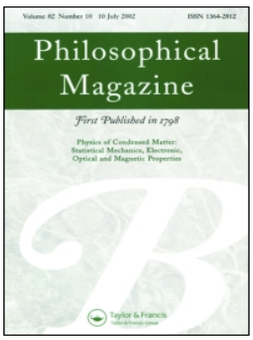

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First-order pre-melting transition of vortex lattices

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ABSTRACT

Vortex lattices in the high-temperature superconductors undergo a first-order phase transition that has thus far been regarded as melting from a solid to a liquid. We point out an alternative possibility of a two-step process in which there is a first-order transition from an ordinary vortex lattice to a soft vortex solid followed by another first-order melting transition from the soft vortex solid to a vortex liquid. We focus on the first step. This pre-melting transition is induced by vacancy and interstitial vortex lines. We obtain good agreement with the experimental transition temperature against field, latent heat, and magnetization jumps for $YBa_2Cu_3O_{7-\delta}$ and $Bi_2Sr_2CaCu_2O_8$.

Phase transitions involving vortex lattices in the high-temperature superconductors is an area of active study (Blatter *et al.* 1994, Brandt 1995). Below a critical value of the magnetic field, vortex lattices in $YBa_2Cu_3O_{7-\delta}$ (YBCO) (Safar *et al.* 1993, Liang *et al.* 1996, Schilling *et al.* 1996, Welp *et al.* 1996) and $Bi_2Sr_2CaCu_2O_8$ (BSCCO) (Zeldov *et al.* 1995, Fuchs *et al.* 1996, Keener *et al.* 1997) undergo a first-order phase transition. This conclusion comes from latent heat measurements (Schilling *et al.* 1996) as well as jumps in the resistivity (Safar *et al.* 1993, Fuchs *et, al.* 1996, Keener *et al.* 1997) and in the magnetization (Zeldov *et al.* 1995, Liang *et al.* 1996, Welp *et al.* 1996). It has generally been assumed that this is a melting transition from a vortex solid to a vortex liquid. In this paper we suggest the possibility that the melting transition actually occurs in two steps as the temperature increases; the first step is a first-order pre-melting transition from an ordinary vortex lattice to a soft solid with a small but finite shear modulus, and the second step is the first-order melting of the soft solid into a vortex liquid. In this paper we focus on the first step. We present an analytic theory of a first-order pre-melting transition in which the shear modulus jumps discontinuously. The transition is induced by interstitial and vacancy line defects in the vortex lattice, which soften the shear modulus c_{66}. We find good agreement with the experimental curve of transition temperature versus field, latent heat and magnetization jumps for YBCO and BSCCO. In the soft solid phase the superconducting phase coherence along the field is destroyed by the wandering of the defect lines which become entangled in the vortices of the soft solid lattice (Nelson 1991, Frey *et al.* 1994). However, since wandering is energetically costly, the superconducting correlation length along the c axis is long. Finally we speculate about the relation between our proposed two-step transition and the well known peak effect (Kwok *et al.* 1994, 1996).

Let us describe our scenario for pre-melting. Our approach follows that of Granato (1992) who showed that interstitial atoms soften the shear modulus of ordinary crystals and lead to a first-order transition. We start with a vortex lattice in a clean layered superconductor with a magnetic field H applied perpendicular to the layers along the c axis. We consider the vortices to be correlated stacks of pancake vortices. We shall assume that the transition is induced by topological defect lines, that is vacancies and interstitials. In a Delaunay triangulation (Preparata and Shamos 1985) a vacancy or an interstitial in a triangular lattice is topologically equivalent to a pair of bound dislocations (Ryu and Stroud 1996) as well as to a twisted bond defect (Kim *et al.* 1996). High-temperature decoration experiments (Kim *et al.* 1996) and Monte Carlo simulations (Ryu and Stroud 1996) have found such defects to be thermally excited. The introduction of these defects softens the elastic moduli. Since the energy to introduce interstitials and vacancies is proportional to the elastic moduli, softening makes it easier to introduce more defects. The softening also increases the vibrational entropy of the vortex lattice, which leads to a pre-melting transition. The transition is driven by the increased vibrational entropy of the vortex lines of the lattice and not by the entropy of the wandering of the defect lines. In fact, Frey *et al.* (1994) showed that a phase transition driven by the entropy of wandering flux lines occurs at a much higher magnetic field than observed experimentally. In the vicinity of the experimentally observed first-order phase transition, wandering in the transverse direction by more than a lattice spacing is energetically quite costly and therefore rare. (The energy scale is set by $\epsilon_0 s$ (Blatter *et al.* 1994, Brandt 1995). Here s is the interplane spacing and ϵ_0 is the energy per unit length of a vortex given by $\epsilon_0 = (\phi_0/4\pi\lambda_{ab})^2$ where ϕ_0 is the flux quantum and λ_{ab} is the penetration depth for currents in the a–b plane.)

Experimentally the resistivity at the transition jumps from zero to a finite value as the temperature increases. This is consistent with our model since the soft solid will have a finite resistivity due to the motion of interstitial (and vacancy) lines. The barrier for the motion of interstitials is very small (Frey *et al.* 1994) and is of order $10^{-3}E_0$ per unit length, where $E_0 = 2\epsilon_0$. The defect lines act like a liquid of lines existing in a soft solid host. Note that, if one tries to measure the shear modulus of such a system using resistivity measurements, only the defect lines would move relative to the pinned soft solid, and one would deduce that the shear modulus was zero (Pastoriza and Kes 1995, Kwok *et al.* 1996, Wu *et al.* 1997).

The first-order transition is nucleated in a small region by a local rearrangement of existing line segments. Slightly above the pre-melting temperature T_p a vortex line can distort and make an interstitial and a vacancy line segment that locally create a soft solid. This is the analogue of a liquid droplet which nucleates melting of a crystal. The role of the surface tension is played by the energy to connect the interstitial segment to the rest of the vortex line. This connection can be a Josephson vortex lying between planes or a series of small pancake vortex displacements spread over several layers. When the length ℓ of the interstitial and vacancy segments equals the critical length ℓ_c the energy gained by pre-melting equals the energy cost of the connections. When $\ell > \ell_c$, it is energetically favourable for the defect segments to grow to the length of the system.

To study pre-melting we assume that we have a vortex lattice with interstitial and vacancy lines extending the length of the lattice. Our goal is to find the free-energy density as a function of the concentration n of defect lines. The free energy density is

$f = f_0 + f_w + f + f_{wan}$, where f_0 is the free-energy density of a perfect lattice, f_w is the work needed to introduce a straight interstitial or vacancy line into the lattice, f_{vib} is the vibrational free-energy density of the system and f_{wan} is the free energy due to the wandering of the defect lines over distances large compared with the lattice spacing. We now examine these terms in detail.

f_0, the free-energy density of a perfect rigid flux lattice, is given by the London term (Frey *et al.* 1994, Tinkham 1996):

$$f_0 = \frac{B^2}{8\pi} + \frac{B\phi_0}{32\pi^2\lambda_{ab}^2} \ln\left(\frac{\eta\phi_0}{2\pi\xi_{ab}^2 B}\right), \quad \frac{\phi_0}{4\pi\lambda_{ab}^2} \ll B \ll H_{c2}, \tag{1}$$

where B is the spatially averaged magnetic induction, ξ_{ab} is the coherence length in the a–b plane and η is 0.130 519 for a hexagonal lattice and 0.133 311 for a square lattice (Frey *et al.* 1994). For B near H_{c2}, f_0 is given by the Abrikosov free energy (deGennes 1989):

$$f_0 = \frac{B^2}{8\pi} - \frac{(H_{c2} - B)^2}{8\pi[1 + (2\kappa^2 - 1)\beta_A]}, \tag{2}$$

where the Ginzburg–Landau parameter $\kappa = \lambda_{ab}/\xi_{ab}$, and the Abrikosov parameter β_A is 1.16 for a triangular lattice and 1.18 for a square lattice.

To calculate f_{vib}, we follow Bulaevskii *et al.* (1992). We denote the displacement of νth vortex pancake in the nth lane from its equilibrium position by $\mathbf{u}(n, \mathbf{r}_\nu)$ where $\mathbf{u} = (u_x, u_y)$ and the pancake position $\mathbf{r} = (r_x, r_y)$. The Fourier transform $\mathbf{u}(\mathbf{k}, q) = \sum_{n\nu} \mathbf{u}(\mathbf{n}, \mathbf{r}_\nu) \exp[i(\mathbf{k}\cdot\mathbf{r}_\nu + \mathbf{qn})]$. $\mathbf{k} = (k_x, k_y)$ and q is the wave-vector along the c axis. $f_{vib} = -(k_B T/V) \ln Z_{vib}$ where V is the volume and the vibrational partition function Z_{vib} is given by

$$\ln Z_{vib} = \sum_{\mathbf{k},\mathbf{q}>0,i} \ln\left[\int \frac{du_R(i\mathbf{k}q)\, du_I(i\mathbf{k}q)}{\pi\xi_{ab}^2} \exp\left(-\frac{\mathcal{F}_{el}}{k_B T}\right)\right] \tag{3}$$

where we have divided by the area $\pi\xi_{ab}^2$ of the normal core of a pancake (Bulaevskii *et al.* 1992). u_R and u_I are the real and imaginary parts of $\mathbf{u}(\mathbf{k}, q)$ and $i \in \{x, y\}$. The elastic free-energy functional associated with these distortions is

$$\mathcal{F}_{el} = \tfrac{1}{2} v_0 \sum_{\mathbf{k}q} \sum_{ij} u_i(q, \mathbf{k}) a_{ij} u_j^*(q, \mathbf{k}), \tag{4}$$

where i and $j \in \{x, y\}$, the volume per pancake vortex is $v_0 = s\phi_0/B$, and s is the interplane spacing. The $\mathbf{k}$ sum is over a circular Brillouin zone $K_0^2 = 4\pi B/\phi_0$. The matrix a_{ij} is given by $a_{ij} = c_B k_i k_j + (c_{66}k^2 + c_{44}Q^2)\delta_{ij}$ where c_B, c_{66} and c_{44} are the bulk, shear and tilt moduli respectively. $c_B = c_{11} - c_{66}$ for a hexagonal lattice. $Q^2 = 2[1 - \cos(qs)]/s^2$. Diagonalizing a_{ij} leads to two eigenvalues: $A_\ell(kq) = c_{11}k^2 + c_{44}Q^2$ and $A_t = c_{66}k^2 + c_{44}Q^2$, where A is the diagonal matrix, the subscript ℓ denotes longitudinal and the subscript t denotes transverse. Using this, we can integrate over u in equation (3); the remaining sums over $\mathbf{k}$ and q are done numerically. At low fields ($b = B/H_{c2} < 0.25$), the elastic moduli are given by

(Blatter *et al.* 1994, Brandt 1995)

$$
\begin{aligned}
c_{66} &= \frac{B\phi_0\zeta}{(8\pi\lambda_{ab})^2}, \\
c_{11} &= \frac{B^2[1+\lambda_c^2(k^2+Q^2)]}{4\pi[1+\lambda_{ab}^2(k^2+Q^2)](1+\lambda_c^2k^2+\lambda_{ab}^2Q^2)}, \\
c_{44} &= \frac{B^2}{4\pi(1+\lambda_c^2k^2+\lambda_{ab}^2Q^2)} + \frac{B\phi_0}{32\pi^2\lambda_c^2}\ln\left(\frac{\xi_{ab}^{-2}}{K_0^2+(Q/\gamma)^2+\lambda_c^{-2}}\right) \\
&\quad + \frac{B\phi_0}{32\pi^2\lambda_{ab}^4Q^2}\ln\left(1+\frac{Q^2}{K_0^2}\right),
\end{aligned}
\tag{5}
$$

where λ_c is the penetration depth for currents along the c axis, $\gamma = \lambda_c/\lambda_{ab}$ is the anisotropy, and $\zeta = 1$. At high fields ($b > 0.5$) (Blatter *et al.* 1994, Brandt 1995), c_{66} is altered by the factor $\zeta \approx (1-0.5\kappa^{-2})(1-b)^2(1-0.58b+0.29b^2)$ and the penetration depths in c_{11} and c_{44} are replaced by $\tilde{\lambda}^2 = \lambda^2/(1-b)$ where λ denotes either λ_{ab} or λ_c. In addition the last two terms of c_{44} are replaced by $B\phi_0/(16\pi^2\tilde{\lambda}_c^2)$. These replacements guarantee that the elastic moduli vanish at H_{c2}. For YBCO the temperature dependence of the penetration depths and coherence lengths are given by $\lambda(T) = \lambda(0)(1-T/T_c)^{-1/3}$ (Kamal *et al.* 1994) and $\xi_{ab}(T) = \xi_{ab}(0)(1-T/T_c)^{-1/2}$ respectively. For BSCCO whose pre-melting field is two orders of magnitude below H_{c2}, $\lambda^2(T) = \lambda^2(0)/[1-(T/T_c)^4]$ and $\xi_{ab}^2(T) = \xi_{ab}^2(0)/[1-(T/T_c)^4]$ (Tinkham 1996).

The free-energy density f_w due to the energy cost of adding a vacancy or interstitial vortex line is difficult to calculate accurately (Frey *et al.* 1994). However, we can write down a plausible form for f_w by noting that a straight line defect parallel to the c axis produces both shear and bulk (but not tilt) distortions of the vortex lattice. For example, if a defect at the origin produces a displacement $\mathbf{u}$ that satisfies $\nabla\cdot\mathbf{u} = v_0\delta(\mathbf{r})/s$, where $\delta(\mathbf{r})$ is a two-dimensional delta function, then $u_\alpha(\mathbf{k}) = ik_\alpha/k^2$ (Frey *et al.* 1994). Inserting this in equation (4), we find that $f_w = (c_{66}+\bar{c}_B)/2$ where $\bar{c}_B = \sum_{\mathbf{k}} c_B(q=0,\mathbf{k})$. Generalizing this to allow for a more complicated distortion and for a concentration n of line defects, we write (Granato 1992)

$$
f_w = \int_0^n dn\,(\alpha_1 c_{66} + \alpha_2 \bar{c}_B), \tag{6}
$$

where α_1 and α_2 are dimensionless constants. We expect the isotropic distortion to be small, that is $\alpha_2 \ll 1$, and the shear deformation to dominate, that is $\alpha_1 \gg \alpha_2$. Integrating over n allows the elastic moduli to depend on defect concentration. We shall assume that c_B is independent of n since we believe that the bulk modulus of the vortex solid is roughly the same as that of the soft solid phase. To find $c_{66}(n)$ (Granato, 1992), we use its definition $c_{66} = \partial^2 f/\partial\varepsilon^2$, where ε is the shear strain. Assuming that c_B has negligible shear strain dependence, we find that $c_{66}(n) = c_{66}(0) + \alpha_1\int_0^n (\partial^2 c_{66}(n)/\partial\varepsilon^2)\,dn$ or

$$
\frac{\partial c_{66}(n)}{\partial n} = \alpha_1 \frac{\partial^2 c_{66}(n)}{\partial\varepsilon^2}. \tag{7}
$$

If we shear the lattice in the a–b plane along rows separated by a distance d, the shear modulus must be periodic in displacements equal to the lattice constant a_0. We describe this with the simplest even periodic function: $c_{66}(u) = c_{66}(u=0)\cos(2\pi u/a_0) = c_{66}(\varepsilon=0)\cos(2\pi d\varepsilon/a_0)$ where $\varepsilon = u/d$. Then $\partial^2 c_{66}(n)/\partial\varepsilon^2 = -\beta c_{66}(n)$, where $\beta = 4\pi^2 d^2/a_0^2$. Combining this with equation (7), we obtain $c_{66}(n) = c_{66}(0)\exp(-\alpha_1\beta n)$. Thus the shear modulus softens exponentially with the defect concentration n. This softening lowers the energy cost to introduce further defects, and increases the vibrational free energy $f_{\rm vib}$ when $c_{66}(n)$ is used in a_{ij}. Substituting $c_{66}(n)$ into equation (6) for $f_{\rm w}$ yields

$$f_{\rm w} = \frac{c_{66}(n=0)}{\beta}[1-\exp(-\alpha_1\beta n)] + \alpha_2\bar{c}_{\rm B}n. \tag{8}$$

The last term that we need to consider is $f_{\rm wan}$, the free energy due to the wandering of the defect lines over distances large compared with the lattice spacing. We can estimate $f_{\rm wan}$ from the following expression (Frey *et al.* 1994):

$$f_{\rm wan} \approx -\frac{k_{\rm B}T}{\ell_z a_0^2}\ln(m_\ell), \tag{9}$$

where $m_\ell = 3$ for a triangular lattice (BSCCO) and $m_\ell = 4$ for a square lattice (YBCO). ℓ_z can be thought of as the distance along the z axis that it takes for the defect line to wander a transverse distance of one lattice spacing a_0. To go from one vacancy or interstitial site to the next, the defect line segment must jump over the barrier between the two positions. This gives ℓ_z a thermally activated form: $\ell_z \sim \ell_0\exp(-E/k_{\rm B}T)$, where $\ell_0 \approx a_0(\epsilon_1/\epsilon_{\rm B})^{1/2}$ and $E \approx a_0(\epsilon_1\epsilon_{\rm B})^{1/2}$. ϵ_1 is the line tension and is given by $\epsilon_1 \sim (\epsilon_0/\gamma^2)\ln(a_0/\xi_{ab})$. Numerical simulations (Frey *et al.* 1994) indicate that the barrier height $\epsilon_{\rm B}$ is small and we use $\epsilon_{\rm B} = 2.5\times10^{-3}\epsilon_0$. $f_{\rm wan}$ itself is quite small compared with the other terms because of the high energy cost of vortex displacements. For example, in the soft solid phase at the transition, $f_{\rm wan}$ is about two orders of magnitude smaller than $f_{\rm w}$ or $f_{\rm vib}$. Thus the transition is not driven by a proliferation of wandering defect lines because near the transition the high energy cost of vortex displacements is not sufficiently offset by the entropy of the meandering line (Frey *et al.* 1994).

Before we plot f against n, we note that the difference between B and H is negligible for YBCO but can be a significant fraction of the pre-melting field $H_{\rm p}$ for BSCCO. To find the value of B to use in the Helmholtz free-energy density f, we minimize the Gibbs free-energy density G, that is $\partial G/\partial B = 0$ where $G = f - \mathbf{B}\cdot\mathbf{H}/4\pi$. Because the concentration dependence of B is negligible, we find B for $n=0$ for each value of H and T. Typical plots of $\Delta f = f(n) - f(0) = f_{\rm w} + \Delta f_{\rm vib}$ against n are shown in the inset of figure 1. The double-well structure of Δf is characteristic of a first-order phase transition. The equilibrium transition occurs when both minima have the same value of Δf. We associate the minimum at $n=0$ with the vortex solid and the minimum at finite n with a soft vortex solid that has a small but finite shear modulus. The defect concentration at the transition is only a few per cent. At higher concentrations, Δf increases with increasing n because introducing defects costs compressional energy which is proportional to the bulk modulus. Thus defects do not proliferate. As an estimate of the softness at the transition, for $n = 5\%$, $c_{66}(n) \approx 0.2c_{66}(0)$ for BSCCO. The strain field $\varepsilon^{\rm d}_{\alpha\beta}(k)$ produced by the defect determines whether the shear modulus is zero in the high temperature phase (Marchetti and Nelson 1990). For dislocation

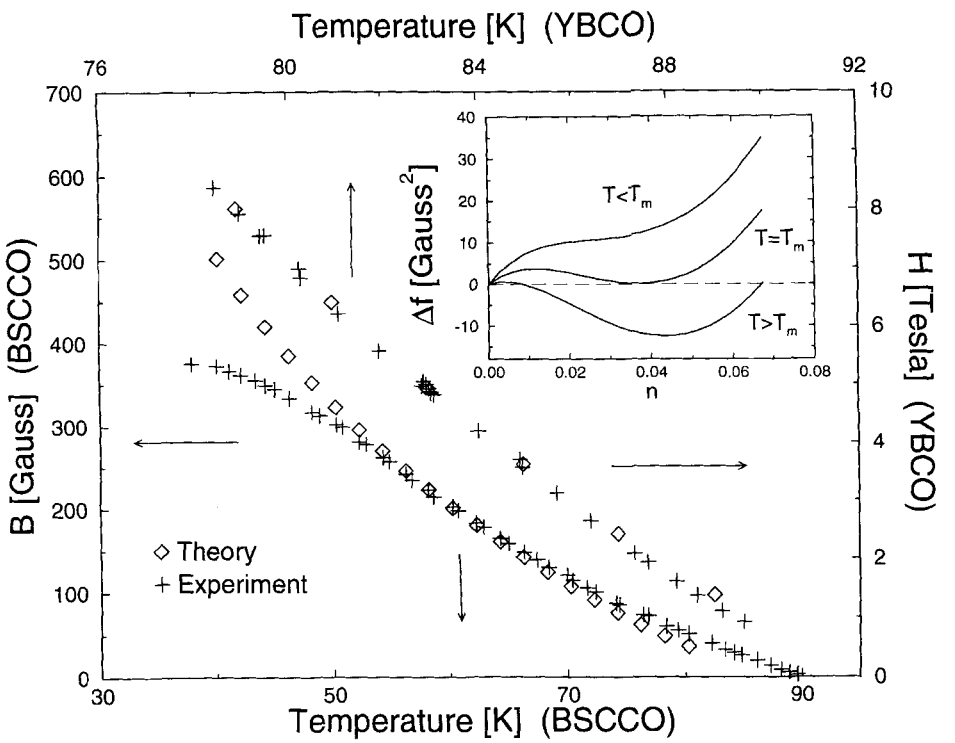

Figure 1. First-order phase transition curves of magnetic field against temperature for YBCO and BSCCO. The parameters used for YBCO are $\alpha_1 = 2.55$, $\alpha_2 = 0.01485$, $\phi = 44.1°$, $\lambda_{ab}(0) = 1186\,\text{Å}$ (Kamal *et al.* 1994), $s = 12\,\text{Å}$, $\xi_{ab}(0) = 15\,\text{Å}$, $\gamma = 5$ and $T_C = 92.74\,\text{K}$. The parameters used for BSCCO are $\alpha_1 = 1.0$, $\alpha_2 = 0.00705$, $\phi = 60°$, $\lambda_{ab}(0) = 2000\,\text{Å}$, $s = 14\,\text{Å}$, $\xi_{ab}(0) = 30\,\text{Å}$, $\gamma = 200$ and $T_C = 90\,\text{K}$. For BSCCO we use the low-field form of the elastic moduli from equation (5) and for YBCO we use the high-field form. For f_0 we use equation (1) for BSCCO and equation (2) for YBCO. (For BSCCO we plot B against T because that is what Zeldov *et al.* (1995) measured). The experimental points for YBCO are from (Schilling *et al.* 1996) and those for BSCCO from Zeldov *et al.* (1995). The inset shows a typical Δf against n plot.

loops, $\varepsilon^{\text{d}}_{\alpha\beta}(k)$ is singular as $k \to 0$, and the shear modulus is zero at $k = 0$ (Marchetti and Nelson 1990). For vacancy and interstitial lines, $\varepsilon^{\text{d}}_{\alpha\beta}(k)$ is finite, and hence the shear modulus is non-zero.

In figure 1 we fit the experimental first-order transition curves in the H–T plane using two adjustable parameters: α_1 and α_2. As expected, $\alpha_1 \gg \alpha_2$ and $\alpha_2 \ll 1$ (see figure 1). The geometrical quantity β can have several values for a given lattice structure, depending on which planes are sheared. We choose $\beta = \pi^2 \tan^2 \phi$ where ϕ is the angle between primitive vectors. Decoration experiments on BSCCO indicate a triangular lattice (Kim *et al.* 1996); so we use $\phi = 60°$. For YBCO we choose $\phi = 44.1°$ which is very close to a square lattice which has $\phi = 45°$. Won and Maki (1996) have argued that the d-wave symmetry of the order parameter yields a square vortex lattice tilted by 45° from the a axis. In experiments (Yethiraj *et al.* 1993, Keimer *et al.* 1994, Maggio-Aprile *et al.* 1995 on YBCO ϕ was found to range from 36° to 45°, (our ϕ equals half the angle cited in their experiments).

We can calculate the jump ΔM in magnetization at the transition using $\Delta M = -\partial(\Delta G)/\partial H|_{T=T_p}$. The jump Δs in entropy is given by $\Delta s = -v_0\,\partial(\Delta G)/\partial T|_{H=H_p}$, where Δs is the entropy change per vortex per layer. The results are shown in figure 2. We have checked that our results satisfy the Clausius–Clapeyron equation $\Delta s = -(v_0\,\Delta B/4\pi)\,\mathrm{d}H_p/\mathrm{d}T$. We obtain good agreement with experiment well below T_c. Near T_c it is thought that the entropy jump is enhanced by microscopic degrees of freedom (Dodgson *et al.* 1997, Rae *et al.* 1997), which are not included in our model.

We can compare our results with the Lindemann criterion by calculating the mean square displacement $\langle|u|^2\rangle$ at the transition using equation (3): $\langle|u|^2\rangle = -(2k_BT/v_0)\sum_{\alpha\mathbf{k}q}\partial(\ln Z_{vib})/\partial A(\alpha\mathbf{k}q)$, where A is the diagonal matrix similar to a_{ij} and α labels the two eigenvalues. Defining the Lindemann ratio c_L by $c_L^2 = \langle|u|^2\rangle/a_0^2$, we find that $c_L \approx 0.25$ for YBCO at $H_p = 5\,\mathrm{T}$ and that $c_L \approx 0.11$ for BSCCO at $H_p = 200\,\mathrm{G}$. Here we have used the same values of the parameters that were used to fit the phase transition curves in figure 1. These values of c_L are consistent with previous values (Houghton *et al.* 1989, Blatter *et al.* 1994, Brandt 1995).

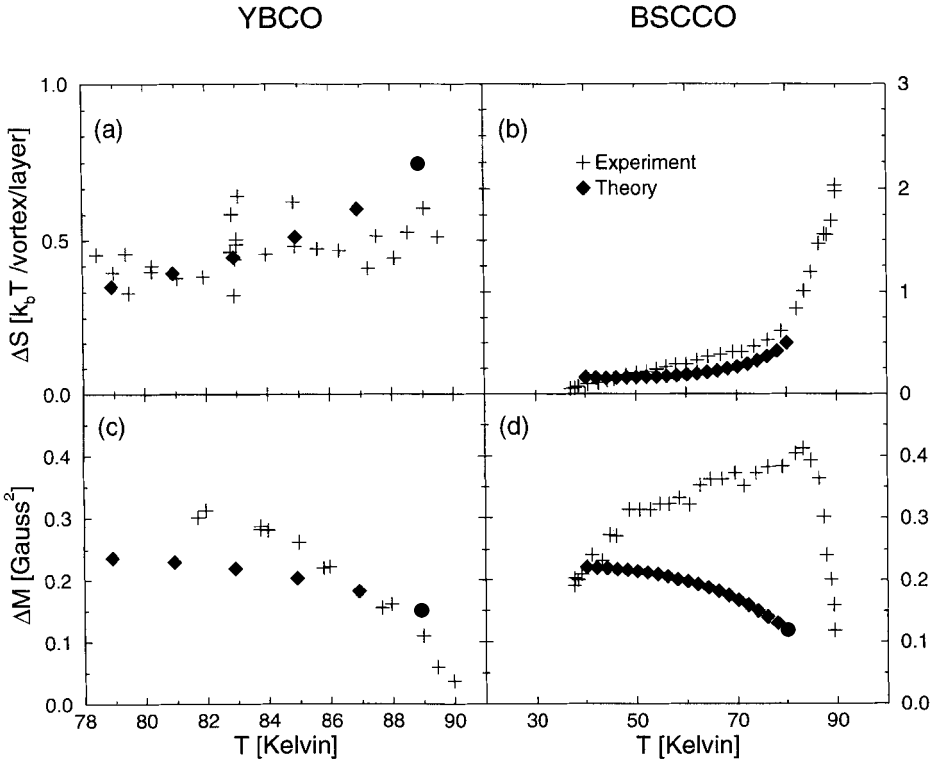

Figure 2. (*a*), (*b*) Entropy jumps Δs per vortex per layer against T_p at the transition for YBCO and BSCCO. The experimental points for YBCO are from Schilling *et al.* (1996) and those for BSCCO are from Zeldov *et al.* (1995). (*c*), (*d*) Magnetization jump ΔM against T_p at the first-order phase transition for YBCO and BSCCO. The experimental points for YBCO are from Welp *et al.* (1996) and those for BSCCO are from Zeldov *et al.* (1995). For the theoretical points the values of the parameters are the same as in figure 1 for all the curves.

Experiments have found little, if any, hysteresis (Safar *et al.* 1993, Zeldov *et al.* 1995, Keener *et al.* 1997). This is consistent with our calculations. We can bound the hysteresis by noting the range of temperatures between which the soft solid minimum appears and the solid minimum disappears. Typical values for the width of this temperature range are 300 mK for YBCO at $H = 5\,\mathrm{T}$ and 1.3 K for BSCCO at $H = 200\,\mathrm{G}$. Another measure of the hysteresis can be found in the plots of Δf against n. The barrier height V_B between the minima is low ($V_B v_0 \approx 30\,\mathrm{mK}$), which is consistent with minimal hysteresis.

In going from the normal metallic phase to the vortex solid, two symmetries are broken: translational invariance and gauge symmetry, which produce the superconducting phase coherence along the magnetic field. In the soft solid phase, longitudinal superconductivity is destroyed by the wandering of the defect lines which become entangled with the soft solid vortices. (A vortex solid with entangled vortex lines has been termed a supersolid (Nelson 1991, Frey *et al.* 1994). Even though line wandering is energetically costly and therefore rare, it does occur. As a result, the correlation length along the c axis will be quite long. This is consistent with measurements in YBCO of the c axis resistivity for which it is found that there is loss of vortex velocity correlations for samples thicker than 100 μm (D. Lopez 1996, private communication, Lopez *et al.* 1996a,b). For samples thicker than the longitudinal correlation length, the loss of longitudinal superconductivity coincides with the pre-melting transition (Chen and Teitel, 1995). This agrees with experiments which indicate that the loss of superconducting phase coherence along the c axis coincides with the first-order transition (D. Lopez 1966, private communication, Lopez *et al.* 1996a,b).

Because the soft solid is a lattice with a few per cent of defect lines, the Fourier transform of the density–density correlation function should exhibit Bragg peaks. Relative to the ordinary vortex solid, the intensity of these peaks would be slightly diminished by the defect lines; so it would be difficult to detect the transition via neutron scattering. In going from the soft solid to the normal metallic state, translational invariance is regained by a first-order melting transition. Thus there are two transitions: the pre-melting transition and the melting of the soft solid. Melting is observable in small-angle neutron scattering experiments (Cubitt *et al.* 1993) in which a rapid decrease in the intensity of the Bragg spots is seen. The region of the phase diagram where the soft solid exists may be quite narrow, of the order a degree Celsius or less in temperature (Kwok *et al.* 1994). There is the intriguing possibility that our scenario of two transitions may be related to the peak effect in which the critical current as a function of temperature or field is observed to increase sharply below the melting transition (Kwok *et al.* 1994). This increase is believed to result from the enhanced pinning of flux lines due to the softening of the shear modulus c_{66} (Larkin *et al.* 1995).

To summarize we have discussed the possibility that a vortex lattice melts in two stages. First it undergoes a first-order pre-melting transition into a soft solid followed by another first-order phase transition into a liquid. The pre-melting transition is induced by vacancy and interstitial vortex lines that soften the shear modulus and enhance the vibrational entropy. The entanglement of these defect lines with the vortex lines of the soft solid leads to the loss of longitudinal superconducting phase coherence. However, the correlation length corresponding to longitudinal superconductivity is quite long because line wandering is energetically costly and therefore rare. We obtain good agreement with the experimentally measured curve of

transition temperature against field, latent heat, and jumps in magnetization for BSCCO and YBCO. The Lindemann ratio c_L is about 11% for BSCCO and about 25% for YBCO. The hysteresis is small.

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