

# Measuring the Rotational Speed of Spiral Galaxies and Discovering Dark Matter



## Activity UCIObs--2 Grade Level: 9-12

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**Standards:** This activity addresses these California Science Content Standards:

*Gr 9-12 Physics – Motion and Forces:*

1c: Students know how to apply the law  $F=ma$  to solve one-dimensional motion problems that involve constant forces (Newton's second law).

1g: Students know circular motion requires the application of a constant force directed toward the center of the circle.

11: \* Students know how to solve problems in circular motion by using the formula for centripetal acceleration in the following form:  $a=v^2/r$ .

*Gr 9-12 Physics – Waves:*

4f: Students know how to identify the characteristic properties of waves: interference (beats), diffraction, refraction, Doppler effect, and polarization.

*Gr 9-12 Earth Sciences:*

2b: Students know galaxies are made of billions of stars and comprise most of the visible mass of the universe.

### What's This Activity About?

In this activity, students will measure the rotation of a spiral galaxy using Doppler shift and learn that galaxies are mostly made up of "invisible" mass called dark matter.

### What Will Students Do?

Students will measure the mass in dark matter in a nearby spiral galaxy by calculating the speeds at which gas clouds rotate around the center of the galaxy. They will (a) use the Doppler shift equation

to convert the observed wavelength of a hydrogen emission line to the line-of-sight velocity, (b) convert line-of-sight velocity to rotation velocity, and (c) use the equation for circular velocity to solve for the 'dynamical' mass of the galaxy. They will then compare the dynamical mass of the galaxy to the mass of the galaxy in stars and gas and learn that 85% of the galaxy's total mass is made of dark matter.

**Required:** Algebra

### **Tips and Suggestions**

This activity should follow the exercise on waves and the Doppler shift, *Activity H-5: The Expanding Universe* in “Universe at Your Fingertips”. This activity will reinforce the lessons learned in *The Expanding Universe* activity by applying

the concepts to real galaxy data. Additionally, this activity will motivate the students by investigating dark matter, a hot topic in modern astronomy & cosmology.

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### **What Will Students Learn?**

**Concepts:** Galaxy Rotation, Dark Matter

**Inquiry Skills:** Applying, Calculating, Comparing, Evaluating, Inferring, Graphing

**Big Ideas:** Matter, Scale, Systems

### **What you will need for each student or team of students:**

Scientific calculator

Scratch paper for the calculations

Ruler

### **Preparation:**

1. Photocopy the Instructions, Table & Graph for each student (pages 7-13).

### **Teacher Instructions:**

1. Hand out the materials.

Go over Section A in the Exercise with the students:

2. Remind students of the different types of galaxies (spirals, ellipticals, and irregulars) and that spiral galaxies can be approximated as flattened circular disks that rotate. Explain that galaxies are made of stars, gas, and dark matter.

Go over Section B in the Exercise with the students:

3. Explain that bright, massive O and B stars emit intense ultraviolet radiation. This UV light ionizes the surrounding gas, most of which is hydrogen. The excited hydrogen electrons later drop back down to lower energy levels. When an electron jumps from the  $n=3$  level to the  $n=2$  level, the H-alpha emission line at 6563 Angstroms ( $\text{\AA}$ ;  $1 \text{ \AA} = 10^{-9} \text{ m}$ ) is emitted. This emission line is in the red region of the visible spectrum. Hydrogen is abundant in the universe, so this can be a very strong emission line if young, massive, hot stars are present, as they are in the disks of spiral galaxies.
4. Review the concept of Doppler shift and how it can be used to measure how fast a source of light is moving toward or away from us ( $V_{\text{LOS}}$ ). Remember that Doppler shift measures only *radial* velocity, not total 3-D space velocity. Also remind students that because the universe is expanding, most galaxies are moving away from us (recession velocity,  $V_{\text{recession}}$ ).

Go over Section C in the Exercise with the students:

5. Review Newton's 2<sup>nd</sup> Law ( $F = ma$ ), Newton's Law of Gravity ( $F = GM_1m_2/r^2$ ), and

centripetal acceleration ( $a = v^2/r$ ) and how these can be combined to derive the equation for circular velocity ( $V_{\text{rotation}} = (GM/r)^{1/2}$ ). In the units relevant to this exercise, the value of G is  $1.33 \times 10^{11} \text{ km}^3 \text{ M}_{\text{sun}}^{-1} \text{ s}^{-2}$ .

$$F = ma = \frac{GMm}{r^2}$$

$$m \left( \frac{v^2}{r} \right) = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$V_{\text{rotation}} = \sqrt{(GM/r)}$$

6. Derive and draw the expected rotation curve for a galaxy:

-Imagine a (spherical) galaxy has a constant density  $\rho$ . Recall that

$$\rho = (\text{Mass} / \text{Volume})$$

$$\text{Volume} = (4/3) \pi r^3$$

$$V_{\text{rotation}} = (GM/r)^{1/2}$$

-The velocity of a star on a circular orbit *inside* the galaxy changes as a function of radius in the following way:

$$\rho = (\text{Mass} / \text{Volume}) = (M / (4/3)\pi r^3) \rightarrow M = (4/3)\pi r^3 \rho \quad \text{where } \rho \text{ is constant.}$$

$$V_{\text{rotation}} = (GM/r)^{1/2} = [(G(4/3)\pi r^3 \rho) / r]^{1/2} = [(4/3)G\pi \rho r^2]^{1/2} \rightarrow V_{\text{rotation}} \propto r$$

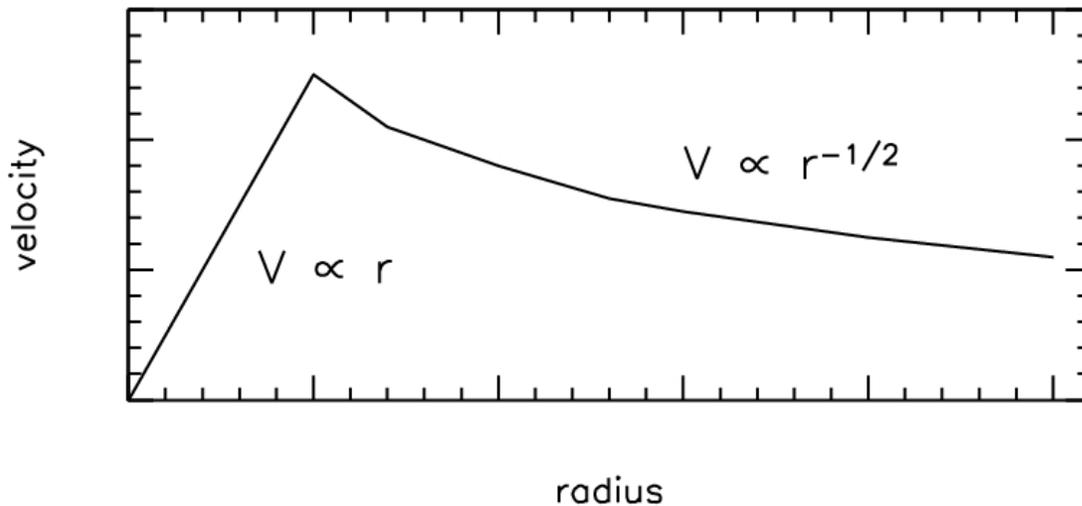
*Inside the galaxy:*  $V_{\text{rotation}} \propto r$  (i)

-The velocity of a star on a circular orbit *outside* most of the mass of the galaxy changes as a function of radius in the following way:

$$V_{\text{rotation}} = (GM_{\text{TOTAL}}/r)^{1/2} \rightarrow V_{\text{rotation}} \propto (1/r)^{1/2}$$

*Outside the galaxy:*  $V_{\text{rotation}} \propto r^{-1/2}$  (ii)

The expected rotation curve of a spiral galaxy therefore looks like:



7. Do the Example Calculation in Section E with the class.
8. To do the activity, you should break the class into groups and assign each group a few of the radii and wavelengths in the data table. First, you will go through the instructions and do the example for the first rotation curve point with the students. Then you will ask them to work with their group to calculate  $V_{\text{LOS}}$ ,  $V_{\text{rotation}}$ , and  $M_{\text{dynamical}}$  and also to plot their rotation curve points.
9. Once the students have finished their calculations, you should have them report their answers to the class so that everyone can fill in the table and plot the rest of the rotation curve points.
10. After the table is filled in, have the students work with their group to calculate  $M_{\text{darkmatter}}$ , the percentage of the total mass of the galaxy that is in dark matter, and the ratio of dark matter mass to visible mass.
11. After the students have calculated  $M_{\text{darkmatter}}$ , collectively discuss the following questions:

**Question #1:**

How does the visible mass of the galaxy ( $M_{\text{visible}} = M_{\text{stars}} + M_{\text{gas}}$ ) compare to the dynamical mass of the galaxy?

**Answer #1:**

$M_{\text{visible}}$  is only ~15% of the total mass of the galaxy.

**Question #2:**

How does the visible mass compare to the dark matter mass ( $M_{\text{darkmatter}}$ )?

**Answer #2:**

There is ~6 times more mass is dark matter than in visible matter.

**Question #3:**

What does this tell you about how much of the galaxy you can directly study with telescopes?

**Answer #3:**

Only a tiny amount of the mass of a galaxy can be detected by telescopes; most of the galaxy is invisible. Note: Gravity acts on both the dark and luminous matter in the exact same way. The luminous matter can be considered test particles. These test particles tell you about the **whole** galaxy even though they are a small percentage of the total mass.

**Question #4:**

How does the shape of the *observed* galaxy rotation curve compare to the *expected* shape of a galaxy rotation curve? Do they look the same or different?

**Answer #4:**

The observed rotation curve rises close to linearly near the center of the galaxy and then has a roughly constant velocity at large radii; the rotation curve does not show a steep drop in velocities at large radii like the expected rotation curve.

**Question #5:**

What does the shape of the rotation curve tell you about how the mass is distributed between the galaxy's central bulge and the halo? Is most of the mass in the bulge of the galaxy, or is there significant non-luminous mass in the halo?

**Answer #5:**

If most of the mass of the galaxy were located in the center of the galaxy, then the outer part of the rotation curve would show declining velocities. The gravitational force on the gas would decrease as you approach the outer edge of the galaxy and as a result, it would rotate more slowly. The gas near the edge of the galaxy, however, is rotating very quickly, and this indicates that there is a substantial amount of non-luminous mass in the galaxy halo.

**Optional Extension:**

While astronomers know that dark matter exists, they do not yet know exactly what it is. They do, however, have a pretty good idea of what it is not. Astronomers have suggested many things that we are familiar with that could be the dark matter (objects like planets or very dim stars), but have so far been able to rule them out. To explore why planets cannot be the dark matter, have the students first calculate the mass of the Sun in units of Earth masses to determine how many Earths it takes to equal the mass of the Sun. Then figure out how many Earths it would take to equal the mass of dark matter in this spiral galaxy.

$$\begin{aligned}M_{sun} &= 2 \times 10^{30} \text{ kg} \\M_{earth} &= 6 \times 10^{24} \text{ kg} \\M_{sun} &= (2 \times 10^{30} \text{ kg}) \cdot \frac{(1 M_{earth})}{(6 \times 10^{24} \text{ kg})}\end{aligned}$$

$$M_{sun} = 3.3 \times 10^5 M_{earth}$$

$$M_{\text{darkmatter}} = 6 \times 10^{10} M_{\text{sun}}$$

$$M_{\text{darkmatter}} = (6 \times 10^{10} M_{\text{sun}}) \cdot \frac{(3.3 \times 10^5 M_{\text{earth}})}{(1 M_{\text{sun}})}$$

$$\boxed{M_{\text{darkmatter}} = 2 \times 10^{16} M_{\text{earth}}} \leftarrow \text{This is a LOT of planets!}$$

This number is *huge* and is one reason why astronomers do not think that dark matter is made of planets. To put this number into perspective, the Milky Way currently has about  $2 \times 10^{10}$  stars. This means that each star in the Galaxy would have 1,000,000 planets! These planets are also ruled out as the dark matter because of the amount of infrared radiation that this many planets would generate. Instead, it is currently thought that dark matter is an undiscovered, new elementary particle such as the WIMP envisioned to exist by theorists, but not yet detected in laboratory experiments.

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

# Measuring the Rotational Speed of Spiral Galaxies and Discovering Dark Matter

## Student Worksheet

In this exercise, you will measure the mass in dark matter in a nearby spiral galaxy (UGC 477). You will use the Doppler shift of a hydrogen emission line to measure the speeds at which gas clouds rotate around the center of the galaxy. Using your measured velocities and the equation for circular speed, you will calculate the total dynamical mass of the galaxy. You will then compare the mass of the galaxy in stars and gas (the mass in normal matter) to the dynamical mass and calculate what percentage of a galaxy's total mass is made of dark matter. Dark matter has mass, but does not interact electromagnetically to produce light. It is currently thought to be an unusual elementary particle that we have yet to detect in laboratory experiments.

### A. Mass Components of a Galaxy

Galaxies are large collections of stars, gas and dark matter that are held together by their own gravity. There are three general types of galaxies: spirals, ellipticals and irregulars. Spiral galaxies are flattened disks in which the stars form a spiral pattern. The stars and gas in a spiral galaxy trace circular orbits around the center of the galaxy. Elliptical galaxies are smooth, featureless galaxies that range in shape from spherical to football-shaped; irregular galaxies have a chaotic, random appearance.

Galaxies are made up of three major mass (M) components: stars, gas, and dark matter such that

$$M_{total} = M_{stars} + M_{gas} + M_{darkmatter} \quad (1)$$

We can detect the stars and gas directly with telescopes (infrared telescopes, optical telescopes, radio telescopes, x-ray telescopes, etc), but we can only indirectly detect the dark matter by the gravitational effect it has on the stars and gas. Because we cannot directly see dark matter, it has been very difficult to figure out what it is.

In units of solar masses ( $M_{sun} = 2 \times 10^{30}$  kg), the mass of your spiral galaxy in stars is  $M_{stars} = 3.8 \times 10^9 M_{sun}$  and the mass in gas is  $M_{gas} = 6.5 \times 10^9 M_{sun}$ . You will calculate the total mass,  $M_{total}$ , and then infer the mass in dark matter,  $M_{darkmatter}$ . Note that this is the first step in trying to identify what might make up dark matter.

### B. Doppler Shift

When a source of light is moving toward or away from an observer, the measured wavelength of the received light ( $\lambda_{observed}$ ) will be different than the wavelength of the light if the source were at rest ( $\lambda_{rest}$ ). The measured wavelength of the received light and the line-of-sight velocity ( $V_{LOS}$ ) of the source are related through the Doppler equation:

$$\frac{(\lambda_{\text{observed}} - \lambda_{\text{rest}})}{\lambda_{\text{rest}}} = \frac{V_{\text{LOS}}}{c} \quad (2)$$

The speed of light  $c$  is  $3 \times 10^5$  km/s.

The hydrogen gas in your spiral galaxy is emitting light with a rest wavelength ( $\lambda_{\text{rest}}$ ) 6563 Å (Angstroms;  $1 \text{Å} = 10^{-9}$  m). This gas rotates around the center of the galaxy. Because we are viewing the galaxy close to edge-on (like looking at the side of a plate), some of the gas is moving away from us and some of it is moving toward us. You can use the observed wavelength of the light ( $\lambda_{\text{observed}}$ ) and the Doppler equation to determine how fast the gas is moving toward or away from us ( $V_{\text{LOS}}$ ) at different distances from the center of the galaxy.

### C. Rotation Velocity

The line-of-sight velocity that is calculated from the Doppler equation tells you how fast the gas is moving directly toward or away from you; what you really need to know, however, is how fast the gas is rotating around the galaxy center. The equation you need for that is:

$$V_{\text{rotation}} = \frac{(V_{\text{LOS}} - V_{\text{recession}})}{\sin(i)} \quad (3)$$

where  $V_{\text{rotation}}$  is the rotation velocity,  $V_{\text{LOS}}$  is the velocity you determined from the Doppler shift,  $V_{\text{recession}}$  is how fast the galaxy is moving away from us, and  $\sin(i)$  is the sine of the inclination angle  $i$  of the galaxy. The inclination angle is a measure of how close to edge-on you are viewing a galaxy. A galaxy viewed face-on (like looking straight down at a dinner plate) has  $i = 0^\circ$ ; a galaxy viewed edge-on (like looking at the edge of a dinner plate) has  $i = 90^\circ$ . Let's take a closer look at this equation.

It is important to recognize that as a whole, the spiral galaxy is moving away from us. This velocity is called recession velocity. In addition to that movement, the gas clouds in the galaxy are rotating around the center of the galaxy with a rotational speed called the rotation velocity. The Doppler shift velocity is a combination of both the recession velocity and the rotation velocity. We need to separate these two velocities.

In Figure 1 is the Hubble Space Telescope image of the spiral galaxy M101 (<http://hubblesite.org/gallery/album/galaxy/spiral/pr2006010a/>). We can assume that the very center of the galaxy (star 2) is not rotating and is only moving in a straight line away from us. The velocity we measure here is the recession velocity. The recession velocity of your spiral galaxy is  $V_{\text{recession}} = 2675$  km/s. The rest of the galaxy is moving away from us *and* rotating. If we view the galaxy close to edge-on, this means the gas near star 1 will be moving toward us, the gas near star 3 will be moving away from us, and the gas near stars 4 and 5 will be moving across our line-of-sight. The measured velocity of a gas cloud near star 3 will therefore be the recession velocity plus a positive rotation velocity. Near star 1 the measured velocities will be the recession velocity plus a negative rotation velocity. Because the gas near stars 4 and 5 do not have any movement *along* our line of sight, the velocities measured there will be only the recession velocity. Going back to the top right side of Equation 3, we see that to isolate



**Figure1:** Motion of gas in a galaxy viewed edge-on. The entire galaxy is moving away from us. Gas near star 1(3) is rotating toward(away) from us. Gas near stars 4 & 5 is moving across our line-of-sight.

the rotational contribution to the measured velocity, we subtract the recession velocity from the measured velocity.

Now let's discuss the bottom part of the right side of Equation 3: the division by  $\sin(i)$ . In this equation,  $i$  is the inclination of the galaxy. The inclination of your spiral galaxy is  $i = 82^\circ$ . Remember that Doppler shift only measures an object's motion *directly* toward or away from you. If an object is moving *across* your line of sight (like the gas near stars 4 and 5), you will not detect any additional Doppler shift over  $V_{\text{recession}}$ . So unless your galaxy is inclined by  $90^\circ$  (and assuming the inclination is not  $0^\circ$ ), the Doppler shift velocity you measure will only be a fraction of the total motion. To correct for this unmeasurable velocity, we divide by  $\sin(i)$ .

#### **D. Dynamical Mass from the Circular Velocity**

The rotation speed of an object ( $V_{\text{rotation}}$ ) depends on the mass of the object being orbited ( $M$ ), the distance between the object and the center of the circular orbit ( $r$ ), and the gravitational constant ( $G$ ):

$$V_{\text{rotation}} = \sqrt{(GM) / r} \quad (4)$$

$G$  is  $1.33 \times 10^{11} \text{ km}^3 \text{ M}_{\text{sun}}^{-1} \text{ s}^{-2}$ . If you know the distance and the velocity, you can use a little bit of algebra to rearrange the equation and solve for mass. Masses that are determined using this technique are called 'dynamical' masses:

$$M_{\text{dynamical}} = \frac{(V_{\text{rotation}})^2 r}{G} \quad (5)$$

The dynamical mass is the *total* mass inside of  $r$ ; it is the mass in stars, gas, and dark matter. If we measure  $V_{\text{rotation}}$  at the edge of the galaxy,  $r_{\text{edge}}$ , then we can calculate the total mass of the galaxy ( $M_{\text{dynamical}} = M_{\text{total}}$ ). If we know the mass of the galaxy in stars and gas ( $M_{\text{stars}}$  and  $M_{\text{gas}}$ ), we can use the total dynamical mass to calculate the mass in dark matter ( $M_{\text{darkmatter}}$ ).

### **E. An Example**

We will define the center of the galaxy to be at  $r = 0$ . Radii measured on the approaching side of the galaxy will be defined to be negative ( $r < 0$ ), and radii on the receding side will be defined to be positive ( $r > 0$ ).

At a radius of  $r = -0.25$  kiloparsecs (kpc;  $1 \text{ kpc} = 3.08 \times 10^{16} \text{ km}$ ), the observed wavelength of the hydrogen gas is  $\lambda_{\text{observed}} = 6621.2 \text{ \AA}$ . A negative radius indicates that the gas is on the side of the galaxy that is moving toward us.

To calculate the line-of-sight velocity, we solve for  $V_{\text{LOS}}$ :

$$V_{\text{LOS}} = \frac{c (\lambda_{\text{observed}} - \lambda_{\text{rest}})}{\lambda_{\text{rest}}}$$

$$V_{\text{LOS}} = \frac{(3 \times 10^5 \text{ km/s})(6621.2 \text{ \AA} - 6563.0 \text{ \AA})}{6563.0 \text{ \AA}}$$

$$V_{\text{LOS}} = 2660.4 \text{ km/s}$$

Using  $V_{\text{LOS}}$ ,  $V_{\text{recession}} = 2675 \text{ km/s}$ , and  $i = 82^\circ$ , calculate  $V_{\text{rotation}}$ :

$$V_{\text{rotation}} = \frac{V_{\text{LOS}} - V_{\text{recession}}}{\sin(i)}$$

$$V_{\text{rotation}} = \frac{2660.4 \text{ km/s} - 2675.0 \text{ km/s}}{\sin(82^\circ)}$$

$$V_{\text{rotation}} = -14.7 \text{ km/s}$$

A negative rotation velocity means the gas is rotating toward us.

Calculate the dynamical mass inside of 0.25 kpc.

Remember that  $1 \text{ kpc} = 3.08 \times 10^{16} \text{ km}$  and  $G = 1.33 \times 10^{11} \text{ km}^3 M_{\text{sun}}^{-1} \text{ s}^{-2}$ .

$$M_{\text{dynamical}} = \frac{(V_{\text{rotation}})^2 r}{G}$$

$$M_{\text{dynamical}} = \frac{(-14.7 \text{ km/s})^2 (0.25 \text{ kpc})}{1.33 \times 10^{11} \text{ km}^3 M_{\text{sun}}^{-1} \text{ s}^{-2}} \frac{(3.08 \times 10^{16} \text{ km})}{(1 \text{ kpc})}$$

$$M_{\text{dynamical}} = 1.25 \times 10^7 M_{\text{sun}}$$

## **F. Calculate the Dark Matter Mass**

Your teacher will assign you and your group a few of the radii and wavelengths in the data table. For each set of measurements, calculate  $V_{\text{LOS}}$ ,  $V_{\text{rotation}}$ , and  $M_{\text{dynamical}}$ . Once the students in your group have finished the calculations, compare your answers and record them in the table. Next, plot your points on the rotation curve. You may assume that your galaxy rotates symmetrically about the center and therefore plot the absolute value of  $r$  and  $V_{\text{rotation}}$ . Inform the teacher of your results.

After the entire class completes the table, use the dynamical mass calculated at the largest radius ( $M_{\text{dynamical}} \text{ at } r_{\text{edge}} = M_{\text{total}}$ ) to determine the mass in dark matter,  $M_{\text{darkmatter}}$ .

**CALCULATIONS:**

*Be sure to show your work and to include units!*

1. The total dynamical mass of the galaxy is:

$$M_{\text{total}} = \underline{\hspace{10cm}}$$

2. Calculate the mass of the galaxy that is in dark matter. Remember that:

$$M_{\text{stars}} = 3.8 \times 10^9 M_{\text{sun}}$$

$$M_{\text{gas}} = 6.5 \times 10^9 M_{\text{sun}}$$

$$M_{\text{total}} = M_{\text{stars}} + M_{\text{gas}} + M_{\text{darkmatter}}$$

$$M_{\text{darkmatter}} = \underline{\hspace{10cm}}$$

3. What percentage of the total mass of the galaxy is in dark matter?

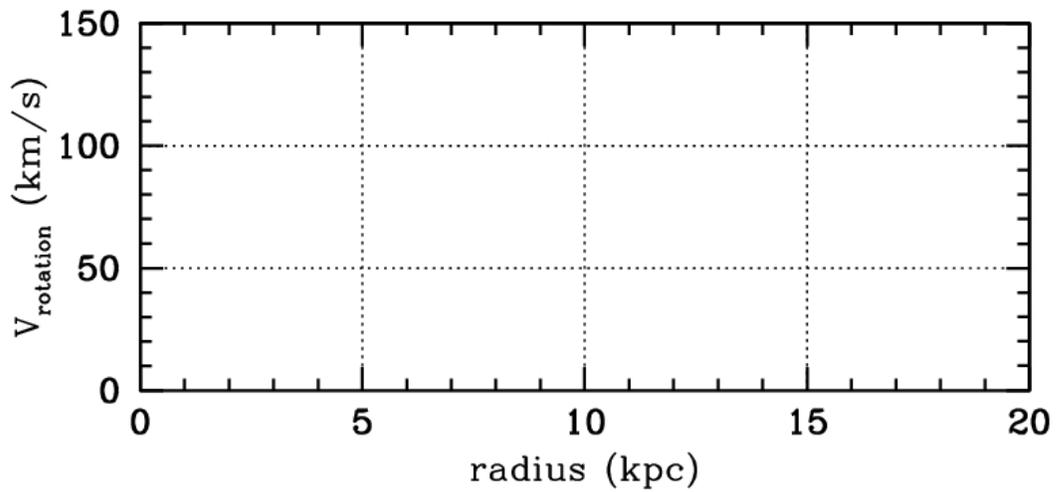
$$\frac{M_{\text{darkmatter}}}{M_{\text{total}}} = \underline{\hspace{10cm}} \times 100 = \underline{\hspace{10cm}} \%$$

4. What is the ratio of dark matter mass to visible mass (stars and gas)?

$$\frac{M_{\text{darkmatter}}}{(M_{\text{stars}} + M_{\text{gas}})} = \underline{\hspace{10cm}} = \underline{\hspace{10cm}}$$

Data Table

R (kpc)	$\lambda_{\text{observed}}$ (Å)	$V_{\text{LOS}}$ (km/s)	$V_{\text{rotation}}$ (km/s)	Enclosed Dynamical Mass ( $M_{\text{sun}}$ )
-0.25	6621.2	2660.4	-14.7	$1.25 \times 10^7$
-0.75	6621.1			
-1.0	6621.0			
-2.0	6620.5			
-3.0	6620.4			
-3.25	6620.3			
-3.5	6620.4			
-3.75	6620.2			
-4.75	6619.9			
-9.0	6619.4			
-10.75	6619.2			
-18.0	6618.7			
1.75	6622.2			
4.0	6622.8			
5.25	6622.9			
5.5	6623.2			
6.0	6623.2			
6.5	6623.3			
7.75	6623.5			
12.0	6623.8			
15.0	6624.0			



**ANSWERS:**

R (kpc)	$\lambda_{\text{observed}}$ (Å)	$V_{\text{LOS}}$ (km/s)	$V_{\text{rotation}}$ (km/s)	Enclosed Dynamical Mass ( $M_{\text{sun}}$ )
-0.25	6621.2	2660.4	-14.7	$1.25 \times 10^7$
-0.75	6621.1	2655.8	-19.4	$6.54 \times 10^7$
-1.0	6621.0	2651.2	-24.0	$1.33 \times 10^8$
-2.0	6620.5	2628.4	-47.1	$1.03 \times 10^9$
-3.0	6620.4	2623.8	-51.7	$1.86 \times 10^9$
-3.25	6620.3	2619.2	-56.3	$2.39 \times 10^9$
-3.5	6620.4	2623.8	-51.7	$2.17 \times 10^9$
-3.75	6620.2	2614.7	-60.9	$3.22 \times 10^9$
-4.75	6619.9	2600.9	-74.8	$6.15 \times 10^9$
-9.0	6619.4	2578.1	-97.6	$1.99 \times 10^{10}$
-10.75	6619.2	2568.9	-107.1	$2.86 \times 10^{10}$
-18.0	6618.7	2546.1	-130.2	$7.07 \times 10^{10}$
1.75	6622.2	2706.1	31.4	$4.00 \times 10^8$
4.0	6622.8	2733.5	59.1	$3.24 \times 10^9$
5.25	6622.9	2738.1	63.7	$4.93 \times 10^9$
5.5	6623.2	2751.8	77.6	$7.67 \times 10^9$
6.0	6623.2	2751.8	77.6	$8.37 \times 10^9$
6.5	6623.3	2756.4	82.2	$1.02 \times 10^{10}$
7.75	6623.5	2765.5	91.4	$1.50 \times 10^{10}$
12.0	6623.8	2779.2	105.2	$3.08 \times 10^{10}$
15.0	6624.0	2788.4	114.5	$4.55 \times 10^{10}$

$$M_{\text{total}} = 7.07 \times 10^{10} M_{\text{sun}}$$

$$M_{\text{darkmatter}} = M_{\text{total}} - M_{\text{stars}} - M_{\text{gas}} = (7.07 \times 10^{10}) - (3.8 \times 10^9) - (6.5 \times 10^9) = 6.04 \times 10^{10} M_{\text{sun}}$$

$$\frac{M_{\text{darkmatter}}}{M_{\text{total}}} = \frac{6.04 \times 10^{10} M_{\text{sun}}}{7.07 \times 10^{10} M_{\text{sun}}} \times 100 = 85\% \text{ of the total mass is in dark matter}$$

$$\frac{M_{\text{darkmatter}}}{(M_{\text{stars}} + M_{\text{gas}})} = \frac{6.04 \times 10^{10} M_{\text{sun}}}{(3.8 \times 10^9 + 6.5 \times 10^9) M_{\text{sun}}} = 5.8 \text{ There is about 6 times more mass in dark matter than visible matter.}$$

ANSWERS:

