

Determining the Sizes & Distances of Stars Using the H-R Diagram



Activity UCIObs – 11 College Level

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What's This Activity About?

In this activity students will explore the reasons why stars are grouped into specific categories in the Hertzsprung-Russell (HR) Diagram based on their luminosities, temperatures and radii. In addition, they will learn how to calculate distances of the stars.

What Will Students Do?

Students will be asked to locate specific stars on an HR Diagram and measure their temperature and luminosity. Using Stefan's Law, which says that an object's luminosity, temperature and size are related, students will calculate the stars' radii. These radii give physical meaning to the categories of stars: supergiants, giants, main sequence, and

white dwarfs. In addition, the students will use the luminosity and apparent magnitude of the stars to compute their distance.

Reading Assignment

Chapter 13 – Taking the Measure of Stars in *"21st Century Astronomy"* by J. Hester et al. (2007).

Tips and Suggestions

This activity requires algebra and logarithms. Students should review basic algebra, scientific notation (i.e., $10^4 = 10,000$), and how to read a logarithmic graph before beginning. For example, see Appendix 1 in *"21st Century Astronomy"* by J. Hester et al. (2007) for a review.

What Will Students Learn?

Concepts: Stellar Properties (Luminosity, Temperature, Radius, Distance), HR Diagram, Stefan's Law, Absolute Magnitude, Apparent Magnitude

Inquiry Skills: Graphing, Computing, Comparing, Evaluating, Classifying

Big Ideas: Scale, Diversity & Unity

What Will a Student Need?

Ruler, Scientific Calculator, Scratch Paper for Calculations

Determining the Sizes & Distances of Stars Using the H-R Diagram

Name: _____ Date: _____ Section: _____

A. Introduction

Do the assigned reading (see previous page for specifics):

Review scientific notation, i.e., $10^4 = 10,000$, $10^{-2} = 1/100 = 0.01$, $10^0 = 1$.

Review how to use a logarithmic graph.

Review the HR Diagram. Note that the temperature axis is reversed with hotter stars on the left and colder stars to the right. More luminous (intrinsically bright) stars lie at the top of the diagram and lower luminosity (intrinsically faint) stars lie at the bottom of the diagram.

Review that stars are grouped into four different categories in the HR Diagram:

- **Main Sequence Stars:** The characteristic of all these stars are that they are all generating energy by nuclear fusion deep in their cores. In low mass stars, fusion proceeds by hydrogen being burned into helium while in high mass stars, fusion proceeds through the carbon-nitrogen-oxygen cycle. In each case the net effect is the conversion of mass to energy, which powers the star's luminosity. For main sequence stars, their luminosity, temperature and radius are set by their mass. The most massive stars are the hottest and most luminous, and the least massive stars are the coolest and least luminous.
- **Giants:** Giants are stars that have begun to evolve after finishing their time as a main sequence star. Their cores have filled with helium. There is not enough hydrogen left in the core so burning occurs in a shell surrounding the core, where more helium resides. In the process, the star grows in radius, becoming **cooler** but **more luminous**.
- **Supergiants:** Supergiants are the more **massive stars** that have begun to evolve after finishing their time as a main sequence star. These stars grow in radius, and can change temperature dramatically, but they do not change much in luminosity.
- **White Dwarfs:** White dwarfs are the end states of stars less than around 6 times the mass of the Sun. The white dwarf is the core of the star that is left after episodes of mass loss causes the star to eject its outer layers, which continue expanding out into space to form a planetary nebulae. For example, a star like the Sun will lose approximately 60% of its mass as it evolves to become a white dwarf. White dwarfs are low in luminosity, because fusion can no longer occur in them. However they are hot because they used to be the core of the star where the temperature was many millions of degrees K, which is much hotter than the outer surface of stars. The white dwarf will slowly cool over time, like a dying ember in a fire, becoming fainter and redder as it radiates away its heat.

In this exercise, you are going to use an Hertzsprung-Russell Diagram to determine the luminosity and temperature of stars. Then you are going to use a relationship called Stephan's Law to calculate the stars' radii. By comparing the radii of different stars you will then find out the reason behind the names of the four categories of stars: white dwarfs, main-sequence stars, giants and supergiants.

Stars

Stars are born with a wide variety of **mass**. The most massive stars are 100 times more massive than the Sun while the least massive ones are only 0.08 times the mass of the Sun. Most stars spend about 90% of their lifetimes shining due to nuclear fusion that goes on in their cores, but after awhile they evolve and begin to die. How long they live and what they evolve to become when they die depends on their mass. In fact, the mass of a star also determines its most important properties: its luminosity, temperature and radius.

A star's **luminosity**, which is how much energy is emitted per second from the star, is measured in Watts or in solar luminosities (L_{\odot}) where $1L_{\odot} = 3.85 \times 10^{26}$ Watt. We determine a star's luminosity by measuring its distance and its apparent brightness, which we call its apparent magnitude. Knowing those two, we can calculate its absolute magnitude, which is how bright the star would be if it were 10 parsecs away from us, and its luminosity relative to the Sun.

A star's **temperature** is the temperature of the gas on the surface of the star. We measure temperature on the Kelvin scale, in which 0 K means that an object has absolutely zero energy. Note that the temperature of the surface of a star is much lower than the temperature in the interior of the star where nuclear reactions happen. For example, the Sun's surface temperature is approximately 6,000 K, but the temperature at the center of the Sun is 15,000,000 K! That is why nuclear reactions only happen in the interiors of stars.

A star's **radius** is simply half the star's diameter. Stars are simply large balls of gas held together by gravity, and they are approximately spherical in shape. Radii of stars can be measured in meters, but because stars are so very large that its much more convenient to measure stellar radii in units of the Sun's radius, where $1 R_{\odot} = 6.96 \times 10^8$ m.

The Hertzsprung-Russell Diagram

A graph of the temperature and luminosity of stars is called the Hertzsprung-Russell Diagram (H-R Diagram), which is named after two famous astronomers. Einar Hertzsprung was a Danish astronomer, and Henry Norris Russell was an American astronomer, and they both did ground-breaking work in measuring and understanding the properties of stars in the early 1900s.

B. Calculating a Star's Radius Using Stefan's Law

Stefan's Law says that for any radiating object its luminosity, temperature and radius are related by this simple formula:

$$\frac{L}{4\pi R^2} = \sigma T^4 \quad \text{EQ \#1}$$

where L is the luminosity, R is the radius, T is the surface temperature, $\pi = 3.141$ and $\sigma = 5.671 \times 10^{-8}$ Watt/m² K⁴. This means that if we measure the luminosity and temperature of a star then we can calculate its radius. Taking the above equation and solving for R gives us

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}} \quad \text{EQ \#2}$$

which is the equation we can use to calculate the radius in meters. Once we have that we can convert it to solar radii by using

$$R \text{ in } R_{\odot} = R \text{ in meters} \times 1 R_{\odot} / 6.96 \times 10^8 \text{ m.} \quad \text{EQ \#3}$$

C. An Example: The Sun

Lets do an example and calculate the Sun's radius. Looking at the HR Diagram, we see that the Sun's luminosity is $1 L_{\odot} = 3.85 \times 10^{26}$ Watt and its temperature is approximately 6,000 K. To calculate its radius, we use EQ #2:

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

$$R = \sqrt{\frac{3.85 \times 10^{26} \text{ Watt}}{4 \times 3.141 \times 5.671 \times 10^{-8} \text{ Watt/m}^2\text{K}^4 \times (6000 \text{ K})^4}}$$

$$R = \sqrt{4.17 \times 10^{17} \text{ m}^2}$$

$$R = 6.46 \times 10^8 \text{ m}$$

Now convert the radius from units of m to units of solar radius:

$$R \text{ in } R_{\odot} = R \text{ in meters} \times 1 R_{\odot} / 6.96 \times 10^8 \text{ m}$$

$$R = 6.46 \times 10^8 \text{ m} \times 1 R_{\odot} / 6.96 \times 10^8 \text{ m}$$

$$R = 0.93 R_{\odot} \approx 1 R_{\odot}.$$

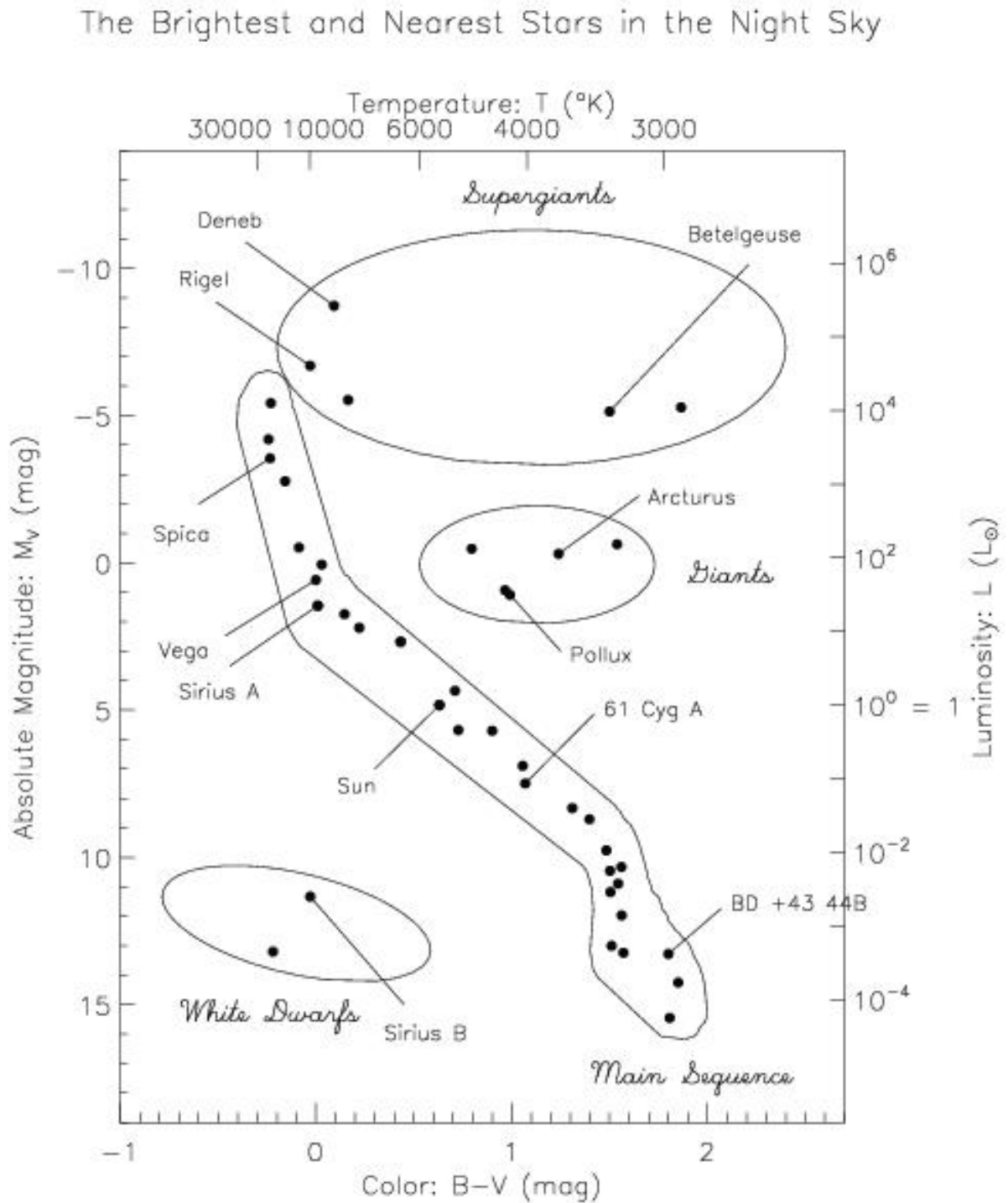
D. Calculating the Radii of the Stars

Using the HR Diagram in Figure 1, estimate the luminosity and temperature of all the stars listed in Table 1 and record their values in Table 1. Now calculate the stars' radii in units of meters and solar radii (see EQs #2 and 3). You must show your calculation explicitly for at least one star in the space below. Enter the radii in units of meters and solar radii in Table 1.

Example Calculation of a Star's Radius

Star Name: _____

Figure 1. The HR Diagram of some of the brightest stars in the Night Sky and stars that are nearest the Sun. Individual stars are identified by name, and stellar categories are shown in script font.



Data & Results

Complete the following table based on the HR Diagram on the previous page. By category, we mean whether the stars are supergiants, giants, main-sequence or white dwarf stars.

Table 1: Stellar Properties

Star Name	Category	L (L_{\odot})	T (K)	R (m)	R (R_{\odot})
Betelgeuse					
Deneb					
Rigel					
Arcturus					
Pollux					
Spica					
Sirius A					
Sun	MS	1.0	6,000	6.5×10^8	1
61 Cyg A					
BD +43 44B					
Sirius B					

Questions:

Complete the following questions below using the results in Table 1. Use complete sentences that clearly demonstrate your understanding of the answer. Answer both qualitatively and quantitatively by citing values of radii in Table 1 that back up your answer.

1. Consider stars along the Main Sequence. How does the radii of stars change going from the most luminous, hotter and most massive (upper left corner of the HRD) to the least luminous, cooler and least massive stars (lower right corner of the HRD)?

E. Stellar Distances

Using the luminosities that you estimated in Table 1, calculate the absolute magnitudes and distances to the stars listed in Table 2.

By definition, the absolute magnitude of a star equals the apparent magnitude the star would have if its distance were 10 pc. The apparent magnitude of a star m is proportional to $-2.5 \log(\text{flux})$ and $\text{flux} = L/4\pi d^2$, where L is the luminosity and d is distance. Because of this, this the absolute magnitude M of a star compared to that of the Sun (whose absolute magnitude is $M_{\odot} = +4.8 \text{ mag}$) is given by

$$M - M_{\odot} = -2.5 \log(L/L_{\odot})$$

Thus using this equation and the luminosities that you measured and listed in Table 1, you can calculate the absolute magnitude of the stars in Table 2.

A star's apparent brightness is a function of its intrinsic luminosity L as well as its distance d . For example, if two stars have the same luminosity, the star that is farther away will appear fainter and have a larger apparent magnitude. Recall that the relationship between distance, apparent magnitude (m) and absolute magnitude (M) is given by the equation:

$$m - M = 5 \log(d / 10 \text{ pc}).$$

Rearrange this equation and solve for d . Show your work below.

Now use the formula you derived to calculate the distances for the stars in Table 2, and record your answers for the absolute magnitudes and distances in Table 2. Give distances in units of parsec (pc). Also record the Category of the star (main sequence, giant, supergiant or white dwarf) in the space provided.

Table 2. Stellar Distances

Star Name	Category	m (mag)	M (mag)	d (pc)
Betelgeuse		0.45		
Deneb		1.25		
Rigel		0.18		
Arcturus		-0.05		
Sirius A		-1.44		
61 Cyg A		5.20		
BD +43 44B		8.09		

Questions:

1. On average, are these giants and supergiants closer to or farther from us than the main sequence stars? What is the reason?

2. Assume that you could travel in a space ship whose velocity is 1% of the speed of light ($c = 3 \times 10^8$ m/s). How many years would it take to travel to the nearest star in Table 2? Recall that the velocity is defined to be $v = d/t$, where d is the distance traveled and t is the time it takes to travel that distance, and $1 \text{ pc} = 3.08 \times 10^{16}$ m.

F. Optional Activity

Reinforce what you have learned by going to the Physical Scales link on the UCI Observatory web page, http://www.physics.uci.edu/~observat/Physical_Scales.html, which shows the relative sizes of some of the stars used in this exercise.