Microscopic Model of Critical Current Noise in Josephson Junctions

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We present a simple microscopic model to show how fluctuating two-level systems in a Josephson junction tunnel barrier of thickness L can modify the potential energy of the barrier and produce critical current noise spectra. We find low frequency $1/f$ noise that goes as L^5. Our values are in good agreement with recent experimental measurements of critical current noise in Al/AlO$_x$/Al Josephson junctions. We also investigate the sensitivity of the noise on the nonuniformity of the tunnel barrier.

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Remarkable progress has been achieved in making high-quality Josephson junction qubits [1–6], though sources of noise and decoherence continue to be problematic. Recent experiments [7,8] indicate that a dominant source of decoherence is two-level systems (TLS) in the insulating barrier of the tunnel junction as well as in the dielectric material used to fabricate the circuit. These TLS fluctuators produce low frequency $1/f$ critical current noise S_{I_c} [9–13]. However, a simple microscopic model showing this is still missing. In a common scenario for S_{I_c} [12], defects in the oxide tunnel barrier affect nm sized conducting channels. But how can defects a few angstroms in width have a noticeable effect on a superconducting wave function with a micron sized coherence length that is orders of magnitude larger than the perturbing defect? The answer is that the tunneling current is exponentially sensitive to perturbations of the tunnel barrier. We have confirmed this with a microscopic calculation of S_{I_c} due to fluctuating TLS in the barrier and obtain good quantitative agreement with experiment.

Previous theoretical work postulated that the qubit was coupled to fluctuating defects by putting a coupling term into the Hamiltonian [7,14–18], but no one has shown how this coupling arises microscopically. In this Letter we calculate the $1/f$ critical current noise S_{I_c} due to thermally fluctuating TLS that have electric dipole moments. We assume that the current I through the Josephson junction is given by $I = I_c \sin\delta$, where I_c is the critical current and δ is the phase difference between the superconductors. In a qubit the junction is small ($\lambda_J \gg \sqrt{A}$, where λ_J is the Josephson penetration depth and A is the area of the junction) so that the phase difference is uniform in the plane of the junction. We start by calculating how a dipole modifies the junction's potential barrier $U(\mathbf{r})$. We then use a WKB formalism to compute the tunneling matrix element $\mathcal{T}_{\rm LR} \sim \exp(-\sqrt{U})$ between the left (L) and right (R) electrodes. The critical current I_c is proportional to $\langle|\mathcal{T}_{\rm LR}|^2\rangle$ averaged over the junction [19]. We consider elastic electron tunneling where different orientations of the dipole correspond to different values of $\mathcal{T}_{\rm LR}$ and hence, I_c. We can obtain S_{I_c} since each fluctuating dipole behaves as a random telegraph variable that has a Lorentzian noise spectrum. By averaging over the standard TLS distribution, and each dipole's orientation and position along the z axis, we obtain S_{I_c}. At low frequencies we find $1/f$ behavior for S_{I_c}, and our values are in good agreement with the corresponding experimental values [13,20,21]. Our model predicts that the noise is very sensitive to the tunnel barrier thickness L and that $S_{I_c} \sim L^5$, implying that the noise can be greatly reduced by decreasing L.

In our model the TLS sits in the insulating tunnel barrier with an electric dipole moment $\mathbf{p}$ consisting of a pair of opposite charges separated by a distance d. The superconducting electrodes are located at $z = 0$ and $z = L$ and kept at the same potential. The angle between $\mathbf{p}$ and the tunneling direction (z axis) is θ_0. The TLS has a double-well potential with a Hamiltonian [22] $H_0 = \frac{1}{2}\times(\Delta\sigma_z + \Delta_0\sigma_x)$, where Δ_0 is a tunneling matrix element, Δ is the energy difference between the right and left wells, and $\sigma_{x,z}$ are the Pauli spin matrices. The energy eigenvalues are $\pm E/2$, where $E = \sqrt{\Delta^2 + \Delta_0^2}$. The TLS couples to the strain field. So an excited two-level system can decay to the ground state by emitting a phonon. The longitudinal relaxation rate is given by [22] $T_1^{-1} = aE\Delta_0^2 \coth(E/2k_BT)$, where the prefactor a is a material dependent constant. The distribution of TLS parameters can be expressed in terms of E and T_1: $P(E, T_1) = P_0/(2T_1\sqrt{1 - \tau_{\min}(E)/T_1})$ [22,23], where P_0 is the TLS density of states. The minimum relaxation time $\tau_{\min}(E) = T_1(E = \Delta_0)$ corresponds to a symmetric double-well potential.

We start by calculating the junction barrier potential energy U when it is distorted by the presence of TLS with electric dipole moments [24,25]. We use cylindrical coordinates (ρ, ϕ, z) with $\hat{\mathbf{z}}$ normal to the plane of the junction. For a square barrier, $U(\rho, \phi, z) = U_0 - eV_{\rm dip}(\rho, \phi, z)$, where $V_{\rm dip}(\rho, \phi, z)$ is the contribution of the electric dipole to the barrier potential and U_0 is the height of the unperturbed uniform square barrier. We use Green's functions [26] to calculate $V_{\rm dip}(\mathbf{r}) = \int G(\mathbf{r}, \mathbf{r}')\alpha(\mathbf{r}')d^3\mathbf{r}'$, where the charge density $\alpha(\mathbf{r}) = \sum_{i=1,2} q_i\delta^3(\mathbf{r} - \mathbf{r}_i)$ with the positive charge $q_1 = |\mathbf{p}|/d$ at $\mathbf{r}_1 = z_0\hat{\mathbf{z}}$, and the negative charge $q_2 = -q_1$ at

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$\mathbf{r}_2 = d(\sin\theta_0)\hat{\mathbf{x}} + (z_0 + d\cos\theta_0)\hat{\mathbf{z}}$. The Green's function is [26] $G(\mathbf{r}, \mathbf{r}') = 1/(\pi\epsilon L) \times \sum_{n=1}^{\infty}\sum_{m=-\infty}^{\infty} \sin(n\pi z/L) \times \sin(n\pi z'/L)\exp[im(\phi - \phi')]I_m(n\pi\rho_</L)K_m(n\pi\rho_>/L)$, where $\rho_< = \min(\rho, \rho')$, $\rho_> = \max(\rho, \rho')$, I_m, and K_m are the m-order modified Bessel functions, and $\epsilon = \epsilon_0\epsilon_r$ is the permittivity of the dielectric. The dipole potential is

$$V_{\rm dip}(\rho, \phi, z) = \frac{p/d}{\pi\epsilon L}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi z}{L}\right)f_n(\rho, \phi; \theta_0, z_0), \quad (1)$$

where $f_n(\rho, \phi; \theta_0, z_0) = K_0(n\pi\rho/L)\sin(n\pi z_0/L) - K_0(n\pi|\boldsymbol{\rho} - \boldsymbol{\rho_0}|/L)\sin[n\pi(z_0 + d\cos\theta_0)/L]$. Here $|\boldsymbol{\rho} - \boldsymbol{\rho_0}| = (\rho^2 + d^2\sin^2\theta_0 - 2\rho d\sin\theta_0\cos\phi)^{1/2}$. In Fig. 1, the barrier potential U is plotted versus $\tilde{z} = z/L$ for different values of $\tilde{\rho} = \rho/L$. The distortion of U decreases as the radial distance $\tilde{\rho}$ increases, eventually becoming negligible when $\tilde{\rho} \simeq 1$. When the dipole flips 180°, U switches from $(U_0 - eV_{\rm dip})$ to $(U_0 + eV_{\rm dip})$.

Next we follow [24,25] and use the WKB approximation to calculate $\mathcal{T}_{\rm LR}^{\pm}$ corresponding to the two orientations of the dipole:

$$|\mathcal{T}_{\rm LR}^{\pm}(\rho, \phi)|^2 \sim \exp\left\{-2\int_0^L dz\sqrt{\frac{2m}{\hbar^2}[\Phi \mp eV_{\rm dip}(\rho, \phi, z)]}\right\}, \quad (2)$$

where $\Phi = U_0 - \varepsilon_{k_z}$ and ε_{k_z} is the energy of the electron incident along the z axis. We assume that $eV_{\rm dip} \ll \Phi$ which allows us to expand the square root in the exponent in powers of $eV_{\rm dip}/\Phi$ and to use the WKB approximation which is valid for a potential that varies slowly along the tunneling direction. To a good approximation Φ is a constant representing the maximum barrier height [25]. We average $|\mathcal{T}_{\rm LR}^{\pm}|^2$ over the junction area A, and we find to lowest order in $(eV_{\rm dip}/\Phi)$:

$$\frac{\langle|\mathcal{T}_{\rm LR}^{\pm}|^2\rangle}{|\mathcal{T}_{\rm LR}^{0}|^2} = \frac{1}{A}\int_0^{2\pi}d\phi\int_{\rho_{\rm min}}^{\rho_{\rm max}}d\rho\,\rho\exp\left\{\pm e\sqrt{\frac{2m}{\hbar^2\Phi}}\mathcal{F}(\rho, \phi)\right\}, \quad (3)$$

where $|\mathcal{T}_{\rm LR}^{0}|^2 \sim \exp(-2L\sqrt{2m\Phi/\hbar^2})$ is the square of the tunneling matrix element in the absence of impurities and $\mathcal{F}(\rho, \phi) = \int_0^L dz V_{\rm dip}(\rho, \phi, z)$. $\mathcal{F}(\rho, \phi)$ will be small due to the oscillation in $V_{\rm dip}$, but it will be nonzero if there is asymmetry in the dipole's position along the z axis or in its orientation θ_0. In the exponent $\mathcal{F}(\rho, \phi)$ can still have a noticeable effect on the noise. Integrating over z yields

$$\frac{\langle|\mathcal{T}_{\rm LR}^{\pm}|^2\rangle}{|\mathcal{T}_{\rm LR}^{0}|^2} = \frac{1}{A}\int_0^{2\pi}d\phi\int_{\rho_{\rm min}}^{\rho_{\rm max}}d\rho\,\rho\exp[\pm\beta W(\rho, \phi; \theta_0, z_0)], \quad (4)$$

where the constant $\beta = [(p/d)/(\pi^2\epsilon)]e\sqrt{2m/(\hbar^2\Phi)}$ and $W(\rho, \phi; \theta_0, z_0) = \sum_{n=1}^{\infty}[1 - (-1)^n]f_n(\rho, \phi; \theta_0, z_0)/n$.

The critical current I_c is proportional to $\langle|\mathcal{T}_{\rm LR}|^2\rangle$ [19]. Hence $I_c^{\pm} = I_c\langle|\mathcal{T}_{\rm LR}^{\pm}|^2\rangle/|\mathcal{T}_{\rm LR}^{0}|^2$, where I_c is the critical current in the absence of any dipoles. The critical current fluctuations, defined as $\Delta I_c(\theta_0, z_0) = I_c^{+}(\theta_0, z_0) - I_c^{-}(\theta_0, z_0)$, are given by

$$\begin{aligned}\frac{\Delta I_c}{I_c} &= \frac{L^2}{A}\int_0^{2\pi}d\phi\int_{\tilde{\rho}_{\rm min}}^{\tilde{\rho}_{\rm max}}d\tilde{\rho}\,\tilde{\rho}\{\exp[\beta W(\tilde{\rho}, \phi; \theta_0, \tilde{z}_0)] \\ &\quad - \exp[-\beta W(\tilde{\rho}, \phi; \theta_0, \tilde{z}_0)]\} \\ &\equiv \frac{L^2\Delta g(\theta_0, \tilde{z}_0)}{A},\end{aligned} \quad (5)$$

where the L^2 factor comes from introducing $\tilde{\rho} = \rho/L$.

To find the critical current noise power S_{I_c}, we assume that each dipole produces a Lorentzian spectrum [27]:

$$\frac{S_{I_c}^{(i)}(f)}{I_c^2} = \left\langle\left(\frac{\Delta I_c}{I_c}\right)^2\right\rangle\frac{4P_+P_-T_1}{1 + \omega^2T_1^2}, \quad (6)$$

where i denotes the ith dipole, $P_{\pm} = \exp(\mp E/2k_BT)/Z$ is the Boltzmann probability of being in the upper (lower) state of the TLS, the partition function $Z = 2\cosh(E/2k_BT)$, and T is temperature. We average over the distribution of TLS to find the low frequency ($\omega\tau_{\rm min} \ll 1$) $1/f$ noise power:

$$\begin{aligned}\frac{S_{I_c}(f)}{I_c^2} &\simeq \int_0^{E_M}dE\int_0^{\infty}dT_1\frac{P_0V}{2T_1}\frac{\langle(\Delta I_c/I_c)^2\rangle}{\cosh^2(\frac{E}{2k_BT})}\frac{T_1}{1 + \omega^2T_1^2} \\ &\simeq \frac{P_0k_BT}{4f}\frac{L^5}{A}\langle\Delta g^2\rangle,\end{aligned} \quad (7)$$

where $V = AL$, $E_M \gg k_BT$, and we can neglect the factor of $1/\sqrt{1 - \tau_{\rm min}(E)/T_1}$ because $\tau_{\rm min}/T_1 \ll 1$ in the region $[E/(2k_BT) > 0.1, \omega T_1 > 0.1]$ that dominates the integral. $\langle\Delta g^2\rangle$ contains the average over each dipole's orientation and position along the z axis: $\langle\Delta g^2\rangle = 1/[2(\tilde{z}_{0_M} - \tilde{z}_{0_m})] \times \int_0^{\pi}d\theta_0\sin\theta_0\int_{\tilde{z}_{0_m}}^{\tilde{z}_{0_M}}d\tilde{z}_0\Delta g^2(\theta_0, \tilde{z}_0)$, where $\tilde{z}_0$ lies between $\tilde{z}_{0_m} = \tilde{d}$ and $\tilde{z}_{0_M} = 1 - \tilde{d}$ to ensure that the dipole lies entirely in the dielectric region. Note that the critical current noise power scales as L^5/A. Although the A^{-1} dependence is well known experimentally [9–12], it would be interesting to check experimentally the L^5 scaling of the noise predicted by our model.

To estimate $\langle\Delta g^2\rangle$, we evaluated the integrals numerically with $\tilde{\rho}_{\rm min} = 0.1$, which is comparable to an atomic radius. As Fig. 1 shows, the lower cutoff $\tilde{\rho}_{\rm min}$ is needed because the effect of the dipole on the tunnel barrier is no longer weak for $\tilde{\rho} \lesssim 0.1$. Figure 2 shows Δg^2 averaged over θ_0 versus position for various values of $\tilde{\rho}_{\rm max}$. The results overlap for $\tilde{\rho}_{\rm max} > 2$. Note that the largest contribution to S_{I_c} comes from dipoles near the electrodes.

Our numerical estimates follow. We obtain $\langle\Delta g^2\rangle = 1.5782 \times 10^{-2}$ for $p = 3.7$ D (which corresponds to the dipole moment of an OH^- impurity [28]), $d = 0.13$ nm, $\epsilon_r = 10$, $\tilde{\rho}_{\rm max} = 4.0$, and $U_0 = 1$ eV [29]. For a Josephson junction with $A = 1\ \mu{\rm m}^2$, $L = 1.5$ nm, $P_0 = 10^{45}\ ({\rm J\,m}^3)^{-1}$ [22], $f = 1$ Hz, and $T = 100$ mK, we obtain a noise power of $S_{I_c}/I_c^2 = 4.13 \times 10^{-14}\ {\rm Hz}^{-1}$, in

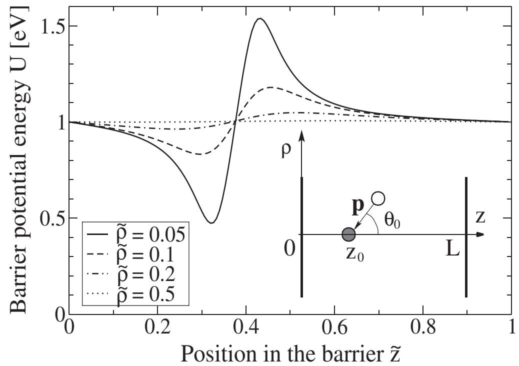

FIG. 1. The potential energy barrier U vs $\tilde{z}$ for different values of $\tilde{\rho}$. The dipole lies along the z axis ($\tilde{z}_0 = 0.334$, $\theta_0 = 0$, and $\tilde{d} = 0.0867$). We have used $p = |\mathbf{p}| = 3.7$ D and $U_0 = 1$ eV [29]. The inset shows the geometry and the parameters of the model.

good agreement with the recent tunnel junction resistance measurements by the Delft group [13] who deduced a value of 2.0×10^{-14} Hz^{-1} for the normalized critical current noise power using similar junction parameters. They also found that the noise varied linearly with temperature in agreement with Eq. (7). Other measurements in large superconducting junctions find larger S_{I_c} than those of the Delft group [9–11]. Using Eq. (7), we find the critical current noise in larger junctions at 4.2 K to be roughly 100 times lower than the experimental values in [10,11]. The noise level difference between the Delft group [13] and those working with larger junctions may be due to differences in fabrication techniques. For example, our calculation indicates that if the effective thickness L of the tunnel barrier in the Nb-AlO$_x$-Nb junctions [9,10] or Nb-NbO$_x$-PbIn junctions [9,11] were larger by only a factor of 2 with respect to our $L = 1.5$ nm, this "discrepancy" would be essentially removed. A T^2 dependence was observed in Ref. [11] which may imply an additional mechanism for the noise [18].

The dipoles that produce critical current noise are also responsible for the fluctuations in the induced charges on the superconducting electrodes that produce charge noise.

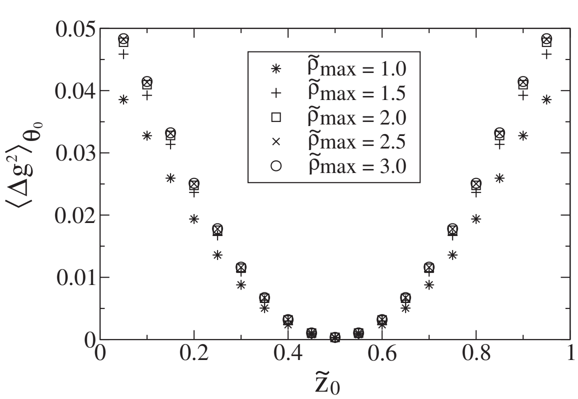

FIG. 2. Δg^2 averaged over the dipole orientation angle θ_0 vs the dipole position $\tilde{z}_0$.

The dipole making 180° flips induces charge fluctuations ($\Delta Q = |2p \cos\theta_0/L|$) on the electrodes. The time series of the charge is a two-state random telegraph signal with a transition rate T_1^{-1} resulting in a Lorentzian charge noise spectrum [27]. The charge noise results from averaging over the TLS density of states [17,27]. At low frequencies one obtains

$$\frac{S_Q(f)}{e^2} = \frac{1}{3} V P_0 k_B T \left(\frac{p}{eL}\right)^2 \frac{1}{f}. \tag{8}$$

We estimate $S_Q/e^2 = 1.83 \times 10^{-3}$ Hz^{-1}, in good agreement with the experimental values ranging from 10^{-4} Hz^{-1} [20] to approximately 4×10^{-4} Hz^{-1} [21]. Using Eqs. (8) and (7), we obtain the ratio between S_Q/e^2 and S_{I_c}/I_c^2 at low frequencies:

$$\frac{S_Q/e^2}{S_{I_c}/I_c^2} = B\left(\frac{pA}{eL^3}\right)^2, \tag{9}$$

where $B = 4/(3\langle \Delta g^2 \rangle)$. For the physical parameters listed above, we obtain $B \simeq 84.5$ and $(S_Q/e^2)/(S_{I_c}/I_c^2) \simeq 4.4 \times 10^{10}$. This is consistent with the value of $\simeq 2 \times 10^{10}$ deduced from experimental measurements of S_Q [21] and S_{I_c} [13], though the S_Q and S_{I_c} measurements were not made on the same samples.

Our calculation of the critical current noise assumed that in the absence of the dipoles, the tunnel barrier is uniform. However, local fluctuations in oxide thickness or barrier height make the tunnel barrier nonuniform and may result in "pinholes" in the barrier [29]. We can investigate the sensitivity of the critical current noise to the nonuniformity of the tunnel barrier by placing the fluctuating dipole in a cylindrical island of radius $\rho_{\rm in}$ with the axis of the cylinder along $\hat{z}$. We set the tunneling barrier of the island to be different from that outside the island: $U(\rho) = U_{\rm in}$ for $0 \leq \rho < \rho_{\rm in}$ and $U(\rho) = U_{\rm out}$ for $\rho_{\rm in} \leq \rho \leq \rho_{\rm max}$. We can model a pinhole by having $U_{\rm in} < U_{\rm out}$. The results in Fig. 3 show that the noise is enhanced as the radius $\tilde{\rho}_{\rm in}$ of the inner cylinder grows because the dipole distortions of the barrier potential are a larger fraction of $U_{\rm in}$ than of $U_{\rm out}$. Unlike before, we do not expand the argument of the square root in Eq. (2). When $\tilde{\rho}_{\rm in} \to \tilde{\rho}_{\rm max}$, we obtain our previous result for a dipole in a square 1 eV barrier.

In the other case where $U_{\rm in} > U_{\rm out}$, and the dipole is on the inner island, then the noise decreases as $\tilde{\rho}_{\rm in}$ increases. This is shown in Fig. 3 with $U_{\rm in} = 10$ eV and $U_{\rm out} = 1$ eV. The large barrier on the inner island reduces the amount of tunneling current through the island and hence limits the noise due to the fluctuating two-level system. In the limit ($\tilde{\rho}_{\rm in} \to \tilde{\rho}_{\rm min}$) that the inner island disappears, we recover our previous result for a dipole in a square barrier of 1 eV. This investigation shows that the noise power can be affected by the nonuniformity of the barrier, and could explain why several experimental groups have measured different values of S_{I_c}/I_c^2 [11–13].

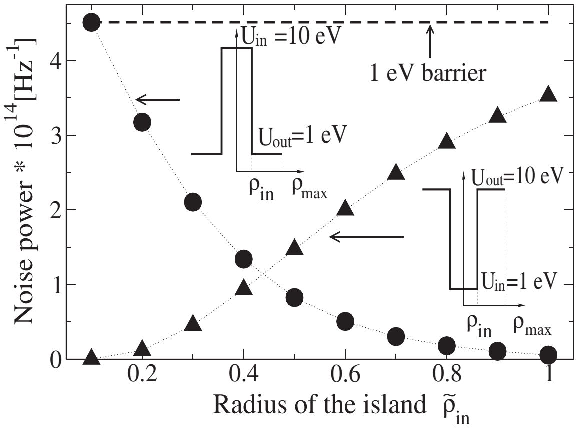

FIG. 3. Normalized critical current noise for a nonuniform tunnel barrier with two islands ($U_{in} = 10$ eV and $U_{out} = 1$ eV and vice versa) vs the radius of the inner island $\tilde{\rho}_{in}$. The dashed line corresponds to noise from a dipole in a square barrier with a height of 1 eV. The critical current fluctuations in all cases are normalized by the critical current through a uniform square barrier of 1 eV with no dipole present.

To conclude, we have used a microscopic model of fluctuating dipoles in the tunnel barrier to calculate the critical current noise in Josephson junctions. By considering a simple tunnel barrier potential with a fluctuating electric dipole associated with the presence of two-level systems in the barrier, we estimated the critical current noise which was in good agreement with recent experiments. We also found that nonuniformities such as pinholes in the barrier can affect the noise. It would be interesting to test our prediction that the critical current noise goes as L^5 and to see if the ratio of the charge noise to critical current noise for the same junction is given by Eq. (9) at low frequencies.

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[1] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002).

[2] Y. Yu, S. Han, X. Chu, S. Chu, and Z. Wang, Science **296**, 889 (2002).

[3] I. Chiorescu, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooij, Science **299**, 1869 (2003).

[4] Y. A. Pashkin, Y. Yamamoto, O. Astafiev, Y. Nakamura, D. V. Averin, and J. S. Tsai, Nature (London) **421**, 823 (2003).

[5] A. J. Berkley, H. Xu, R. C. Ramos, M. A. Gubrud, F. W. Strauch, P. R. Johnson, J. R. Anderson, A. J. Dragt, C. J. Lobb, and F. C. Wellstood, Science **300**, 1548 (2003).

[6] R. McDermott, R. W. Simmonds, M. Steffen, K. B. Cooper, K. Cicak, K. D. Osborn, S. Oh, D. P. Pappas, and J. M. Martinis, Science **307**, 1299 (2005).

[7] R. W. Simmonds, K. M. Lang, D. A. Hite, S. Nam, D. P. Pappas, and J. M. Martinis, Phys. Rev. Lett. **93**, 077003 (2004).

[8] J. M. Martinis, K. B. Cooper, R. McDermott, M. Steffen, M. Ansmann, K. D. Osborn, K. Cicak, S. Oh, D. P. Pappas, and R. W. Simmonds *et al.*, Phys. Rev. Lett. **95**, 210503 (2005).

[9] F. C. Wellstood, Ph.D. thesis, University of California, Berkeley, 1988.

[10] B. Savo, F. C. Wellstood, and J. Clarke, Appl. Phys. Lett. **50**, 1757 (1987).

[11] F. C. Wellstood, C. Urbina, and J. Clarke, Appl. Phys. Lett. **85**, 5296 (2004).

[12] D. J. Van Harlingen, T. L. Robertson, B. L. T. Plourde, P. A. Reichardt, T. A. Crane, and J. Clarke, Phys. Rev. B **70**, 064517 (2004).

[13] J. Eroms, L. C. van Schaarenburg, E. F. C. Driessen, J. Plantenberg, K. Huizinga, R. N. Schouten, A. H. Verbruggen, C. J. P. M. Harmans, and J. E. Mooij, Appl. Phys. Lett. **89**, 122516 (2006).

[14] L.-C. Ku and C. C. Yu, Phys. Rev. B **72**, 024526 (2005).

[15] A. Shnirman, G. Schon, I. Martin, and Y. Makhlin, Phys. Rev. Lett. **94**, 127002 (2005).

[16] L. Faoro, J. Bergli, B. L. Altshuler, and Y. M. Galperin, Phys. Rev. Lett. **95**, 046805 (2005).

[17] L. Faoro and L. B. Ioffe, Phys. Rev. Lett. **96**, 047001 (2006).

[18] L. Faoro and L. B. Ioffe, Phys. Rev. B **75**, 132505 (2007).

[19] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. **10**, 486 (1963).

[20] G. Zimmerli, T. M. Eiles, R. L. Kautz, and J. M. Martinis, Appl. Phys. Lett. **61**, 237 (1992).

[21] A. B. Zorin, F. J. Ahlers, J. Niemeyer, T. Weimann, H. Wolf, V. A. Krupenin, and S. V. Lotkhov, Phys. Rev. B **53**, 13 682 (1996).

[22] W. A. Phillips, *Amorphous Solids* (Springer-Verlag, Berlin, 1981).

[23] S. Hunklinger and A. K. Raychaudhuri, Prog. Low Temp. Phys. **9**, 265 (1986).

[24] F. W. Schmidlin, J. Appl. Phys. **37**, 2823 (1966).

[25] D. J. Scalapino and S. M. Marcus, Phys. Rev. Lett. **18**, 459 (1967).

[26] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1999), 3rd ed.

[27] S. Kogan, *Electronic Noise and Fluctuations in Solids* (Cambridge University Press, Cambridge, England, 1996).

[28] B. Golding, M. v. Schickfus, S. Hunklinger, and K. Dransfeld, Phys. Rev. Lett. **43**, 1817 (1979).

[29] L. S. Dorneles, D. M. Schaefer, M. Carara, and L. F. Schelp, Appl. Phys. Lett. **82**, 2832 (2003).