

# Determining the Sizes of Stars Using the H-R Diagram



**Activity UCIObs – 4**  
**Grade Level: 8 – 12**

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**Standards:** This activity addresses these California Science Content Standards:

*Gr 9-12 Earth Sciences:*

2d. Students know that stars differ in their life cycles and that visual, radio, and X-ray telescopes may be used to collect data that reveal those differences.

*Gr 8 Physical Science:*

4b. Students know that the Sun is one of many stars in the Milky Way galaxy and that stars may differ in size, temperature, and color.

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## What's This Activity About?

In this activity students will explore the reasons why stars are grouped into specific categories in the Hertzsprung-Russell (HR) Diagram based on their luminosities, temperatures and radii.

## What Will Students Do?

Students will be asked to locate specific stars on an HR Diagram and measure their temperature and luminosity. Using Stefan's Law, which says that an object's luminosity, temperature and size are related, students will calculate the stars' radii. These radii give physical meaning to the categories of stars:

supergiants, giants, main sequence, and white dwarfs.

## Tips and Suggestions

Review how to use a graph and scientific notation with the students (i.e.,  $10^4 = 10,000$ ) before they begin.

Some of the stars marked on the HR Diagram are some of the brightest stars in the night sky, which can easily be seen without a telescope even in an urban setting, thus you can combine this activity with a star-gazing assignment. Details are given in Optional Activity B.

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## What Will Students Learn?

**Concepts:** Stellar Properties, HR Diagram, Stefan's Law

**Inquiry Skills:** Graphing, Computing, Comparing, Evaluating, Classifying

**Big Ideas:** Scale, Diversity & Unity

### What you will need for each student or team of students:

Ruler

Scientific calculator

Scratch paper for the calculations

### Preparation:

1. Photocopy the Instructions and HR Diagram for each student (pages 5 - 9).

### Teacher Instructions:

1. Hand out the materials.
2. Review scientific notation, i.e.,  $10^4 = 10,000$ ,  $10^{-2} = 0.01$ ,  $10^0 = 1$
3. Review how to use a graph.
4. Explain the HR Diagram. Point out that the temperature axis is reversed with hotter stars on the left and colder stars to the right. More luminous (intrinsically bright) stars lie at the top of the diagram and lower luminosity (intrinsically faint) stars lie at the bottom of the diagram.
5. Explain that stars are grouped into four different categories:
  - **Main Sequence Stars:** The characteristic of all these stars are that they are all generating energy by nuclear fusion deep in their cores. In low mass stars, fusion proceeds by hydrogen being burned into helium while in high mass stars, fusion proceeds through the carbon-nitrogen-oxygen cycle. In each case the net effect is the conversion of mass to energy, which powers the star's luminosity. For main sequence stars, their luminosity, temperature and radius are set by their mass. The most massive stars are the hottest and most luminous, and the least massive stars are the coolest and least luminous.
  - **Giants:** Giants are stars that have begun to evolve after finishing their time as a main sequence star. Their cores have filled with helium. There is not enough hydrogen left in the core so burning occurs in a shell surrounding the core, where more helium resides. In the process, the star grows in radius, becoming **cooler** but **more luminous**.
  - **Supergiants:** Supergiants are the more **massive stars** that have begun to evolve after finishing their time as a main sequence star. These stars grow in radius, and can change temperature dramatically, but they do not change much in luminosity.
  - **White Dwarfs:** White dwarfs are the end states of stars less than around 6 times the mass of the Sun. The white dwarf is the core of the star that is left after episodes of mass loss causes the star to eject its outer layers, which continue expanding out into space to form a planetary nebulae. For example, a star like the Sun will lose approximately 60% of its mass as it evolves to become a white dwarf. White dwarfs are low in luminosity, because fusion can no longer occur in them. However they are hot because they used to be the core of the star where the temperature was many millions of degrees K, which is much hotter than the outer surface of stars. The white dwarf will slowly cool over time, like a dying ember in a fire, becoming fainter and redder as it radiates away its heat.
6. Explain that the names of the four categories contain information on the sizes of the stars. In general, from smallest to largest are: white dwarfs, main sequence stars, giants and supergiants.
7. Go through the instructions with the students and do the example calculation of the radius of the Sun with them.

8. Split students up into groups and have them calculate the radius for their given star or stars.
9. While students are working, draw the blank data table on page 11 on the board so students can come up and fill in their answers once they are finished.
10. When a group has finished comparing the results of all the students in the group, resolved any discrepancies, and agreed on the correct radius of their assigned star(s), you should have a representative from the group come up and fill in their data in the table on the board, and all students should write the radius next to the star's name on their HR Diagram.
11. When all groups have finished, discuss the results. Ask the class to look for unifying patterns in the radii.
  - Consider stars along the Main Sequence. Question: How does the radii of stars change going from the most luminous, hotter and most massive (upper left corner of the HRD) to the least luminous, cooler and least massive stars (lower right corner of the HRD)? Answer: The radii of main-sequence stars increases as mass increases.
  - Consider the Giants. Question: Do the giants have smaller or larger radii compared to Main Sequence stars? Answer: Giants have larger radii compared to main-sequence stars of the same luminosity.
  - Consider the Supergiants: Question: Which are larger: Supergiants or Giants? Answer: Supergiants.
  - Consider the White Dwarfs. Question: How does the radius of Sirius B compare to those of main-sequence stars like Vega or Sirius A that have the same temperature? Answer: Sirius B has a much smaller radius.
12. Discuss how the names of the four categories reflect their ordering in radii. Ask the class what they would have called these categories if they had a choice.

## **Optional Activities:**

### **A. Physical Scale Images**

If your classroom has access to the internet, you can reinforce the lesson by showing students the Physical Scales link at the UCI Observatory web page:

[http://www.physics.uci.edu/%7Eobservat/Physical\\_Scales.html](http://www.physics.uci.edu/%7Eobservat/Physical_Scales.html)

which contains some beautiful images showing the relative sizes of some of the stars used in this activity. We do not know who the author of the images are, but they are very colorful and a very useful addition to this exercise. We have color versions of these as posters at the UCI Observatory and we use them when classes visit.

### **B. Star Gazing Assignment**

In addition, you can assign a star-gazing activity in which you ask students to search for these specific stars if they are up in the night sky. To get a free night sky map for the current month, we recommend using:

<http://skymaps.com/downloads.html>

But be aware that only half the celestial sphere is visible at any given time. The other half is hidden by the Earth. So students may only be able to see a subset of the stars used in this

exercise. Which stars they can see will depend on the month of the year when you do the exercise.

Give night sky maps for the current month (for examples, see <http://skymaps.com>) and ask students to locate the stars in the night sky and observe their apparent color. For example, do Rigel and Deneb look bluer than Betelgeuse or Arcturus? Consult the HR Diagram and the tables you created to learn their temperatures.

Most of the stars students can see in the night sky are those with the highest intrinsic luminosity. Are they mostly main sequence stars, supergiants or giants? Or is there a mix of each category?

You can ask students to record the apparent brightness and colors of stars using the key on the sky map. The size of the dot marking the star tell you its apparent magnitude; 0 mag are the brightest stars in the night sky and 6<sup>th</sup> mag are the faintest stars in the night sky (or 4 or 5 if you live in a city with a bright night sky background). The stars that are hotter are bluer, and the stars that are cooler are redder. Remember that star's apparent brightness is a function of their intrinsic luminosity as well as their distance. For stars with the same luminosity, the star that is farther away will appear fainter. If students want to calculate a star's distance, they can use the sky map to estimate the apparent magnitude of a star ( $m$ ) and the HR Diagram to determine its absolute magnitude ( $M$ ) and calculate the distance using the equation,

$$m - M = 5 \log( d / 10 \text{ pc} )$$

where 1 parsec (pc) =  $3.085 \times 10^{16}$  m. You can tell students that stars distances are usually given in units of parsec rather than in meters, because a parsec is the typical distance between stars in the neighborhood of the Sun.

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Per: \_\_\_\_\_

## Determining the Sizes of Stars Using the H-R Diagram Exercise

In this exercise, you are going to use an Hertzsprung-Russell Diagram to determine the luminosity and temperature of stars. Then you are going to use a relationship called Stephan's Law to calculate the stars' radii. By comparing the radii of different stars you will then find out the reason behind the names of the four categories of stars: white dwarfs, main-sequence stars, giants and supergiants.

### A. Stars

Stars are born with a wide variety of **mass**. The most massive stars are 100 times more massive than the Sun while the least massive ones are only 0.08 times the mass of the Sun. Most stars spend about 90% of their lifetimes shining due to nuclear fusion that goes on in their cores, but after awhile they evolve and begin to die. How long they live and what they evolve to become when they die depends on their mass. In fact, the mass of a star also determines its most important properties: its luminosity, temperature and radius.

A star's **luminosity**, which is how much energy is emitted per second from the star, is measured in Watts or in solar luminosities ( $L_{\odot}$ ) where  $1L_{\odot} = 3.85 \times 10^{26}$  Watt. We determine a star's luminosity by measuring its distance and its apparent brightness, which we call its apparent magnitude. Knowing those two, we can calculate its absolute magnitude, which is how bright the star would be if it were 10 parsecs away from us, and its luminosity relative to the Sun.

A star's **temperature** is the temperature of the gas on the surface of the star. We measure temperature on the Kelvin scale, in which 0 K means that an object has absolutely zero energy. Note that the temperature of the surface of a star is much lower than the temperature in the interior of the star where nuclear reactions happen. For example, the Sun's surface temperature is approximately 6,000 K, but the temperature at the center of the Sun is 15,000,000 K! That is why nuclear reactions only happen in the interiors of stars.

A star's **radius** is simply half the star's diameter. Stars are simply large balls of gas held together by gravity, and they are approximately spherical in shape. Radii of stars can be measured in meters, but because stars are so very large that its much more convenient to measure stellar radii in units of the Sun's radius, where  $1 R_{\odot} = 6.96 \times 10^8$  m.

### B. The Hertzsprung-Russell Diagram

A graph of the temperature and luminosity of stars is called the Hertzsprung-Russell Diagram (H-R Diagram), which is named after two famous astronomers. Einar Hertzsprung was a Danish astronomer, and Henry Norris Russell was an American astronomer, and they both did ground-breaking work in measuring and understanding the properties of stars in the early 1900s.

### **C. Calculating a Star's Radius Using Stefan's Law**

Stefan's Law says that for any radiating object its luminosity, temperature and radius are related by this simple formula:

$$\frac{L}{4\pi R^2} = \sigma T^4 \quad \text{EQ \#1}$$

where L is the luminosity, R is the radius, T is the surface temperature,  $\pi = 3.141$  and  $\sigma = 5.671 \times 10^{-8} \text{ Watt/m}^2 \text{ K}^4$ . This means that if we measure the luminosity and temperature of a star then we can calculate its radius. Taking the above equation and solving for R gives us

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}} \quad \text{EQ \#2}$$

which is the equation we can use to calculate the radius in meters. Once we have that we can convert it to solar radii by using

$$R \text{ in } R_{\odot} = R \text{ in meters} \times 1 R_{\odot} / 6.96 \times 10^8 \text{ m.} \quad \text{EQ \#3}$$

### **D. An Example: The Sun**

Lets do an example and calculate the Sun's radius. Looking at the HR Diagram, we see that the Sun's luminosity is  $1 L_{\odot} = 3.85 \times 10^{26} \text{ Watt}$  and its temperature is approximately 6,000 K. To calculate its radius, we use EQ #2:

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

$$R = \sqrt{\frac{3.85 \times 10^{26} \text{ Watt}}{4 \times 3.141 \times 5.671 \times 10^{-8} \text{ Watt/m}^2 \text{ K}^4 \times (6000 \text{ K})^4}}$$

$$R = \sqrt{4.17 \times 10^{17} \text{ m}^2}$$

$$R = 6.46 \times 10^8 \text{ m}$$

Now convert the radius from units of m to units of solar radius:

$$R \text{ in } R_{\odot} = R \text{ in meters} \times 1 R_{\odot} / 6.96 \times 10^8 \text{ m}$$

$$R = 6.46 \times 10^8 \text{ m} \times 1 R_{\odot} / 6.96 \times 10^8 \text{ m}$$

$$R = 0.93 R_{\odot} \approx 1 R_{\odot}.$$

### E. Calculate the Radius for Your Star

Star Name: \_\_\_\_\_

Star's Category (Circle One):    White Dwarf    Main Sequence    Giant    Supergiant

Using the HR Diagram, record below your star's luminosity in solar units and its temperature in degrees K, but remember that the temperature axis is non-linear and backward so that hotter objects are to the left and cooler objects are to the right.

Luminosity,            L = \_\_\_\_\_  $L_{\odot}$

Temperature,            T = \_\_\_\_\_ K

Convert your star's luminosity from units of solar luminosities to Watts:

Luminosity,            L = \_\_\_\_\_  $L_{\odot} \times 3.85 \times 10^{26} \text{ Watt} / 1L_{\odot}$

L = \_\_\_\_\_ Watt

Now calculate the star's radius using EQ #2:

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

$$R = \sqrt{\frac{\text{_____ Watt}}{4 \times 3.141 \times 5.671 \times 10^{-8} \text{ Watt/m}^2 \text{K}^4 \times (\text{_____ K})^4}}$$

$$R = \sqrt{\text{_____ m}^2}$$

$$R = \text{_____ m}$$

Now convert the radius from units of meters to units of solar radius using EQ #3,

$$R \text{ in } R_{\odot} = R \text{ in meters} \times 1 R_{\odot} / 6.96 \times 10^8 \text{ m}$$

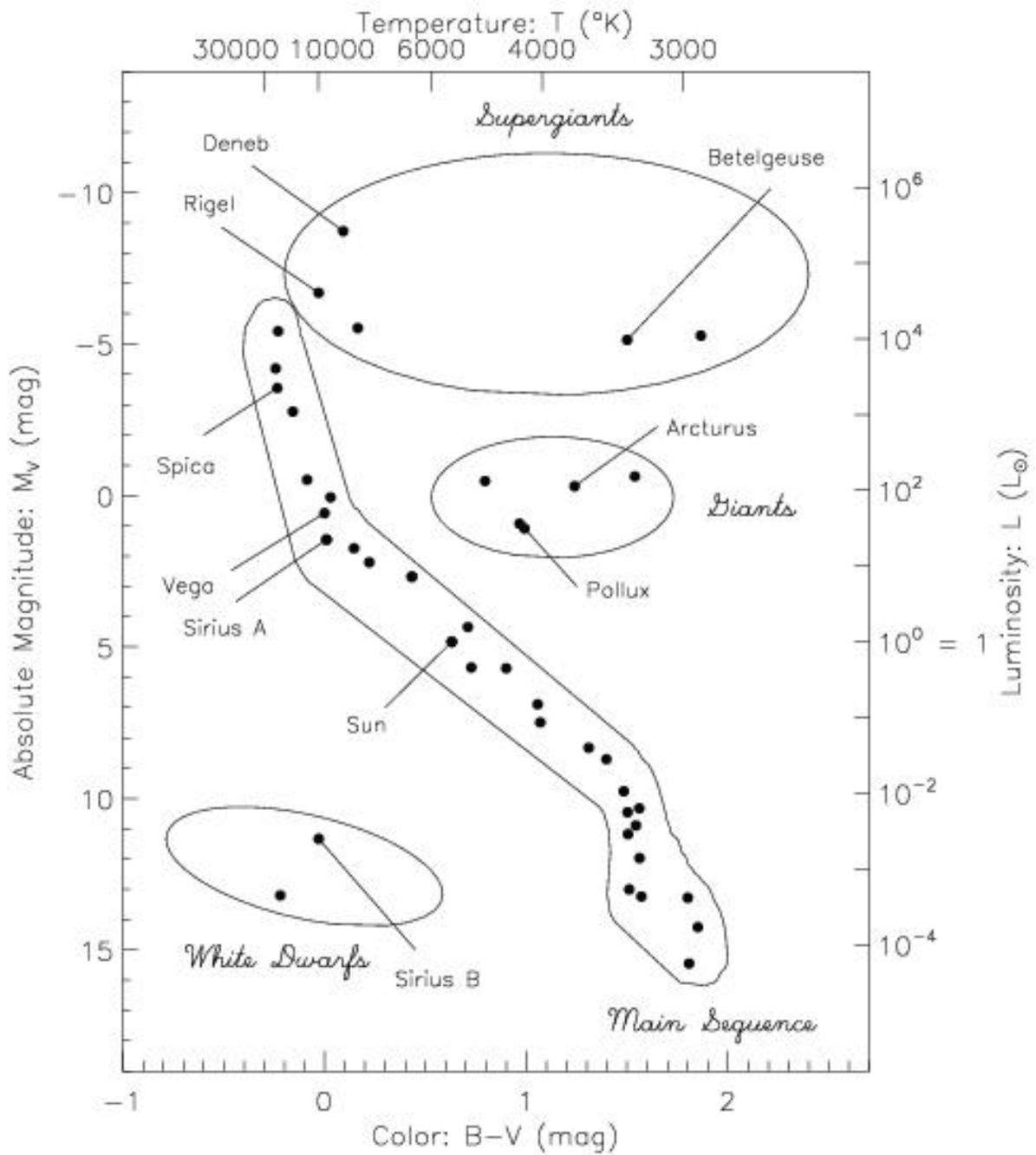
$$R = \underline{\hspace{2cm}} \text{ m} \times 1 R_{\odot} / 6.96 \times 10^8 \text{ m}$$

$$R = \underline{\hspace{2cm}} R_{\odot}$$

### **F. Compare Your Results**

Once the students in your group have finished calculating the radius, compare your answers, resolve any differences and inform the teacher of your results. Your teacher will collect the results and then you will analyze them to understand how the radii of stars vary among the different categories of stars.

# The Brightest and Nearest Stars in the Night Sky



## Data & Results

You can write a blank version of the table (see below) on the blackboard and have a representative from each group come up and fill in the entries for their star. Then have the class examine the trends in the radii. The answers are on the following page.

<b>Star Name</b>	<b>Category</b>	<b>L (<math>L_{\odot}</math>)</b>	<b>T (K)</b>	<b>R (m)</b>	<b>R (<math>R_{\odot}</math>)</b>
Betelgeuse					
Deneb					
Rigel					
Arcturus					
Pollux					
Spica					
Vega					
Sirius A					
Sun					
Sirius B					

**Answers:**

<b><i>Star Name</i></b>	<b><i>Category</i></b>	<b><i>L (L<sub>☉</sub>)</i></b>	<b><i>T (K)</i></b>	<b><i>R (m)</i></b>	<b><i>R (R<sub>☉</sub>)</i></b>
Betelgeuse	Supergiant	10,000	3,500	1.9 x 10 <sup>11</sup>	273
Deneb	Supergiant	250,000	9,000	1.4 x 10 <sup>11</sup>	206
Rigel	Supergiant	40,000	10,000	4.7 x 10 <sup>10</sup>	67
Arcturus	Giant	100	3,800	1.6 x 10 <sup>10</sup>	23
Pollux	Giant	25	4,200	6.6 x 10 <sup>9</sup>	9.5
Spica	MS	2,000	25,000	1.7 x 10 <sup>9</sup>	2.4
Vega	MS	50	10,000	1.6 x 10 <sup>9</sup>	2.4
Sirius A	MS	20	9,500	1.2 x 10 <sup>9</sup>	1.7
Sun	MS	1	5,800	6.9 x 10 <sup>8</sup>	1
Sirius B	White Dwarf	0.0025	10,000	1.2 x 10 <sup>7</sup>	0.017