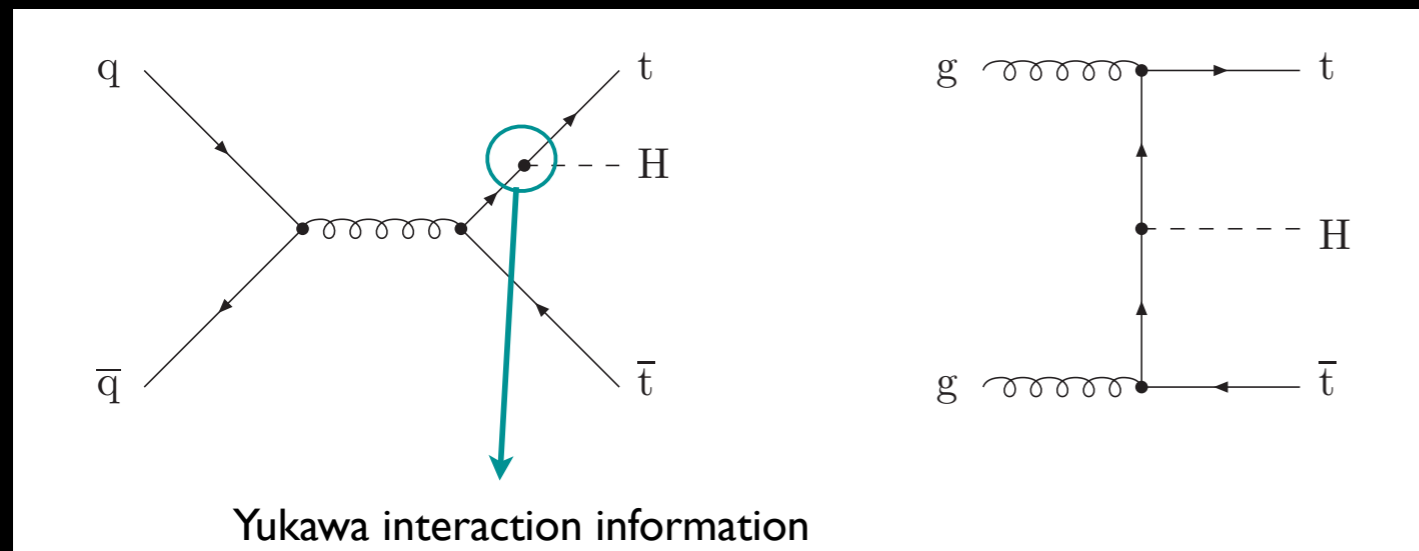


$$\sigma(W, ZH) \approx 0.6 \text{ pb at } 7 \text{ TeV}$$

$$\sigma(W, ZH) \approx 1.5 \text{ pb at } 14 \text{ TeV}$$

for  $M_H = 125 \text{ GeV}$

## the **$t\bar{t}H$** channel - Higgs in association with a top quark



$$\sigma(t\bar{t}H) \approx 88 \text{ fb at } 7 \text{ TeV}$$

$$\sigma(t\bar{t}H) \approx 611 \text{ fb at } 14 \text{ TeV}$$

for  $M_H = 125 \text{ GeV}$

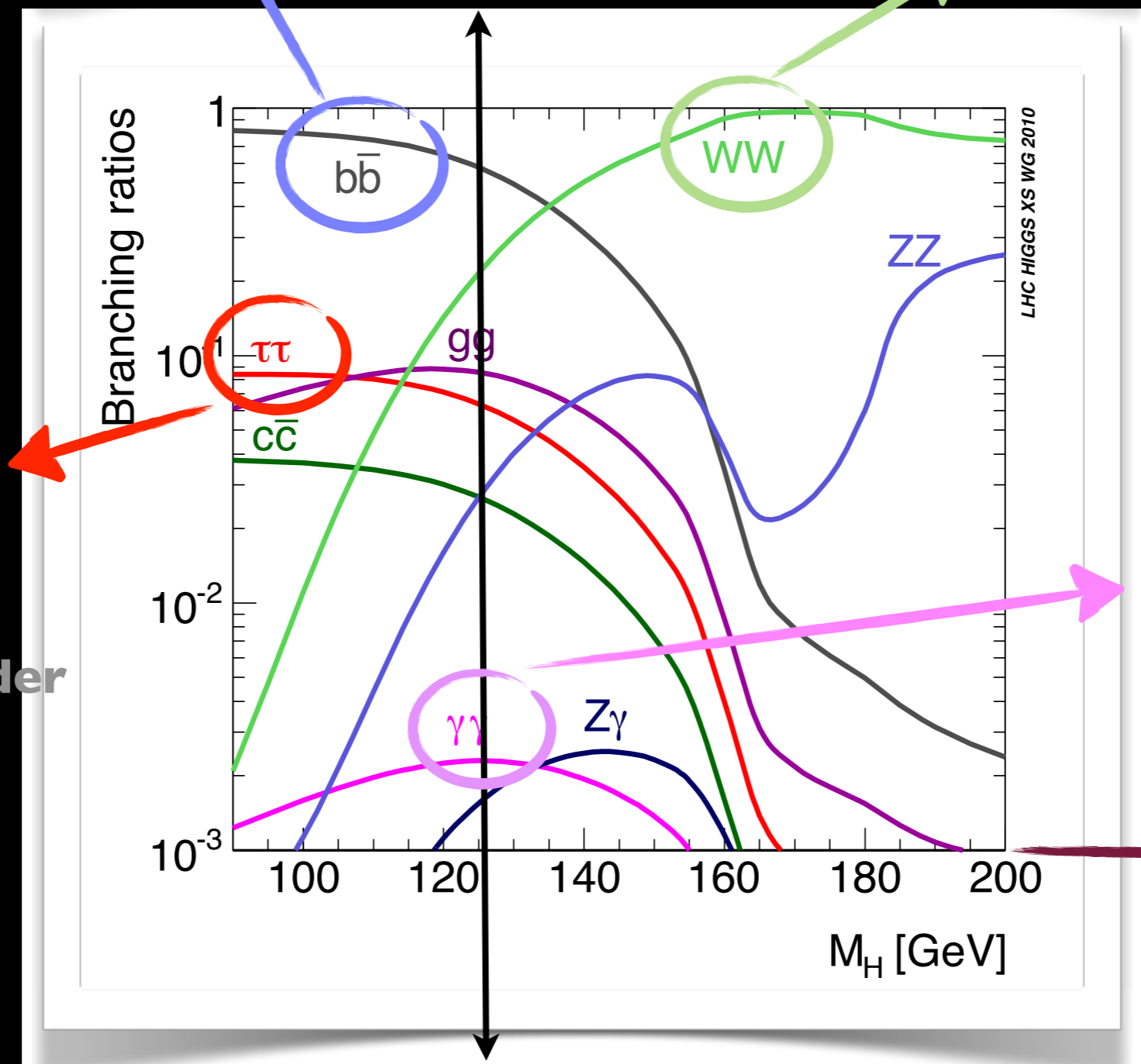
# SM Higgs Decay at the LHC

high decay rate, but  
b tagging efficiency  
at 60%  
 $H \rightarrow b\bar{b}$

$WW \rightarrow l\nu l\nu$

clean and  
efficient,  
background under  
control

$\tau \rightarrow l\nu\nu$   
clean and  
efficient,  
background under  
control

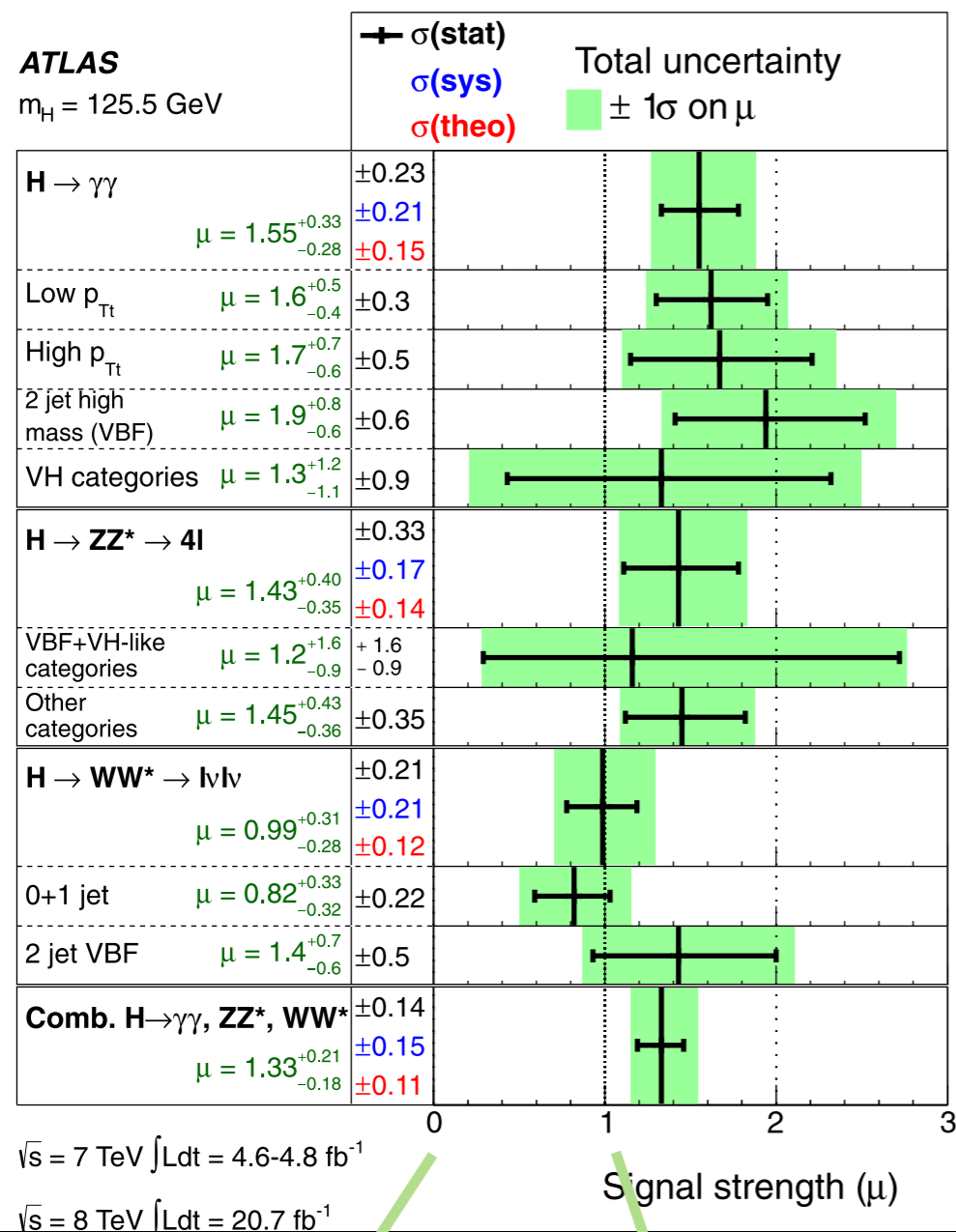


comparatively  
low rate, but very  
clean and efficient

$H \rightarrow t\bar{t}$

kicks in at 350  
GeV

# LHC Higgs-like Boson Discovery



**combined mass measurement:**

**ATLAS:**

$$m_H = 125.5 \pm 0.2(\text{stat})^{+0.5}_{-0.6}(\text{sys}) \text{ GeV}$$

**CMS:**

$$m_H = 125.3 \pm 0.4(\text{stat}) \pm 0.5(\text{sys}) \text{ GeV}$$

**combined signal strength measurement:**

**ATLAS:**

$$\mu = 1.33 \pm 0.14(\text{stat}) \pm 0.15(\text{sys})$$

**CMS:**

$$\hat{\mu} = \frac{\sigma}{\sigma_{\text{SM}}} = 0.87 \pm 0.23 \quad \text{for } M_H = 125 \text{ GeV}$$

$\mu = 0$   
background only  
hypothesis

$\mu = 1$   
SM Higgs boson  
hypothesis

**consistent with SM Higgs  
hypothesis**

# Theoretical Uncertainties in Higgs Measurement

large systematic uncertainty from **higher order QCD calculations matched to parton shower** - common to both ATLAS and CMS

## ATLAS<sup>1</sup>

Source (theory)	Uncertainty (%)
QCD scale	$\pm 8$ (ggF), $\pm 1$ (VBF, VH), $^{+4}_{-9}$ (ttH)
PDFs + $\alpha_s$	$\pm 8$ (ggF, ttH), $\pm 4$ (VBF, VH)

## CMS<sup>2</sup>

Source	Range (%)
Integrated luminosity	2.2-4.4
Lepton identification and trigger efficiency (per lepton)	3
Z( $\nu\nu$ )H triggers	2
Jet energy scale	2-3
Jet energy resolution	3-6
Missing transverse energy	3
b-tagging efficiency	3-15
Signal cross section (scale and PDF)	4
Signal cross section ( $p_T$ boost, EWK/QCD)	5-10/10
Statistical precision of signal simulation	1-5
Backgrounds estimated from data	10
Backgrounds estimated from simulation	30

include more EW and QCD higher-order corrections, **resum EW Sudakov logs in VHbb ...**

better match parton shower to existing NLO and NNLO and implement in simulation tool **SHERPA, MC@NLO, POWHEG, MADGRAPH ...**

# The Fox-Wolfram Moments<sup>1</sup>

a rotationally invariant set of observables  
constructed from Legendre polynomials

$$H_\ell = \sum_{i,j} \frac{|\vec{p}_i| |\vec{p}_j|}{s} P_\ell(\cos \Omega_{ij})$$

correlations  
between  
hadrons, jets,  
calorimeter  
entries...

weight factor

total angle between  
objects

$$\cos \Omega_{ij} = \cos \theta_i \cos \theta_j + \sin \theta_i \sin \theta_j \cos(\phi_i - \phi_j)$$

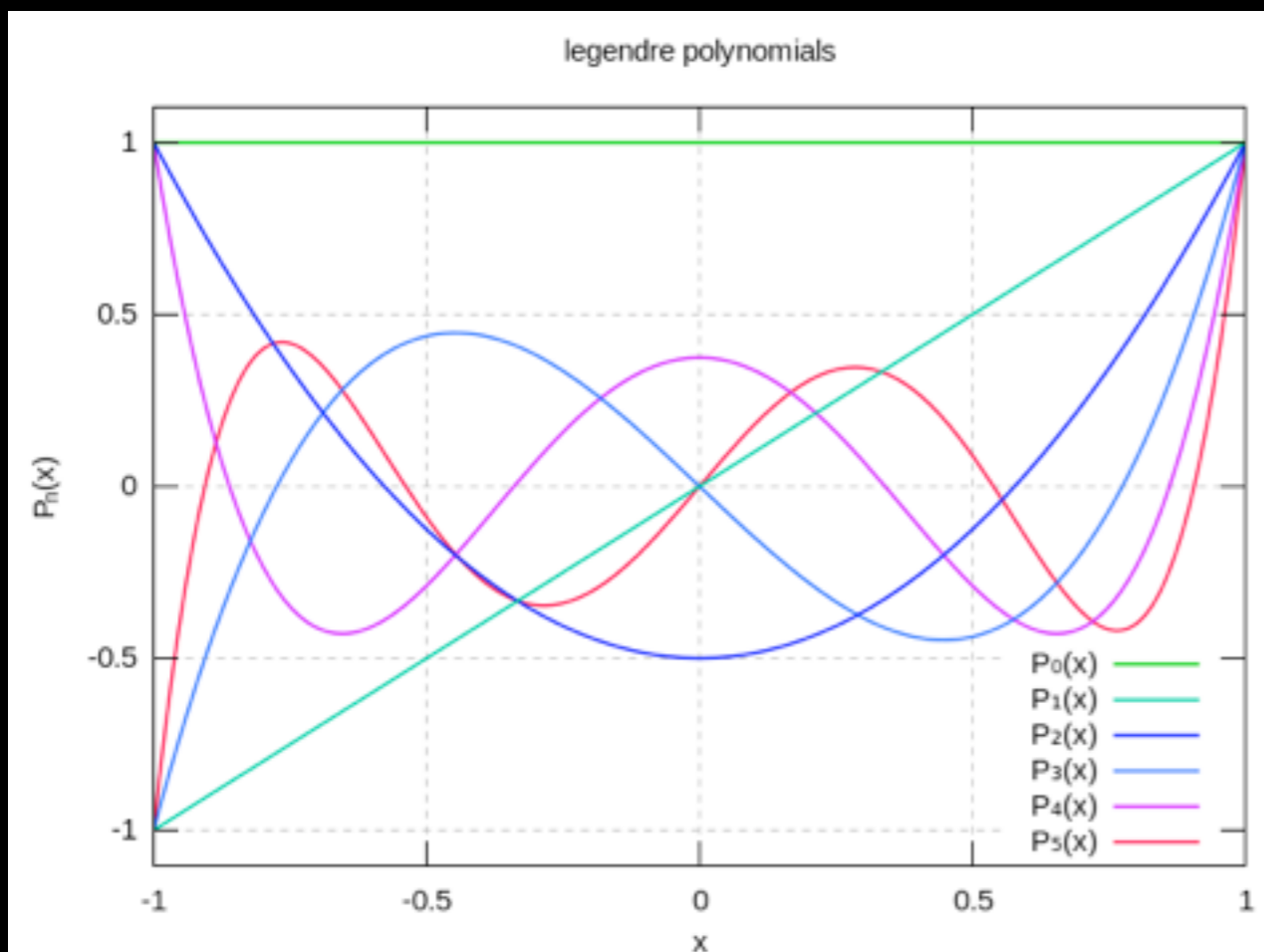
<sup>1</sup>Fox, Wolfram, PRL 1978

# Legendre Polynomials

occur as series solution to Laplace's equation in spherical coordinates

$$\frac{d}{dx} \left[ (1 - x^2) \frac{d}{dx} P_n(x) \right] + n(n + 1) P_n(x) = 0$$

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2 - 1)^n]$$



$$P_0(x) = 1, \quad P_1(x) = x$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x)$$

$\vdots$

$$P_7(x) = \frac{1}{16}(429x^7 - 693x^5 + 315x^3 - 35x)$$

# **The Fox-Wolfram Moments**

an event shape observable describing correlations  
between four-momentum objects

- ✦  **$e^+ e^-$  to jets**

**Fox, Wolfram Nucl. Phys. B 149 (1979) 413-496**

- ✦ **Top Quark signal at Tevatron**

**Field, Kanev, Tayebnejad PRD 55, 9 (1997)**

- ✦ **B meson decays at Belle:**

**Toru Iijima, hep-ex 0105005 (2001)**

- ✦ **Higgs physics at the LHC: VBF H  $\tau\tau$  vs  $Z+2j$  and Top Pair**

**C.B., Buschmann, Butter, Plehn PRD 87, 073014 (2013)**

- ✦ **A Multivariate study of Fox-Wolfram Moments for Higgs Analyses at the LHC**

**C.B., Mellado, Plehn, Ruan, Schichtel, in preparation**

# The Fox-Wolfram Moments

$$H_\ell = \sum_{i,j} \frac{|\vec{p}_i| |\vec{p}_j|}{s} P_\ell(\cos \Omega_{ij})$$

“weight factor”

$$0 \leq H_\ell \leq 1$$

$$W_{ij}^T = \frac{p_{Ti} p_{Tj}}{p_{T,\text{tot}}^2}$$

transverse  
momentum  
weight

$$W_{ij}^U = 1$$

unit  
weight

$$W_{ij}^p = \frac{|\vec{p}_i| |\vec{p}_j|}{|\vec{p}|_{\text{tot}}^2}$$

magnitude  
momentum  
weight

# Fox-Wolfram Moments - 2 jet properties

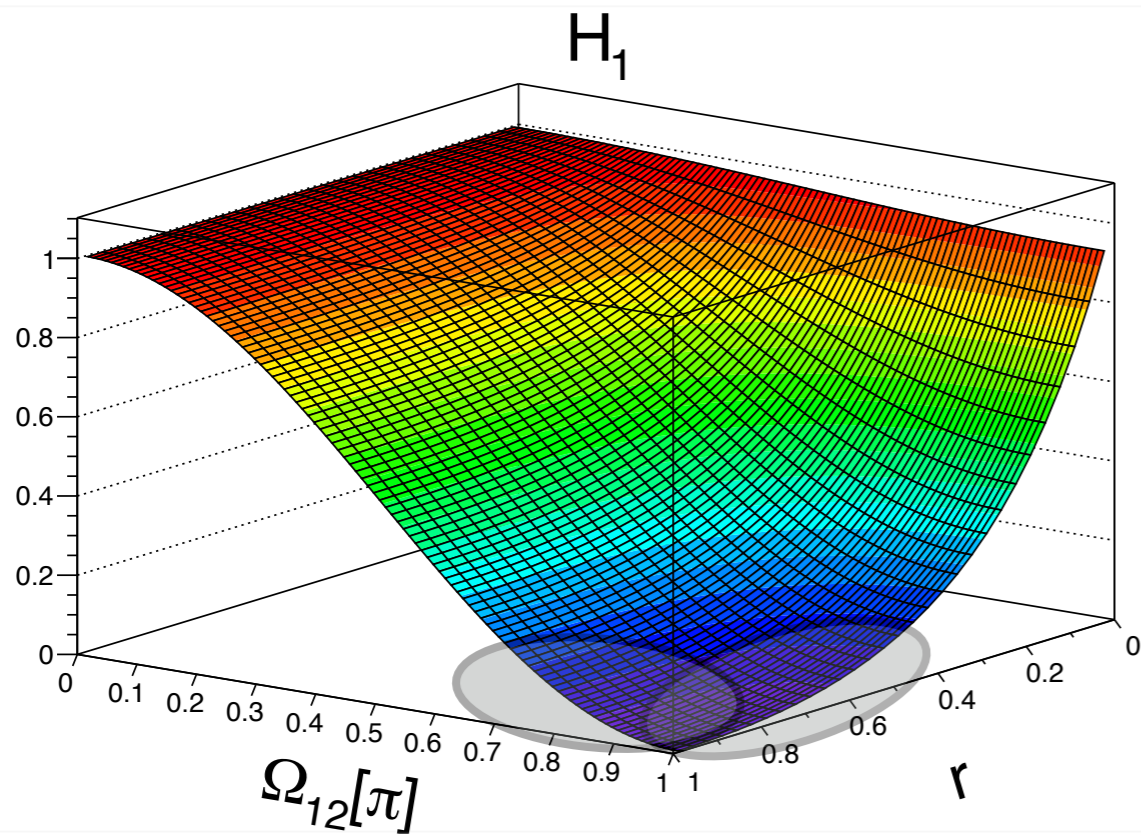
$$\begin{aligned} H_\ell &= \sum_{i,j=1}^2 \frac{W_i W_j}{W_{\text{tot}}^2} P_\ell(\cos \Omega_{ij}) \\ &= \frac{1}{(W_1 + W_2)^2} \left[ W_1^2 P_\ell(\cos 0) + W_2^2 P_\ell(\cos 0) \right. \\ &\quad \left. + W_1 W_2 P_\ell(\cos \Omega_{12}) \right] \\ &= 1 + \frac{2W_1 W_2}{(W_1 + W_2)^2 P_\ell(\cos \Omega_{12})} \\ &= \frac{1 + 2r P_\ell(\cos \Omega_{12})}{1 + 2r + r^2} \end{aligned}$$

$$r = \frac{W_2}{W_1}$$

$$0 \leq r \leq 1$$

# Fox-Wolfram moments - 2 jet properties

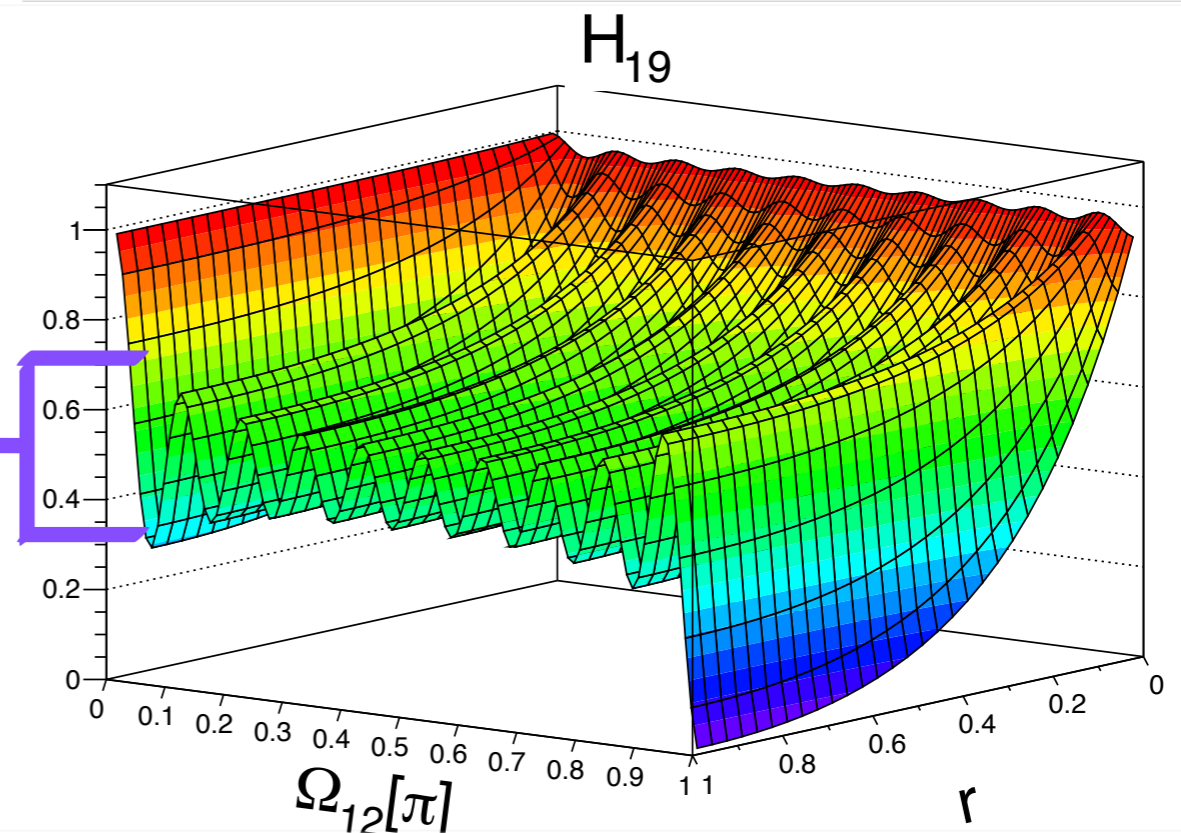
**odd moments - best for discriminating back-to-back jets,  
higher moments resolve larger angular  $j_1 j_2$  separation**



$$r = \frac{W_2}{W_1}$$

**multivalued function, no  
resolution to intermediate  
values of  $\Omega_{12}$**

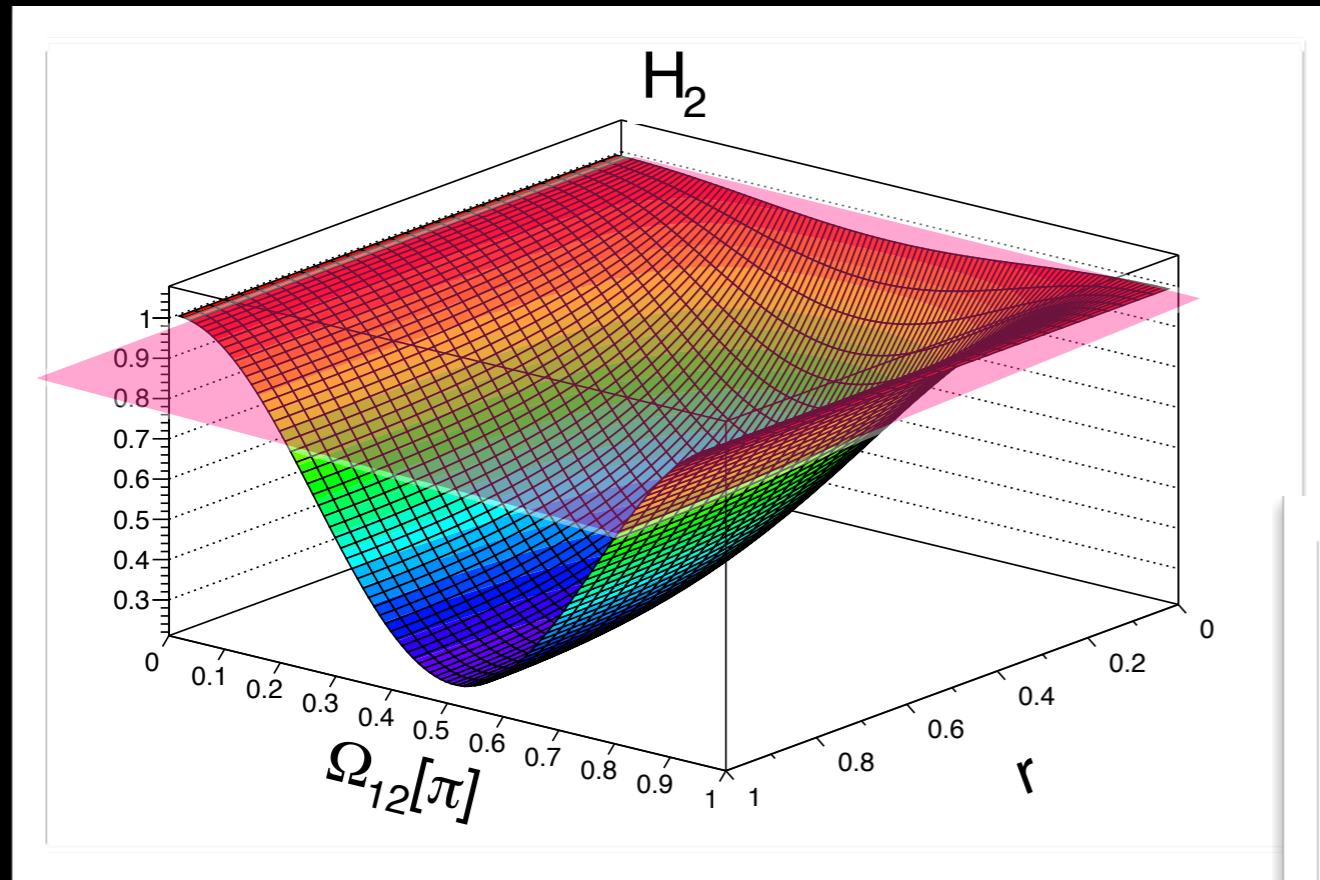
$$H_\ell \rightarrow 0 \quad \text{for} \quad \Omega_{12} \rightarrow \pi$$



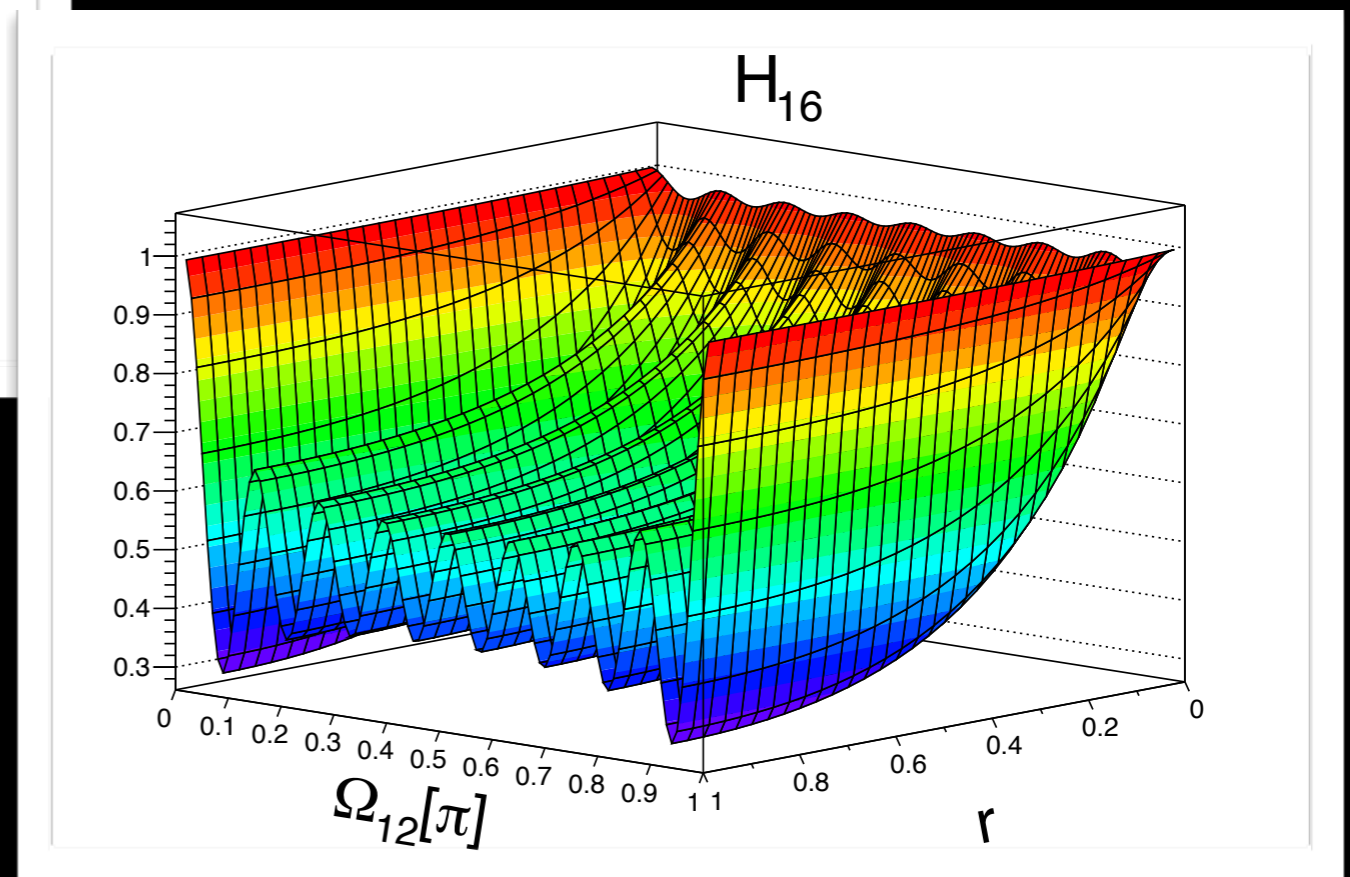
# Fox-Wolfram moments - 2 jet properties

**even moments - symmetry of even function  
reduces discriminatory power**

$$H_\ell \rightarrow 1 \quad \text{for} \quad \Omega_{12} \rightarrow 0 \quad \text{AND} \quad \Omega_{12} \rightarrow \pi$$



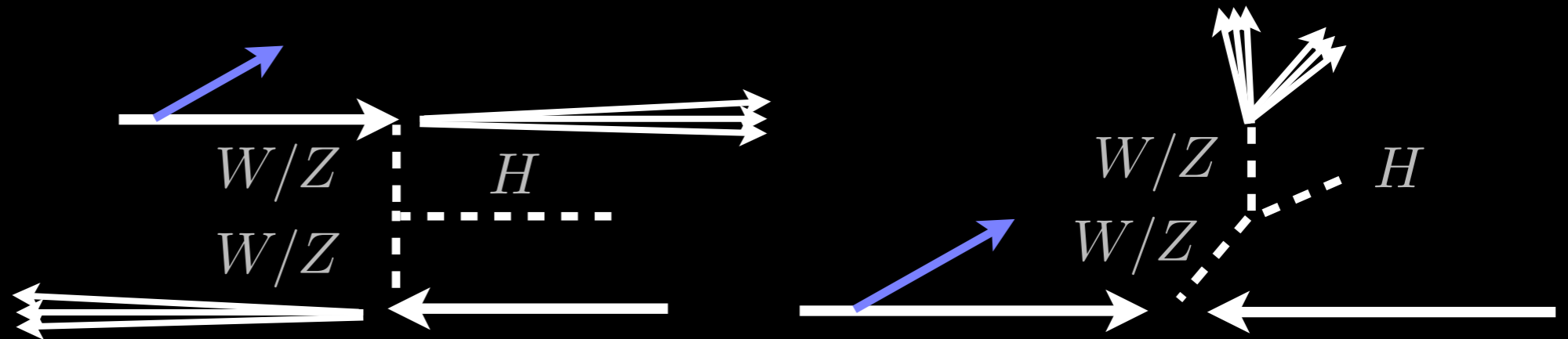
low, even moments may  
discern non forward-  
backward jets



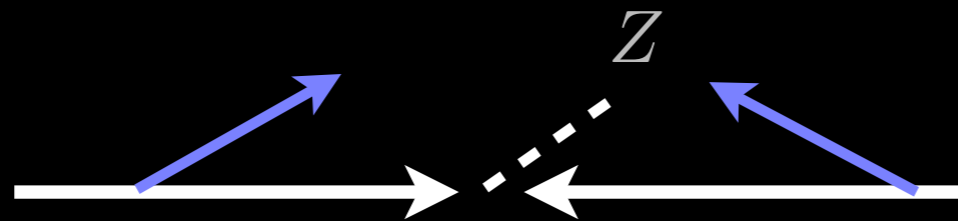
# Analysis for $H \rightarrow \tau \tau$

(process + hard jet) x PS with CKKW using SHERPA

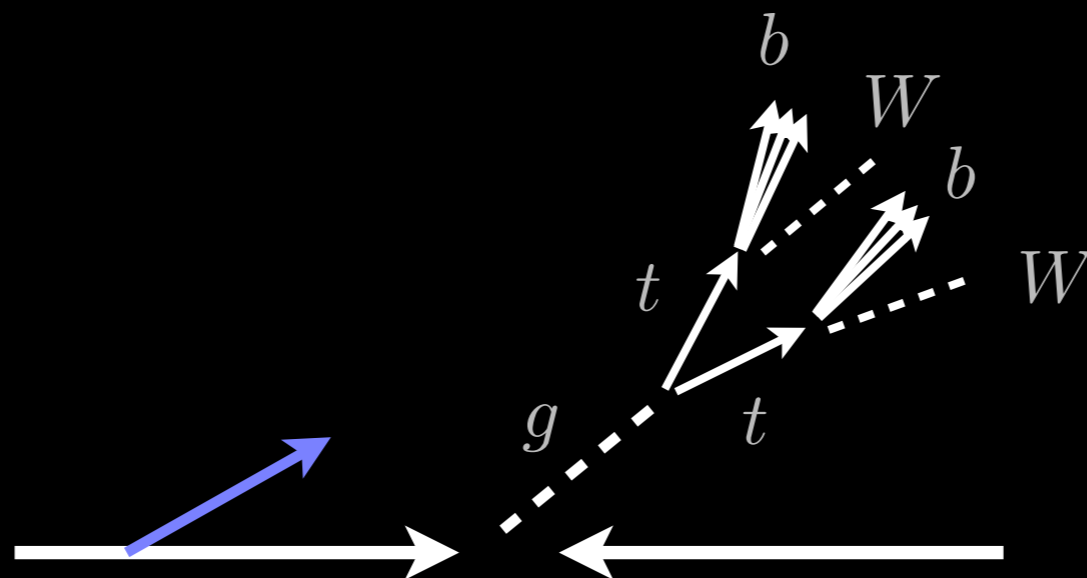
signal WBF



background  
QCD ZJJ



background  
Top Pair



Fastjet anti-  
kT algorithm  
with  $R = 0.4$ ,  
8TeV

# Cutflow Analysis

acceptance	WBF + 1 jet		QCD ZJJ		Top Pair		S / B
	% fail	XS (fb)	% fail	XS (fb)	% fail	XS (fb)	
		18.7		115000		17200	1/7070
$p_{Tj_1,j_2} > 20 \text{ GeV}$	29.4	13.2	93.2	7820	9.63	15500	1/1767
$ y_{j_1,j_2}  < 5.0$	1.49	13.0	0.97	7740	0.182	15500	1/1788
$\Delta R_{j_1 j_2} > 0.7$	2.73	12.6	3.84	7440	2.32	15100	1/1789
$m_{j_1 j_2} > 600 \text{ GeV}$	68.9	3.92	96.6	253	95.8	634	1/226
$b - veto$	NA	3.92	NA	253	54.0	292	1/139
$y_1 \cdot y_2 < 0$	1.41	3.86	9.17	230	13.8	252	1/125
$ y_{j_1} - y_{j_2}  > 4.4$	13.9	3.32	31.8	157	66.1	85.4	1/73

can cuts on FWM replace or be added to current cuts used for VBF event selection?

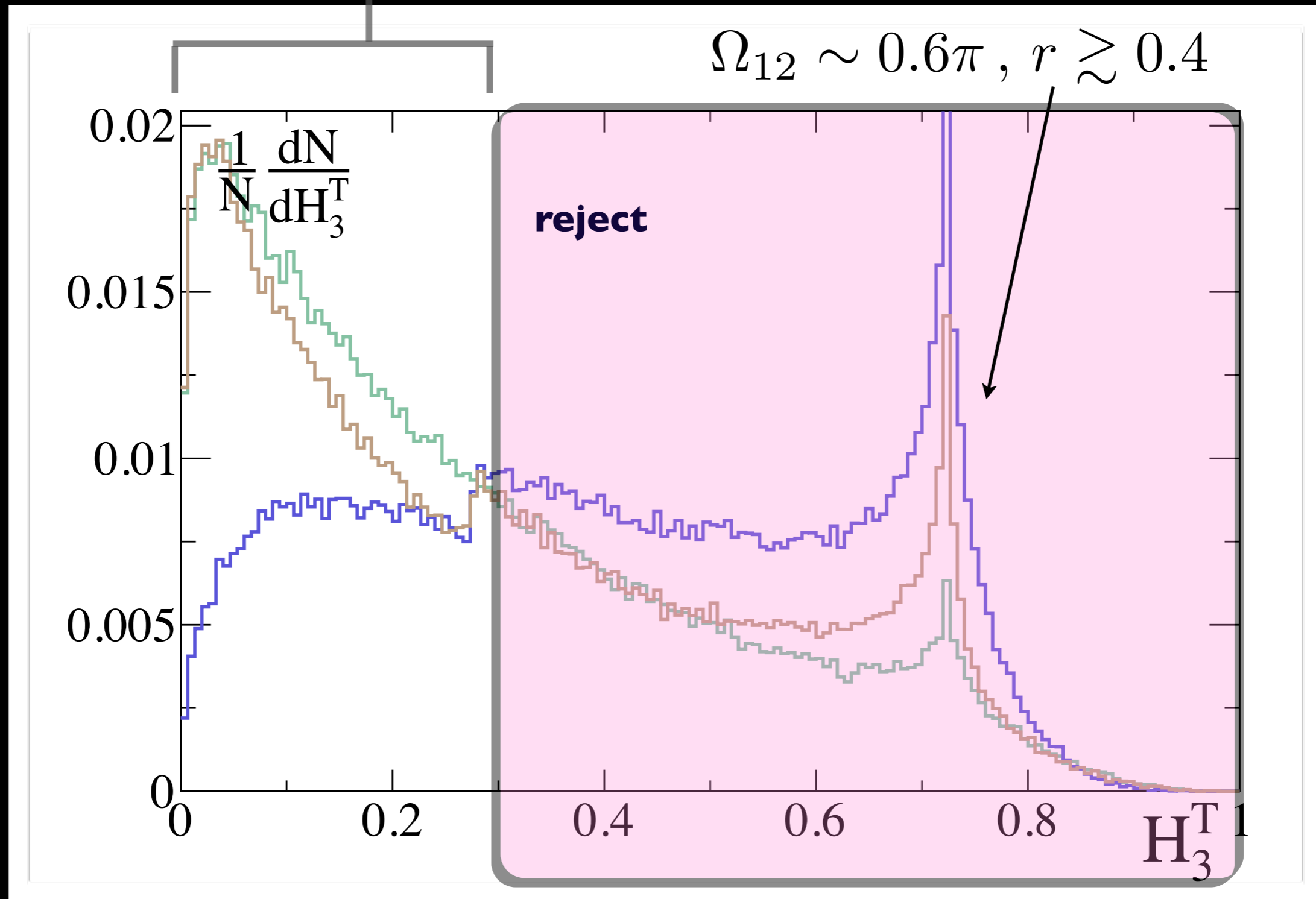
# Cuts on FWM Distributions

$$\Omega_{12} \gtrsim 0.8\pi, r \gtrsim 0.4$$

WBF + l jet

QCD ZJJ

Top Pair + l jet



# Cuts on FWM Distributions

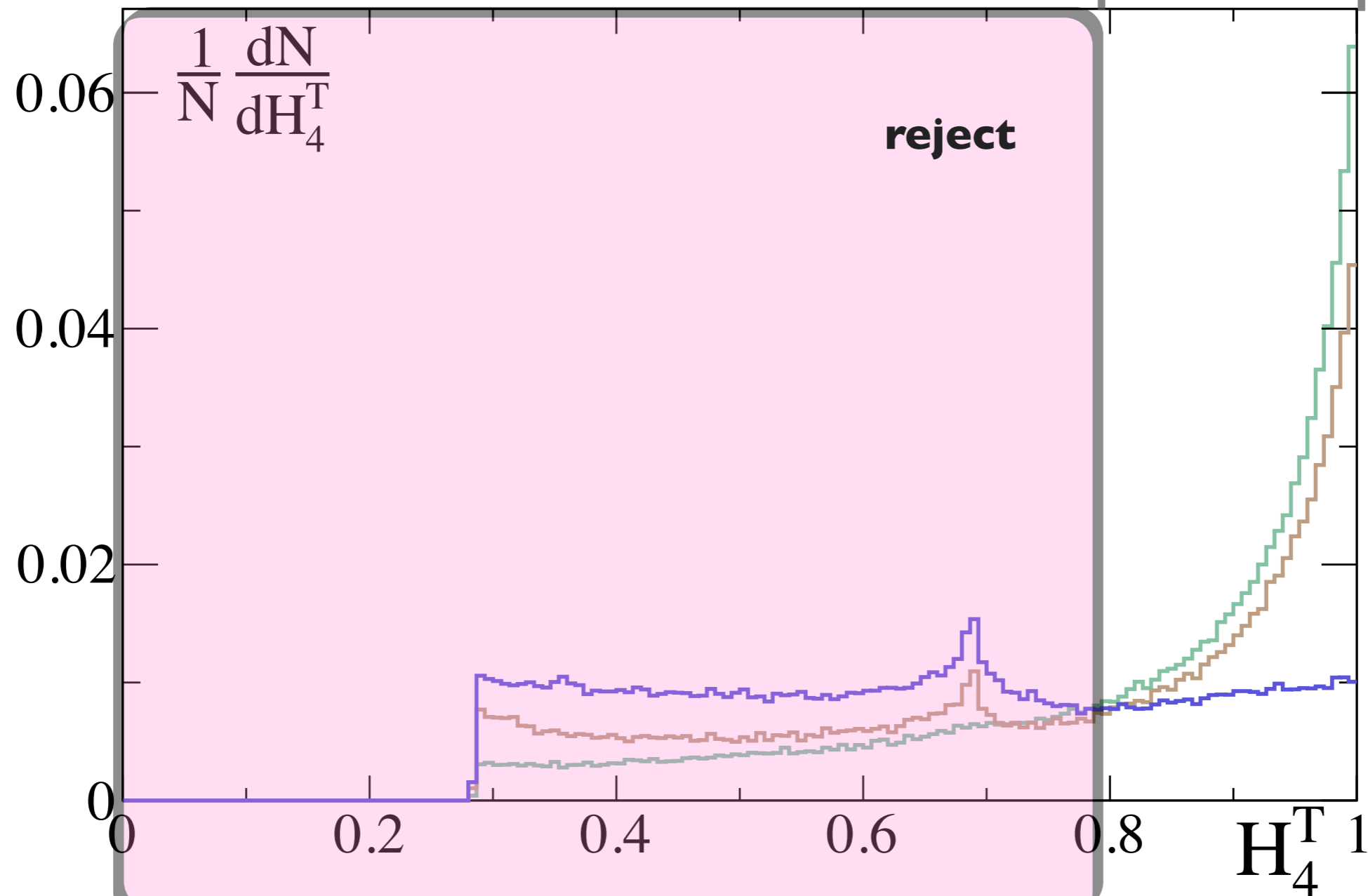
**OR**

$$\begin{aligned} &\Omega_{12} \sim 0, \pi \text{ any } r \\ &r \leq 0.3 \text{ any } \Omega_{12} \end{aligned}$$

WBF + 1 jet

QCD ZJJ

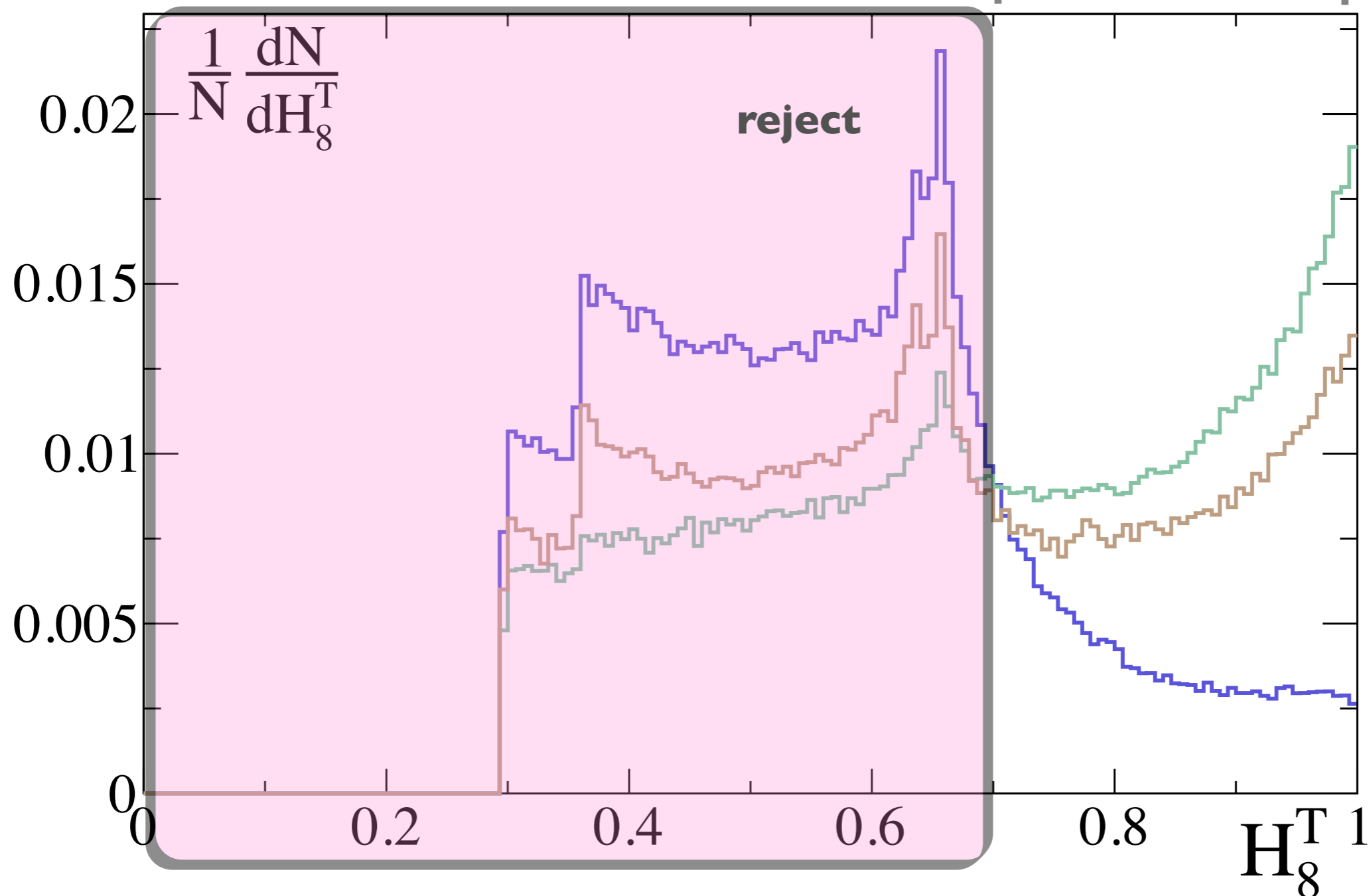
Top Pair + 1 jet



# Cuts on FWM Distributions

**OR**  $\Omega_{12} \sim 0, \pi$  any  $r$   
 $r \leq 0.3$  any  $\Omega_{12}$

WBF + l jet  
QCD ZJJ  
Top Pair + l jet



# Cuts on FWM Distributions<sup>1</sup>

acceptance	WBF + 1 jet		QCD ZJJ		Top Pair		S / B
	% fail	XS (fb)	% fail	XS (fb)	% fail	XS (fb)	
min cuts + b-veto		3.92		253		292	1/139
$H_3^T < 0.3$	38.4	2.41	44.4	141	64.6	103	1/101
$H_4^T > 0.8$	35.8	2.52	48.1	131	73.3	78.0	1/83
$H_8^T > 0.8$	50.1	1.96	60.5	100	81.6	53.7	1/78
$H_{12}^T > 0.7$	64.5	1.39	73.0	68.3	88.0	35.0	1/74
rapidity gap	13.9	3.32	31.8	157	66.1	85.4	1/73

<sup>1</sup>C.B. et.al, PRD 87, 073014 (2013)

# Analysis - Cutting on FWM

after typical WBF cuts are exhausted, can the moments help?

acceptance	WBF + 1 jet		QCD ZJJ		Top Pair		S / B
	% fail	XS (fb)	% fail	XS (fb)	% fail	XS (fb)	
		18.7		115000		17200	1/7070
minimal cuts + b veto	NA	3.92	NA	253	54.0	292	1/139
central jet cuts	13.9	3.32	31.8	157	66.1	85.4	1/73
					$H_{12}^T > 0.7$		1/57

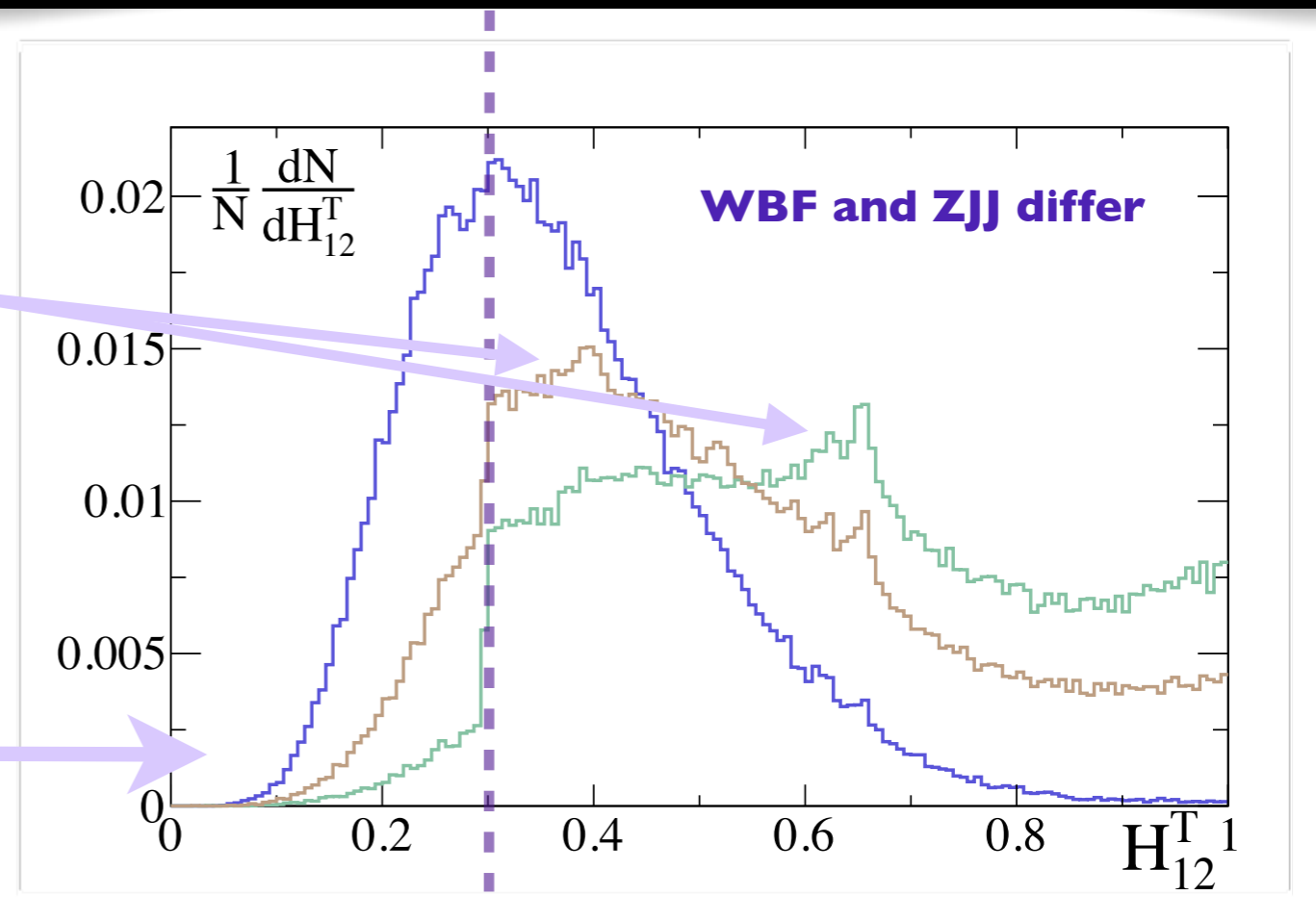
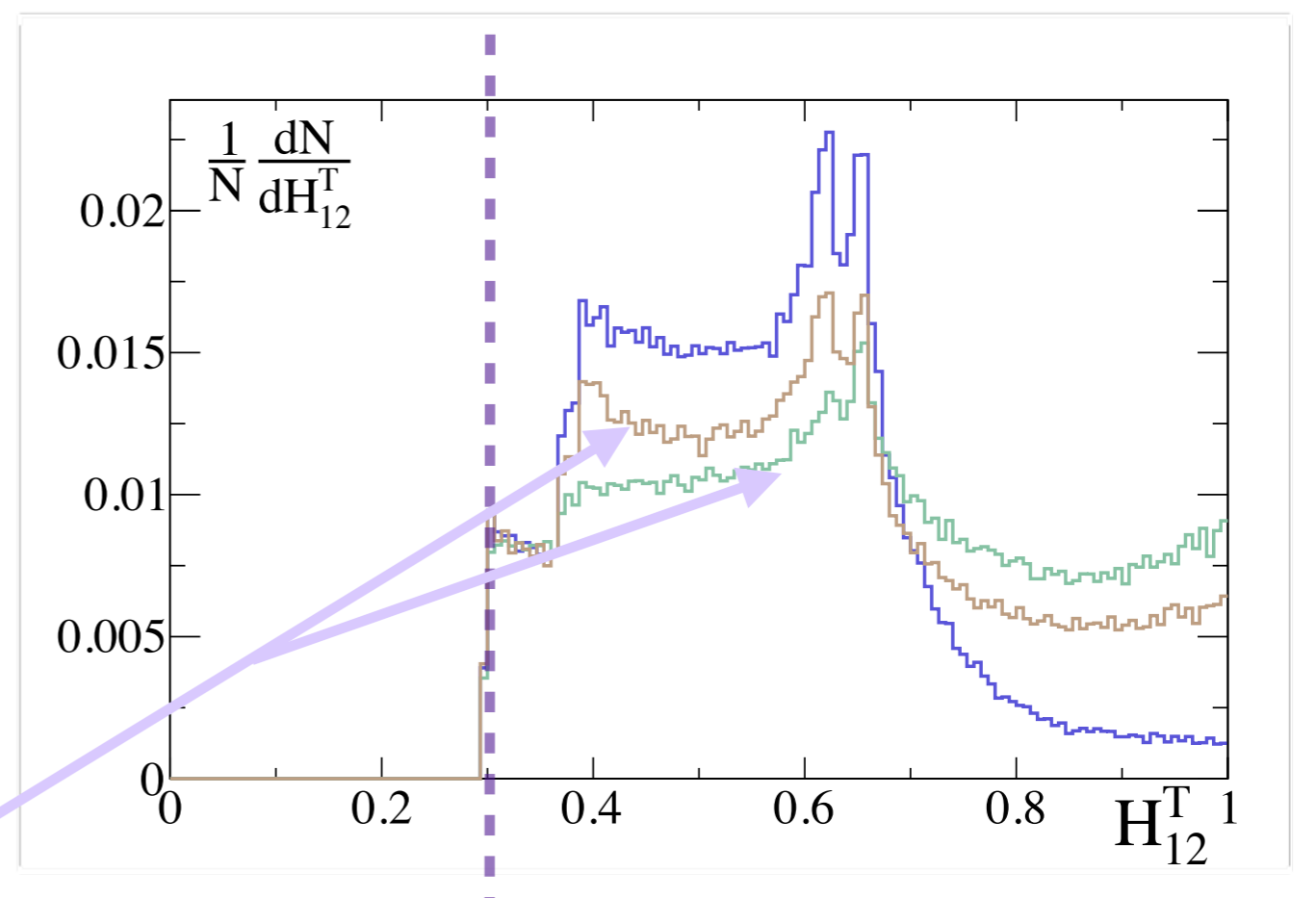
top pair background can be further suppressed based on tagging jet correlations rephrased into FWM

# Inclusive FWM:

require *at least 2* tagging jets satisfying minimal cuts

more power to discern WBF from ZJJ (3rd and higher jets have more drastically differing weights)

$H_\ell < 0.3$  region populated



# Classification Rule

A “classifier” is a rule for determining which class an instance of a set belongs to

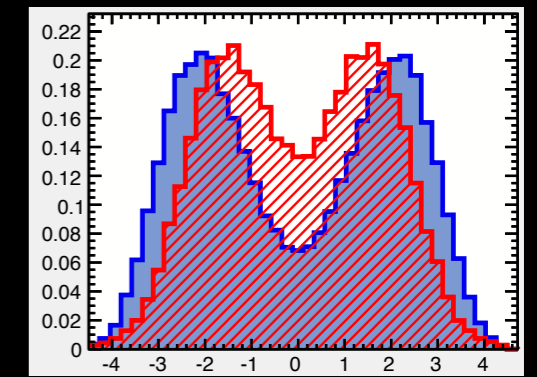
sig/bkg

event data or MC sample

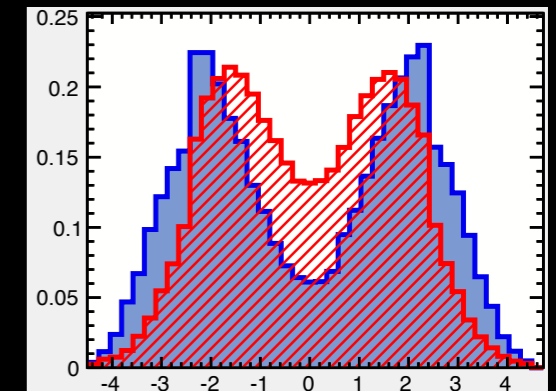
instance	$y_{j1}$	$y_{j2}$	$\Delta y_{12}$	$m_{12}$	class
event 1	2.79854	-1.33015	4.12869	264.056 GeV	S
event 2	1.5059	-1.09764	2.60354	156.285 GeV	B
⋮	⋮	⋮	⋮	⋮	⋮
event n	-1.10029	1.83929	2.93958	209.104 GeV	S

$$y(\vec{x}) \in \mathbb{R}$$

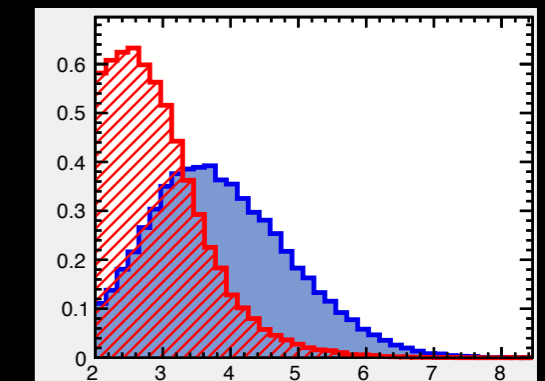
map all information of an event onto a real number - the “scalar output” of the classifier



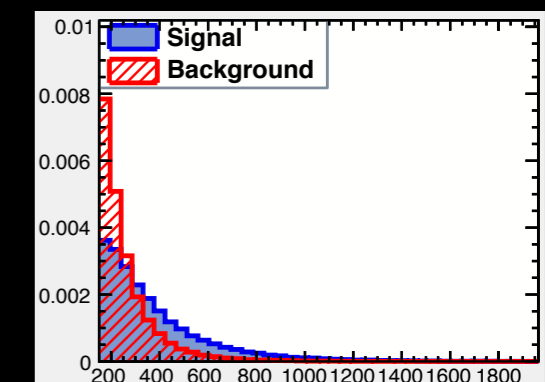
$y_{j1}$



$y_{j2}$

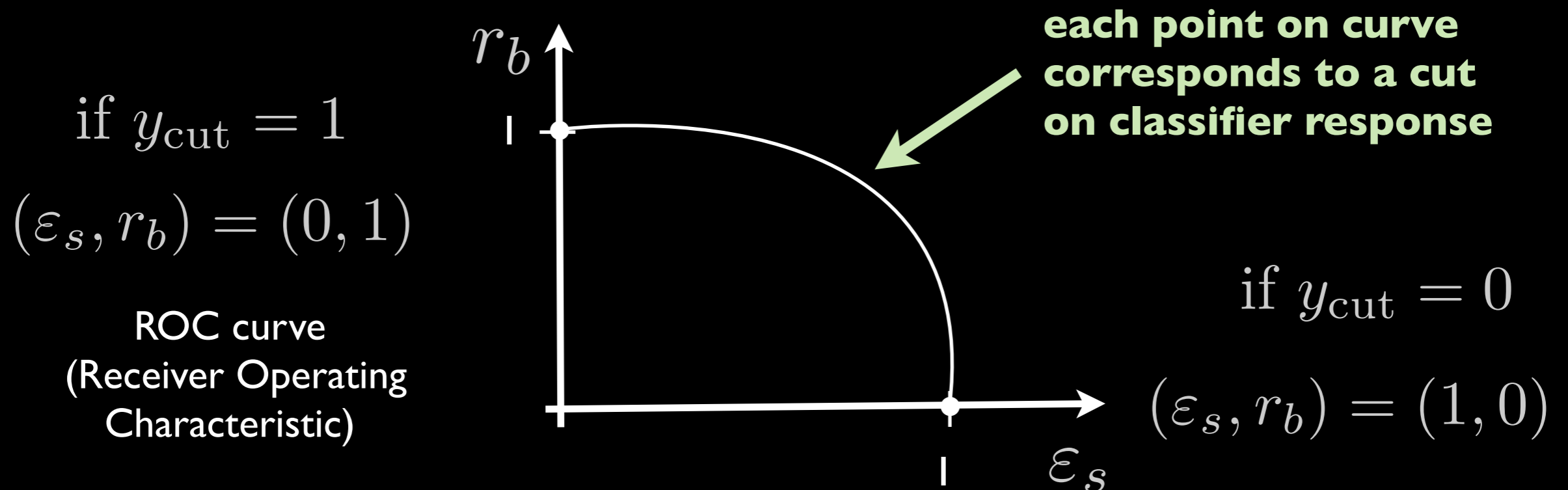
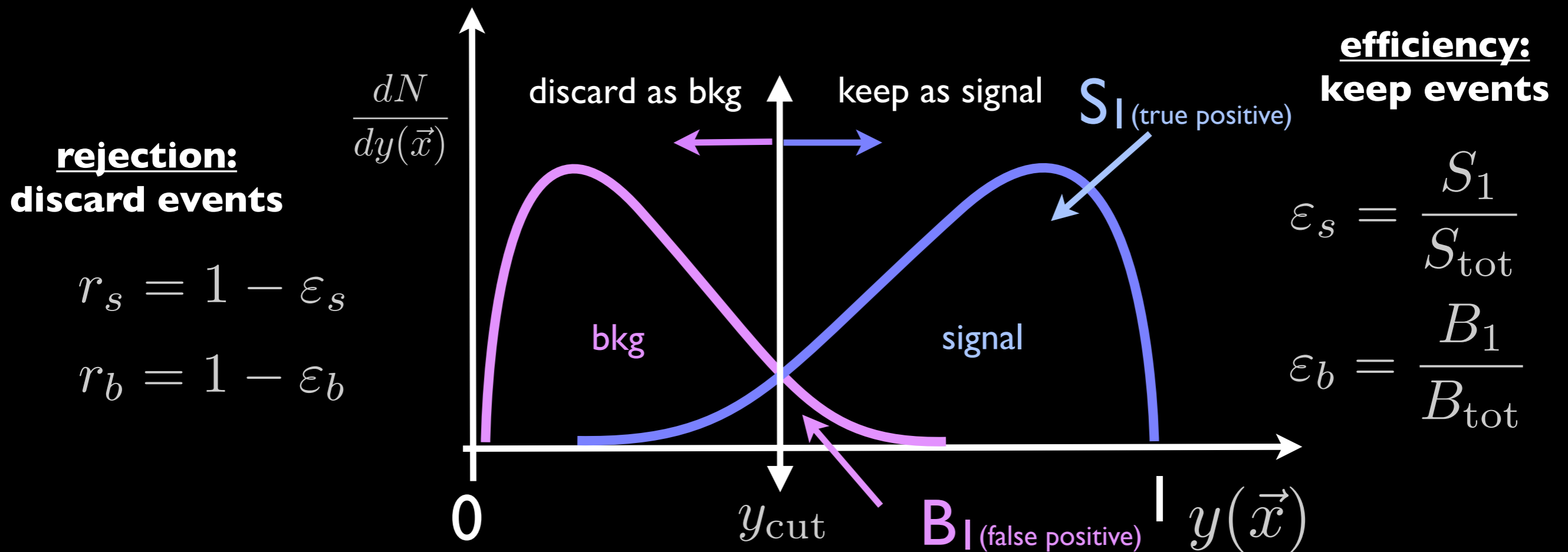


$\Delta y_{12}$



$m_{12}$

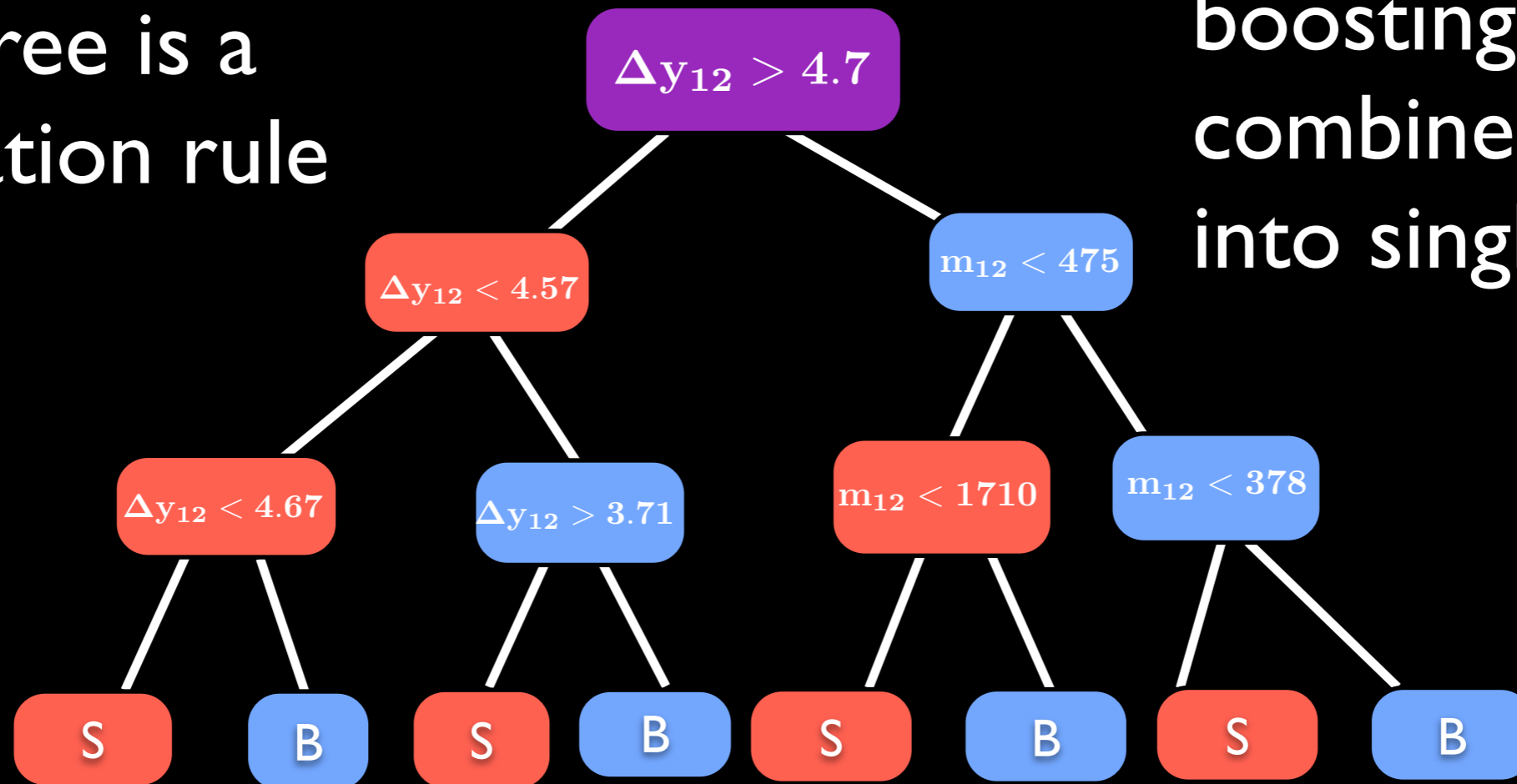
# Classification Response and ROC Curves



# Boosted Decision Trees

each tree is a  
classification rule

boosting:  
combine trees  
into single rule



Adaptive Boost Algorithm:

$$y(\vec{x}) = \frac{1}{N_{\text{boost}}} \sum_i^{N_{\text{boost}}} \ln(\alpha_i) h_i(\vec{x})$$

events misclassified are reweighted, another  
tree is built, misclassification rate is updated,  
event is reweighted, etc...

$$h_i(\vec{x}) = +1 \text{ (sig)}, -1 \text{ (bkg)}$$

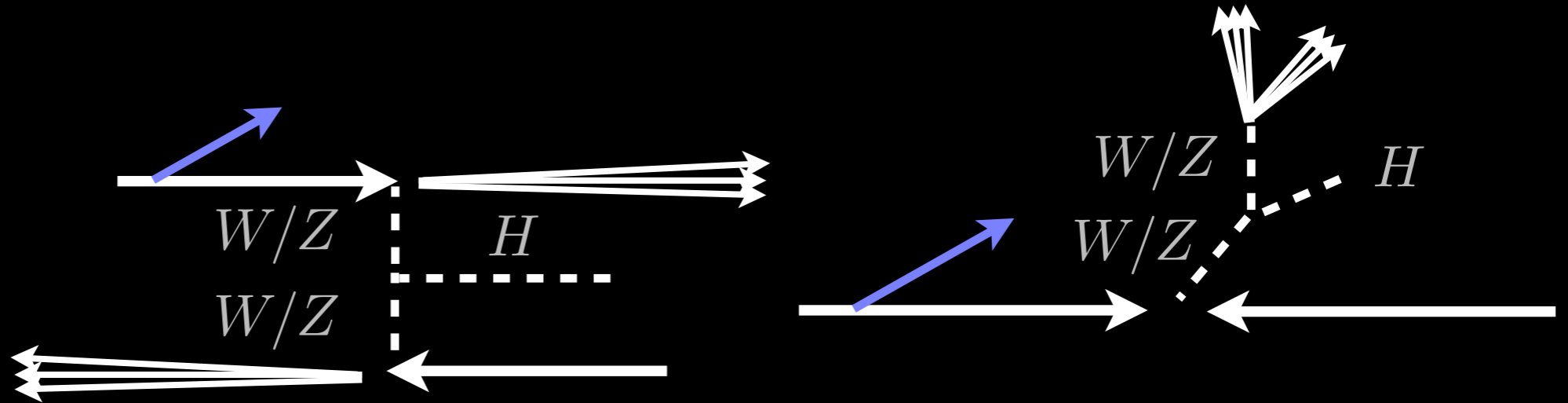
$$\alpha_i = \frac{1 - \text{err}_i}{\text{err}_i}$$

$\text{err}_i$  = misclassification rate

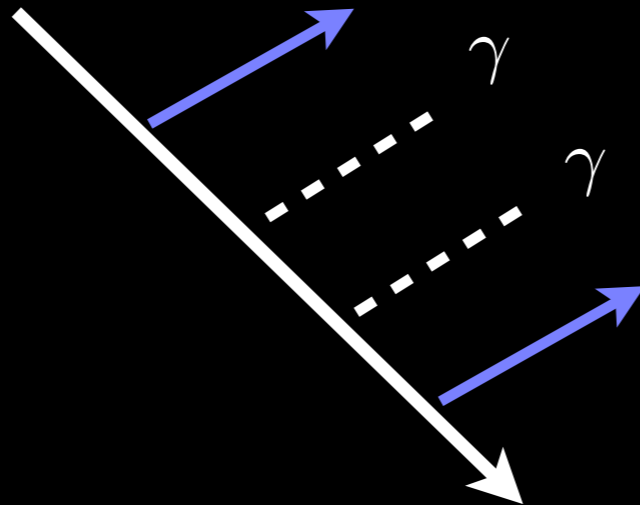
# Analysis for $H \rightarrow \gamma\gamma$

(process + hard jet) x PS with CKKW using SHERPA

signal VBF +  
1 matrix  
element  
level jet



background  
diphoton +  
2 matrix  
element  
level jets



Fastjet anti-  
kT algorithm  
with  $R = 0.4$ ,  
8TeV

# BDT Analysis with only Tagging Jet Correlations

**use FWM after  
applying  
acceptance  
criteria for jets:**

$$p_{Tj} > 25 \text{ GeV} \quad \text{for} \quad |y_j| < 2.4$$

$$p_{Tj} > 30 \text{ GeV} \quad \text{for} \quad 2.4 \leq |y_j| < 4.5$$

$$|\Delta y_{j_1 j_2}| \geq 2 \quad \text{and} \quad m_{j_1 j_2} > 150 \text{ GeV}$$

**compare FWM  
with tagging jet  
correlations  
used by ATLAS**

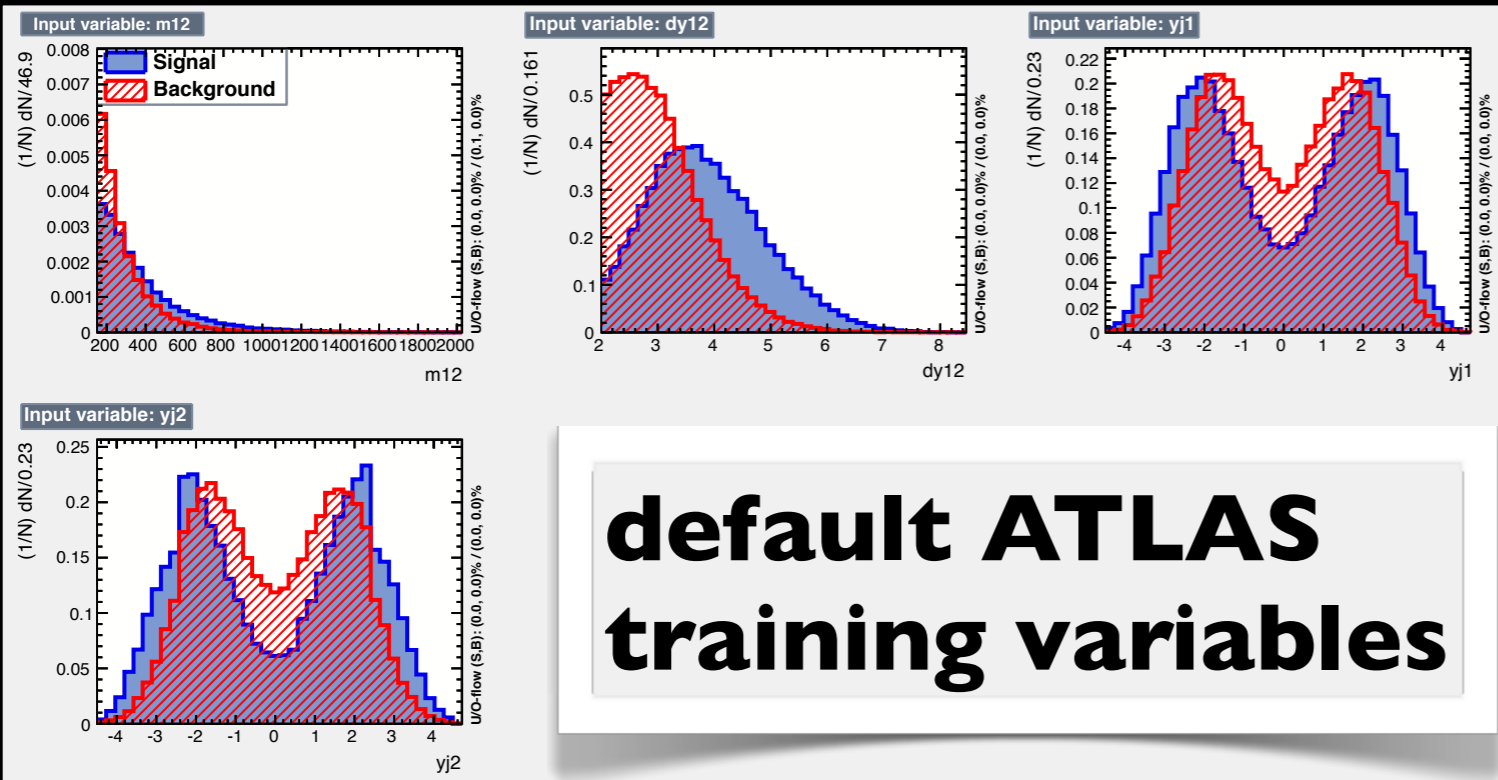
$$\{m_{j_1 j_2}, y_{j_1}, y_{j_2}, \Delta y_{j_1 j_2}\}$$

**Decision Tree  
Settings<sup>1</sup>:**

$$N_{\text{train}}, N_{\text{test}} = 100\text{K}, 50\text{K}$$
$$N_{\text{trees}}, N_{\text{layers}} = 400, 3$$

<sup>1</sup>Hoecker et.al., Toolkit for Multivariate Analysis , <http://tmva.sourceforge.net>

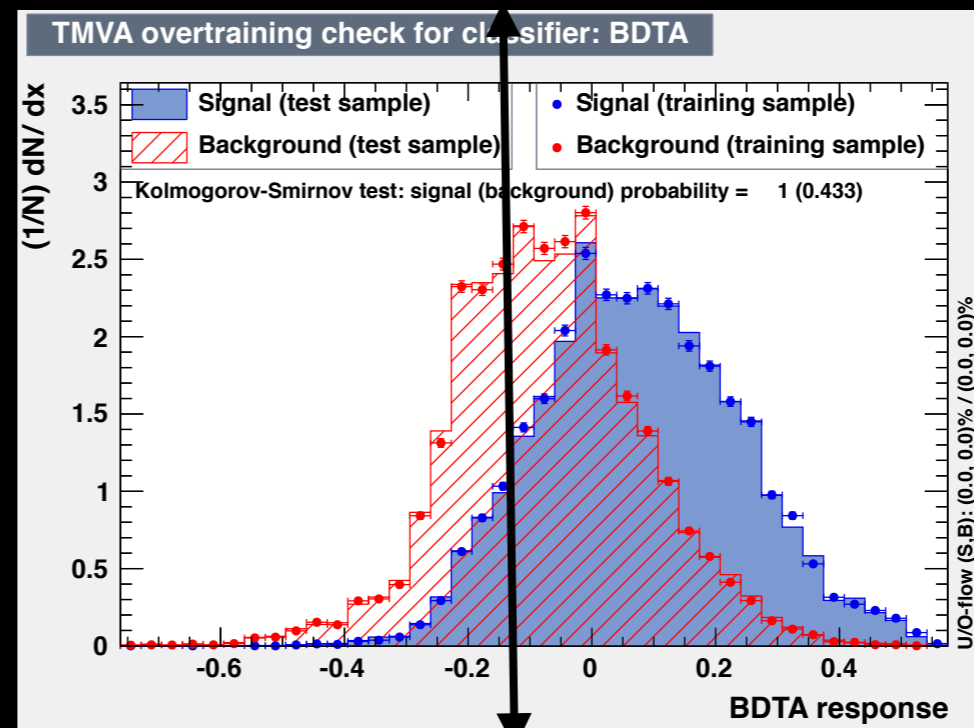
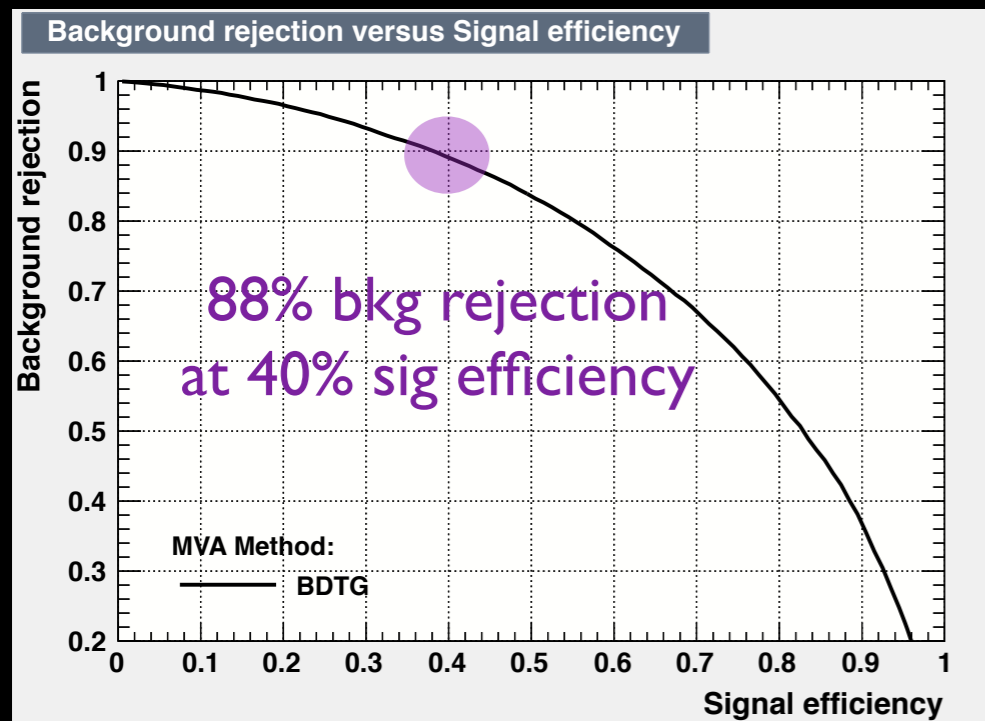
# Results of BDT Analysis Including FWM



**default ATLAS  
training variables**

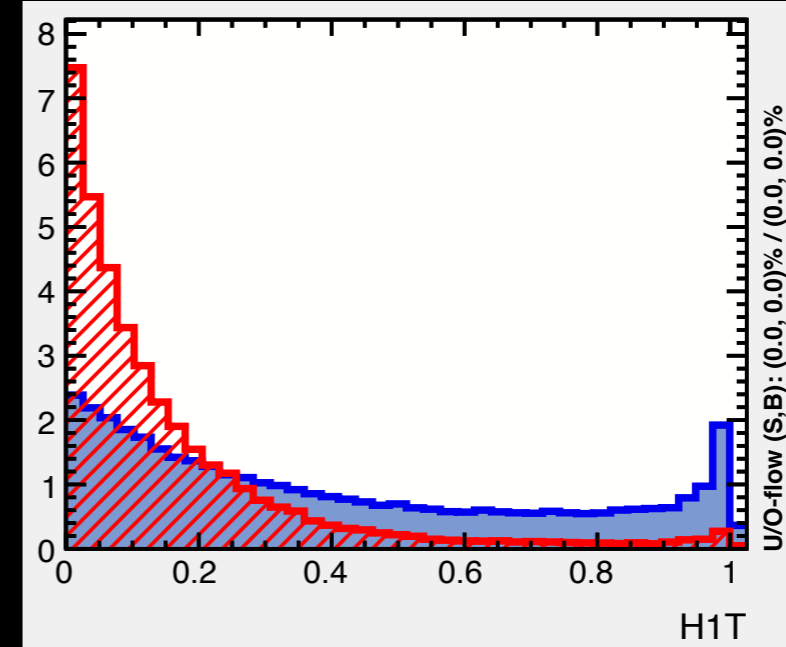
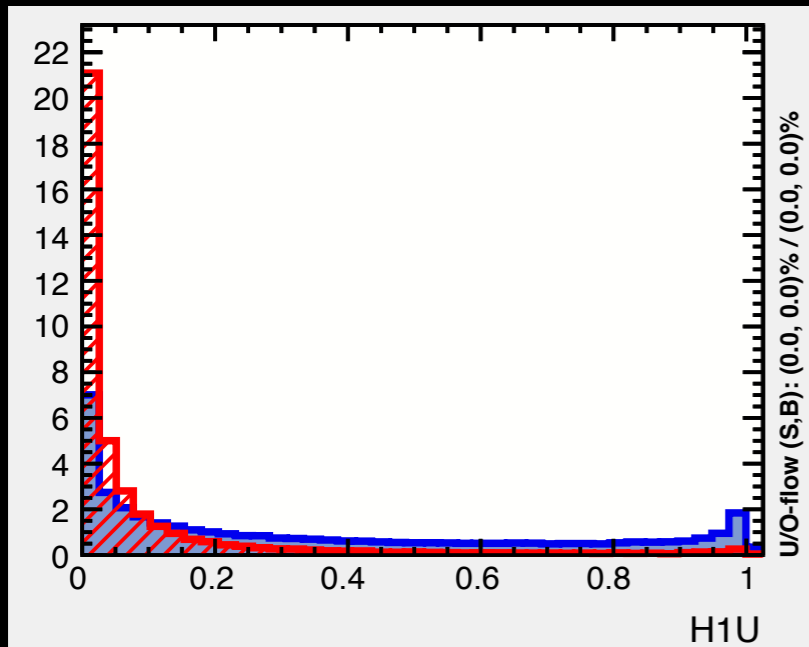
$$\frac{S}{\sqrt{S+B}} = 198.7$$

for cut at  $y = -0.14$



# Results of BDT Analysis Including FWM<sup>1</sup>

in addition to default, train with:  $H_\ell^{x,\phi} = \sum_{i,j=1}^N W_{ij}^x P_\ell(\cos \Delta\phi_{ij})$



$$H_1^{U,\phi} = \frac{1}{2} + \frac{1}{4} \cos \Delta\phi_{12}$$

$$H_1^{T,\phi} = \frac{p_{T1}^2 + p_{T2}^2}{p_{T\text{tot}}^2} + \frac{p_{T1}p_{T2}}{p_{T\text{tot}}^2} \cos \Delta\phi_{12}$$

	$B$ rejection	$\frac{S}{\sqrt{S+B}}$	improvement
ATLAS default	88.7%	198.7 (-0.14)	
$H_1^{T,\phi}, H_1^{U,\phi}$	95.2%	209.166 (-0.07)	5.3%
$H_1^{T,\phi}$	94.9%	206.703 (-0.08)	4.0%
$H_1^{U,\phi}$	95.2%	208.821 (-0.08)	5.1%
$\cos \Delta\phi_{12}$	95.2%	208.821 (-0.08)	5.1%

<sup>1</sup>Bernaciak, Mellado, Plehn, Ruan, Schichtel, in preparation

# Results of BDT Analysis Including FWM<sup>1</sup>

improvement with  
redefinition of FWM:

$$H_{\ell}^{x,\phi} = \sum_{i,j=1}^N W_{ij}^x P_{\ell}(\cos \Delta\phi_{ij})$$

	$B$ rejection	$\frac{S}{\sqrt{S+B}}$	improvement
ATLAS default	88.7%	198.7 (-0.14)	
$H_1^{T,\phi} \rightarrow H_{20}^{T,\phi}, H_1^{U,\phi} \rightarrow H_{20}^{U,\phi}$	95.0%	208.901 (-0.07)	5.1%
$H_1^{T,\phi}, H_3^{T,\phi}, H_1^{U,\phi}, H_3^{U,\phi}$	95.3%	209.115 (-0.08)	5.3%
$H_1^{T,\phi}, H_2^{T,\phi}, H_2^{U,\phi}, H_2^{U,\phi}$	95.2%	209.132 (-0.08)	5.3%
$H_1^{T,\phi}, H_1^{U,\phi}$	95.2%	209.166 (-0.07)	5.3%
$H_1^{T,\phi}$	94.9%	206.703 (-0.08)	4.0%
$H_1^{U,\phi}$	95.2%	208.821 (-0.08)	5.1%
$\cos \Delta\phi_{12}, W_{12}^T$	95.3%	209.299 (-0.08)	5.3%
$\cos \Delta\phi_{12}$	95.2%	208.821 (-0.08)	5.1%

redefinition of FWM offer modest improvement over ATLAS default variables

<sup>1</sup>Bernaciak, Mellado, Plehn, Ruan, Schichtel, in preparation

# Conclusions - Future Work

- ✦ **FWM** suitable for both cut-based and decision tree analysis - offer consistent 5% improvement for azimuthal angle definition
- ✦ combinations of **U** and **T** weighted moments are better than **T** alone, **U** may be sufficient alone
- ✦ total angle moments - offer 1% improvement for **MVA** as opposed to **Cut-Based Analysis** - need to understand why
- ✦ the **FWM** are an interesting addition to the variables currently used in **Higgs** analyses

## Work Underway

- ✦ compare with **Neural Network MVA**
- ✦ incorporate 3rd jet and its scale uncertainty into this analysis
- ✦ can moments be used as a modified jet veto?