Why study 1/*f* noise in Coulomb glasses

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We briefly review 1/f noise in Coulomb glasses. We then argue that measurements of the second spectrum of the noise in Coulomb glasses could help to determine if electron fluctuations are correlated.

An electron glass is an amorphous insulator in which the electrons see a random potential and are localized at random positions. If the electrons interact with one another via long range Coulomb interactions, then we have a special case of an electron glass known as a Coulomb glass.

In a broad sense there have been two theoretical points of view of electron hopping in Coulomb glasses. The single particle approach models the electronic excitations as quasiparticles. Transport is described by variable range hopping which depends on the single particle density of states. The Coulomb interactions between localized electrons produce a Coulomb gap in the single particle density of states that is centered at the Fermi energy. The Coulomb gap makes the ground state stable with respect to single electron hops [1–3]. This approach has been very successful when compared to experiment. It finds that the conductivity goes as $\exp\left[-(T_o/T)^{1/2}\right]$ which has been seen experimentally. Tunneling experiments have also seen the Coulomb gap [4].

In the other approach the interactions between electrons produces many body excitations [5]. As a result transport is accomplished by correlated electron hopping, e.g., sequential hops as well as simultaneous or collective hopping. This approach is more difficult to deal with theoretically and is harder to verify experimentally. In this paper we argue that noise measurements could help to determine if correlated electron motion is involved in transport in Coulomb glasses.

To an experimentalist, noise is a nuisance at best and a serious problem hindering measurements at worst. However noise comes from the fluctuations of microscopic entities and it can act as a probe of what is happening physically at the microscopic scale. Let us set up our notation and define what we mean by noise. Let $\delta I(t)$ be a fluctuation in some quantity I at time t. If the processes producing the fluctuations are stationary in time, i.e., translationally invariant in time, then the autocorrelation function of the fluctuations $\langle \delta I(t_2)\, \delta I(t_1) \rangle$ will be a function $\psi_I(t_2 - t_1)$ of the time difference. In this case the Wiener–Khintchine theorem can be used to relate the noise spectral density $S_I(f)$ to the Fourier transform $\psi_I(f)$ of the autocorrelation function [6]: $S_I(f) = 2\psi_I(f)$ where f is the frequency. 1/f noise, which is ubiquitous and dominates at low frequencies, corresponds to $S_I(f) \sim 1/f$.

In Coulomb glasses electron hopping can occur on very long time scales which can produce low frequency noise. Experimental studies on doped silicon inversion layers have shown that low frequency $1/f$ noise is produced by hopping conduction [7]. Shklovskii had suggested that $1/f$ noise is caused by fluctuations in the number of electrons in an infinite percolating cluster [8]. These fluctuations are caused by the slow exchange of electrons between the percolating conducting cluster and

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small isolated donor clusters. Subsequent theoretical [9–13] and experimental [14–16] work found $1/f$ noise at low temperatures and low frequencies in Coulomb glasses.

However, it is still unclear what the temperature dependence of the noise amplitude is. Experiments on doped silicon (Si : B) at temperatures above 1.5 K with a fixed bias current of 4.5 mA found that the noise increased with increasing temperature [14]. On the other hand measurements on ion implanted silicon (Si : B : P) at low temperatures ($T < 0.5$ K) found that the noise decreased with increasing temperature [15]. Other measurements on doped silicon done between 2 K and 20 K find that the noise is independent of temperature at lower temperatures and then decreases with temperature at higher temperatures [16]. The crossover temperature depends on the current bias. This suggests that the temperature dependence is sensitive to the amount of current bias as well as the temperature [16].

Theoretically, the picture is equally murky. Starting from the model that noise comes from fluctuations δN_P in the number of electrons in the percolating cluster due to electron exchange with small isolated clusters [9], Shtengel and Yu [13] found that the noise increases with increasing temperature due to the increase in the thermally activated electron hopping. However Shklovskii [12] argued that the noise decreases with increasing temperature because the probability of finding a isolated cluster decreases with increasing temperature. (A cluster is isolated if it has no neighbors to which it can hop within a certain amount of time.) Another temperature dependent factor is the normalization of the noise amplitude by the size of the percolating cluster N_P:

$$\frac{S_I(f)}{I^2} = \frac{2\langle \delta N_P(t_2)\, \delta N_P(t_1)\rangle_f}{N_P^2} \,. \tag{1}$$

where I is the average DC current, and $\langle \ldots \rangle_f$ is the Fourier transform of the autocorrelation function. The size of the percolating cluster increases with increasing temperature. To understand this, note that two sites have a bond if electrons can hop between them faster than a certain rate which includes thermally activated hopping. Preliminary calculations [17] on noninteracting electrons with a flat density of states indicates that $N_P \sim T^3$. If this is included in the calculations of Shtengel and Yu [13] where the isolated clusters were single sites, the noise amplitude decreases with increasing temperature. In short, the temperature dependence of the noise is still an open question. Setting this issue of temperature dependence aside, measurements of the 1/f noise spectrum in Coulomb glasses are not really able to tell if the fluctuations were single particle or collective. A more relevant measurement would be the so–called second spectrum of the noise. To understand the second spectrum, consider the following. Suppose we take a time series on Monday and calculate the first spectrum, i.e., the noise spectrum $S(f)$. Then we do the same thing on Tuesday, Wednesday, etc. (In practice one would want to take sequential spectra as close together in time as possible.) So now we have a set of noise spectra $S(f)$ taken at different times t_2. The second spectrum is the power spectrum of the fluctuations of $S(f)$ with time, i.e., the Fourier transform of the autocorrelation function of the time series of $S(f)$ [18–20]. To calculate the second spectrum, we can divide a first spectrum into octaves. An octave is a range of frequencies from f_L to f_H where typically $f_H = 2f_L$. We can discretize the first spectrum by associating each octave with the total noise power in that octave. We do this for each data set. For each octave this gives us a set of numbers with one number from each data set labeled by t_2. Now we can calculate the fluctuations in the noise power in a given octave labeled by frequency f. Then we can calculate the autocorrelation function of these fluctuations, Fourier transform it and obtain the noise power $S_2(f_2,f)$ which is the second spectrum.

The second spectrum looks for correlations in the fluctuations that produce the first spectrum. So the second spectrum can tell us if the fluctuators are correlated or independent. If the second spectrum is white (independent of f_2) and equal to the square of the first spectrum [19–21], the fluctuators are not correlated. Such noise is called Gaussian. If the fluctuators are correlated, then the noise is non-Gaussian, and $S_2 \sim 1/f_2^{1-\beta}$ where the exponent $(1-\beta) > 0$. It is often useful to plot the second spectrum $S_2(f_2,f)$ as a function of the ratio f_2/f because over a given time interval the high frequency fluctuations get averaged more than the low frequency fluctuations.

The second spectrum has been used to differentiate between the hierarchical model and the droplet model of spin glasses [19, 20] because these two models assume different correlations between the fluctuators. These models were originally developed for short range spin glasses. We will take the liberty of adopting their qualitative features for the case of electron glasses. In the droplet model, clusters or droplets of rearranging electrons produce fluctuations [22–24]. There are fewer large droplets than small droplets, and the big droplets rearrange more slowly than the small droplets. So the large clusters contribute to the low frequency noise and the small fast clusters contribute to the high frequency noise. In the simplest case, the droplets are noninteracting and produce a white second spectrum. A more sophisticated version has interacting droplets. Large droplets are more likely to interact with other droplets than are small droplets. So non-Gaussian noise and the second spectrum will be larger at lower frequencies f_1.

In the hierarchical model [19, 20, 25–30] the states (or electron arrangements) of the electron glass lie at the endpoints of a bifurcating hierarchical tree (which looks more like the roots of a tree). The Hamming distance D between two states is the fraction of electrons that must rearrange to convert one state into another. It turns out that D corresponds to the minimum height to which one must go in the tree in order to get from one endpoint (state) to another. The farther apart 2 states are, the longer the time to go between them. The tree structure is self similar. As a result, the hierarchical model predicts that S_2 will be scale invariant and will only depend on f_2/f and not on the frequency f, while the interacting droplet model predicts that for fixed f_2/f, S_2 will be a decreasing function of f [19, 20]. Because of this, the second spectrum can differentiate between the droplet model and the hierarchical model. Measurements of resistance fluctuations in the spin glass CuMn find that its behavior is consistent with the hierarchical model [19, 20].

Measurements of the second spectrum of the noise in silicon inversion layers in MOSFETs in the vicinity of the metal–insulator transition have found that the exponent $(1-\beta)$ changes from being approximately zero in the metallic phase to a finite value of order unity [31] in the glassy or insulating phase. Similar results were found for doped silicon crystals Si:P(B) where $(1-\beta)$ was small in the metallic phase and became greater than 1 in the insulating phase [16]. The frequency dependence of S_2 indicates that the electronic fluctuations are correlated. Furthermore in the experiments on the Si MOSFETs, second spectra plots versus f_2/f for different values of f fall along one curve, implying that the hierarchical picture is better suited to describing the insulating phase [31].

Returning to our original question, we see that the measurements of the second spectra imply that the correlated electron fluctuations are important. We then need to how to reconcile this with the single quasiparticle picture that has been so successful. One possibility is that correlated electron motion produces conductance and tunneling characteristics similar to those predicted by the single quasiparticle picture. Another possibility is the dynamical current redistribution (DCR) model [32] in which non-Gaussian noise statistics can result from statistically independent fluctuators. As a simple example of this, consider two fluctuating resistors in parallel. The amplitude of the fluctuation of the total resistance due to one resistor depends on the state of the other resistor. The DCR model is most effective near the percolation threshold where a small number of large fluctuators produce frequency dependence in S_2 over a range of 1 or 2 decades. This bandwidth is determined by the frequency of the independent fluctuators. It could be that single quasiparticle hopping produces non-Gaussian noise according to the DCR model if one allows for a large number of quasiparticles with a broad distribution of hopping rates. A third possibility is that the observed non-Gaussian noise arises from nonequilibrium aging which would result in the first spectrum $S(f)$ deviating from $1/f$. For example, if the fluctuations had a drift that increased linearly in time, then $S(f)$ would go as $1/f^2$. Even though the Si MOSFET experiments do see a first spectrum that approaches $1/f^2$ with decreasing electron density, the experimentalists were careful to rule out aging [33].

In any event it is clear that more work needs to be done both theoretically and experimentally to understand the implications of the second spectrum. Noise measurements should be done on the insulating side further away from the metal-insulator transition. Theoretical modeling and simulations could shed some light on the implications of the second spectra measurements.

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