Comparison of Ising Spin Glass Noise to Flux and Inductance Noise in SQUIDs

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Recent experiments implicate spins on the surface of metals as the source of flux and inductance noise in SQUIDs. We present Monte Carlo simulations of 2D and 3D Ising spin glasses that produce magnetization noise S_M consistent with flux noise. At low frequencies S_M is a maximum at the critical temperature T_C in three dimensions, implying that flux noise should be a maximum at T_C. The second spectra of the magnetization noise and the noise in the susceptibility are consistent with experimentally measured SQUID inductance noise.

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While remarkable progress has been achieved in making high quality superconducting qubits, noise and decoherence continue to be a problem. Low frequency $1/f$ flux noise [1] in superconducting quantum interference devices (SQUIDs) is one of the dominant sources of noise in superconducting flux [2,3] and phase [4] qubits. Recent experiments indicate that there is a high density ($\sim 5 \times 10^{17}$ m^{-2}) of unpaired spins on the surface of thin films of normal metals [5] and superconductors [6]. These spins may arise from local moments in localized states at the metal-insulator interface [7]. It is thought that fluctuations of these spins produce $1/f$ flux noise. To further probe these fluctuations, experiments have measured the inductance L of SQUIDs, and found that the inductance noise S_L goes as $1/f^{\alpha}$ increasing with decreasing temperature [8]. Correlations between fluctuations in the flux and inductance imply that both arise from fluctuations of the surface spins. There is some experimental indication of interactions between the spins [6] leading to the theoretical suggestion that the flux noise is the result of spin diffusion via RKKY interactions between the spins [9]. This raises the possibility of a low temperature spin glass phase. However, the surface spins may be in two dimensions while the lower critical dimension is 3 for RKKY spin glasses [10], implying that the surface spins would not have a low temperature RKKY spin glass phase. Still they may undergo a finite temperature spin glass transition if the interactions are of another type. For example, power law spin glass interactions that go as $r_{ij}^{-\sigma}$ with $d/2 < \sigma < d$ can result in a finite critical temperature in d dimensions [11].

How does the flux noise in SQUIDs compare with the magnetization noise in spin glasses found in previous work? Measurements of spin glass magnetization noise $S_M(f)$ [12–14] find a low frequency $1/f$ noise spectrum that is a maximum near the spin glass transition temperature T_g [14]. $S_M(f) \sim 1/f$ is consistent with the $1/f$ flux noise spectrum. Theoretically, the infinite range (mean field) spin glass models predict a magnetization noise spectrum $S_M(f) \sim f^{-\alpha}$ with $\alpha \leq 1/2$ in the spin glass phase ($T \leq T_C$) [15–19], while the droplet model predicts $S_M(f) \sim (\ln f)/f$ [20], and the hierarchical model predicts $S_M(f) \sim 1/f$ [21]. Monte Carlo simulations of the 2D [22,23] and 3D [24] Ising ($\pm J$) spin glass model above the critical temperature T_C find that the low frequency magnetization noise spectrum goes from white (flat) at high temperatures to $1/f$ as the temperature is lowered. However, there have been no theoretical or computational results that can be compared to the SQUID inductance noise.

In this Letter we address the question of whether the flux and inductance noise of a SQUID can be produced by an interacting spin system. We will compare the flux noise with the magnetization noise of a spin glass system. Since the inductance L is proportional to the linear magnetic susceptibility χ, we will also compare the noise in the linear susceptibility of a spin glass to the SQUID inductance noise.

We address these issues with simulations of the 2D and 3D Ising spin glasses, the latter having a finite temperature second order spin glass transition. Although the 2D Ising spin glass is more similar to surface spins, it has a critical temperature $T_C = 0$ that does not allow us to examine noise near or below T_C. The behavior above T_C is similar in 2D and 3D Ising spin glasses. Above T_C we find a paramagnetic susceptibility that goes as $1/T$, which is consistent with experiment [5,6]. As in previous simulations [12,23,24], we find that the low frequency magnetization noise $S_M(f)$ goes from white to $1/f$ as the temperature decreases. In 3D we find that at low frequencies $S_M(f)$ is a maximum at T_C, implying that a maximum in the SQUID flux noise can be used to identify T_C.

To relate our results to the SQUID inductance noise, we use the fact that $L(f_1) \propto \chi(f_1)$, where f_1 is the probe frequency at which L and χ are measured. We calculate the noise $S_{\chi'}(f_2; f_1)$ in the real part of the linear susceptibility as well as the second spectrum $S_M^{(2)}(f_2; f_1)$ of the magnetization noise because $S_M^{(2)}(f_2; f_1)$ is proportional to the noise $S_{\chi''}(f_2; f_1)$ in the imaginary part of the linear susceptibility, and hence, to noise $S_{L''}(f_2; f_1)$ in $L''(f_1)$. As the temperature T decreases, we find that both spectra go from flat to $1/f_2^y$ where $y < 1$ increases with decreasing temperature. This is consistent with the behavior found for SQUID inductance noise [8]. Unlike the experiment that found long lived correlations between fluctuations in the flux and inductance [8], the spin glass simulations show no correlation between fluctuations in the magnetization and susceptibility, which is consistent with time reversal symmetry.

We start with the Hamiltonian of the Ising spin glass:

$$H = -\sum_{\langle i,j\rangle} J_{ij} s_i s_j, \tag{1}$$

where $\langle i, j\rangle$ denotes nearest neighbor sites i and j, and the spins $s_i = \pm 1$. The random exchange constants J_{ij} are chosen from a Gaussian distribution:

$$P(J_{ij}) = \frac{1}{\sqrt{2\pi J^2}} e^{-J_{ij}^2/(2J^2)}, \tag{2}$$

where the variance $J^2 = 1$. The 3D Ising spin glass has a second-order phase transition at $T_c = 0.95$ [25]. Temperatures are measured in units of J. We study systems of N spins ($N = 4^3$, 6^3, 8^3, and 10^3 in three dimensions; $N = 8^2$ and 16^2 in two dimensions) with periodic boundary conditions. We use parallel tempering [26,27] to equilibrate the systems at a set of temperatures ranging from 0.7 to 1.8 in three dimensions, and from 0.1 to 1.8 in two dimensions. (The 2D systems were not properly equilibrated below 0.5.) In parallel tempering, simulations at chosen temperatures are run between 10^5 and 2×10^5 Monte Carlo time steps per spin (MCS) using a Metropolis algorithm. At 100 time step intervals we attempt to switch the configurations of two neighboring temperatures using the following Metropolis test that ensures that the energies of the configurations sampled at any given temperature have a Boltzmann distribution. Let β_1 and β_2 be two neighboring inverse temperatures, and let U_1 and U_2 be the corresponding potential energies of the configurations at these temperatures. If $\Delta = (\beta_1 - \beta_2) \times (U_2 - U_1)$, then the switch is accepted with probability unity if $\Delta \leq 0$ and with probability $\exp(-\Delta)$ if $\Delta > 0$. For the system size $N = 8^3$ associated with the results in the figures, switches are accepted between 33% and 62% of the time.

To check for equilibration, we use the link overlap q_l method [28]. Once the system is equilibrated at all the chosen temperatures, we stop switching the configurations between different temperatures. We then use a Metropolis algorithm to run the system at each temperature for 1.5×10^6 Monte Carlo time steps per spin, and record the time series for M, the magnetization per spin. From the time series, we calculate the linear susceptibility $\chi' = N\sigma_M^2/(k_B T)$ where σ_M^2 is the variance of M. We also calculate the spectral density of the time series $M(t)$: $S_M(\omega) = 2\int_{-\infty}^{\infty} dt e^{i\omega t}\langle \delta M(t)\delta M(0)\rangle$, where $\delta M(t) = [M(t) - \langle M\rangle]$. $S_M(\omega)$ is the first spectrum of the noise, and is related to $\chi''(\omega)$, the imaginary part of the susceptibility, by the fluctuation-dissipation theorem:

$$S_M(\omega) = 4k_B T \frac{\chi''(\omega)}{\omega}. \tag{3}$$

This relation has been shown to be experimentally valid both above and below the freezing temperature [13,14]. We normalize the noise power by setting the total noise power per time step equal to the variance: $S_{\text{tot}} = (1/N_\tau)\sum_{\omega=0}^{\omega_{\max}} S_M(\omega) = \sigma_M^2$, where N_τ is the duration of the time series. This allows us to compare power spectra from time series of different durations.

To compare our results to noise in the inductance L, we relate the inductance to the susceptibility χ of spins in a layer of thickness d on the surface [29]: $L = \mu_0 \chi d(R/r)$ where μ_0 is the permeability of the vacuum, R is the radius of the inductance loop, and r is the radius of the wire. The experimentally measured inductance noise is dominated by the noise in the imaginary part $L''(f_1)$ [8,29]. The noise $S_{L''}$ in the inductance is related to the noise $S_{\chi''}$ in $\chi''(f_1)$, and, from Eq. (3), to noise in the noise spectrum $S_M(f_1)$. So we need to look at the fluctuations $\delta S_M(t, f_1)$ as a function of time, and calculate the associated noise spectrum which is known as the second spectrum $S_M^{(2)}(f_2; f_1)$ of the noise [30]. To do this, we divide the time series of $M(t)$ into equal segments, calculate the noise spectrum $S_M(f)$ of each segment, and compute the octave sum by summing the noise power between frequencies f_a and f_b, producing a new time series of octave sums $S_M(t_i, f_1)$, where t_i is the time of the ith segment, and $f_1 = (f_a + f_b)/2$. The power spectrum $S_M^{(2)}(f_2; f_1)$ of this time series is called the second spectrum. This is related to the noise $S_{\chi''}(f_2; f_1)$ in $\chi''(f_1)$ by $S_M^{(2)}(\omega_2; \omega_1) = (4k_B T/\omega_1)^2 S_{\chi''}(\omega_2; \omega_1)$.

To relate our results to noise in the real part of the inductance, we find the noise in the real part χ' of the susceptibility per spin by dividing the magnetization time series into equal segments of length $\Delta\tau$, and calculating the susceptibility $\chi'(t_i) = N\sigma_M^2/(k_B T)$ of each segment to produce a time series in the susceptibility. $S_{\chi'}(f_2)$ is the noise power of this time series. (The effective probe frequency $f_1 = 1/\Delta\tau$.)

We now present our results. We find that the real part of the linear susceptibility χ' is paramagnetic and goes as $1/T$ above T_C. This is consistent with the measurements of the flux and susceptibility that also find $1/T$ behavior [5,6]. The linear susceptibility of a spin glass has a cusp at T_C and does not diverge. We find a maximum in χ' in three dimensions at $T \approx T_C$, but in two dimensions the maximum occurs at $T \simeq 0.5 > T_C$, implying that the 2D system is not equilibrated for $T < 0.5$.

We now consider noise spectra. Figure 1 shows that the first spectrum $S_M(f)$ of the magnetization noise goes as $1/f$ in three dimensions. Similarly, in two dimensions,

$S_M(f) \sim f^{-\alpha}$ with $\alpha = 1.1$ for frquencies of order 10^{-3} to 10^{-2} MCS^{-1} in the temperature range from $T = 0.5$ to $T = 1.0$. This is consistent with $1/f$ flux noise due to spins on the surface of metals [1,6]. If we examine the low frequency noise at different temperatures, it has its maximum value at the transition temperature T_C (see Fig. 1) due to critical fluctuations [31]. This is consistent with experiments on spin glasses that found a maximum in the low frequency magnetization noise near the spin glass transition temperature T_g [14]. The maximum in $S_M(f)$ also implies that the low frequency flux noise will be a maximum at the spin glass transition temperature. Having a signature of the phase transition in the noise spectra was observed in previous simulations on systems without quenched disorder [31] that found that first and second order phase transitions can have a maximum in the low frequency noise at T_C.

The second spectrum of the magnetization $S_M^{(2)}(f_2; f_1)$, shown in Fig. 2 for two dimensions, reflects the noise $S_{\chi''}$ in χ''. We have normalized the second spectra plots by dividing by $\langle S_M(t, f_1)\rangle_t^2$, the square of the average of the octave sums. Here $\langle \ldots \rangle_t$ is the average over the time series. If the noise $S_{\chi''}(f_2; f_1)$ (or $S_{L''}(f_2; f_1)$) in the imaginary part of the susceptibility (or inductance) is normalized in a similar way, then $S_M^{(2)}(f_2; f_1)/\langle S_M(t, f_1)\rangle_t^2 = S_{\chi''}(f_2; f_1)/\langle \chi''(t; f_1)\rangle_t^2 = S_{L''}(f_2; f_1)/\langle L''(t; f_1)\rangle_t^2$. The log-log plots for 3D are similar and continue to get steeper as the temperature drops below T_C. At low frequencies in both 2D and 3D, $S_M^{(2)}(f_2; f_1)/\langle S_M(t, f_1)\rangle_t^2 \sim 1/f_2^y$ where the exponent y increases as the temperature decreases, and $y < 1$ for all the temperatures that we studied. (Notice that y increases even above T_C.) This increase is qualitatively consistent with the measured SQUID inductance noise that goes as $1/f_2^{y'}$ where y' increases with decreasing temperature as the inductance noise rises above the Johnson white noise background. Experimenally, y' is between 1.0 and 1.1 at the lowest temperatures between 100 and 200 mK [8,29]. The increase in y with decreasing T indicates that there are slow processes changing the characteristic spin relaxation times that dictate the first spectrum of the noise. These processes could be due to the system slowly exploring metastable states in the energy landscape. Our simulations also show that y decreases as the frequency $f_1 = (f_a + f_b)/2$ of the first spectrum decreases. For example, in 2D at $T = 0.5$ with $N = 16^2$ and $f_b = 2f_a$, $y = 0.88$ for $f_a = 0.25$ MCS^{-1}, and $y = 0.54$ for $f_a = 0.015\,625$ MCS^{-1}.

As shown in Fig. 3 for two dimensions, the noise $S_{\chi'}(f_2)$ in the real part of the susceptibility χ' behaves similarly to $S_M^{(2)}(f_2; f_1)$. The 2D and 3D plots are similar. The low frequency noise goes as $1/f_2^{v}$ where v increases as the temperature decreases. This is consistent with measurements of the noise in the real part of the SQUID inductance where the noise goes as $1/f_2^{v'}$ with v' increasing as the temperature drops [8]. In our simulations the exponent v increases slightly in three dimensions with the length of the time segment $\Delta\tau$ into which the magnetization time series is divided at a given temperature. In two dimensions we find that v is independent of $\Delta\tau$.

We find that the cross spectra $\langle \delta M^*(\omega)\delta\chi(\omega)\rangle / [|\delta M(\omega)||\delta\chi(\omega)|]$ between fluctuations in the magnetization and the susceptibility are zero at all frequencies and temperatures, implying that time reversal symmetry is preserved in our simulations. ($\delta M^*(\omega)\delta\chi(\omega)$ goes as the third power of δM, and so is odd under time reversal.) SQUID experiments find a high degree of correlation between flux and inductance at low temperatures, implying a common source of flux and inductance noise [8].

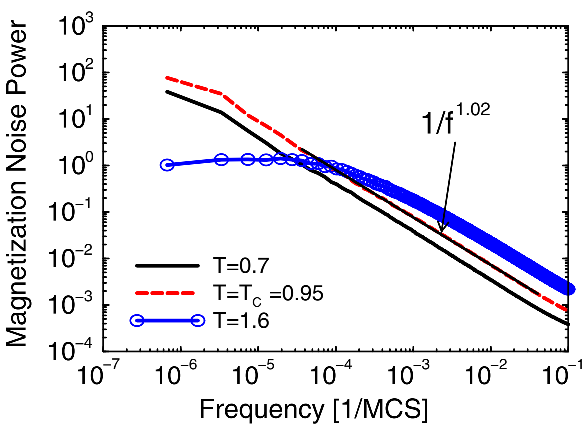

FIG. 1 (color online). Log-log plot of the magnetization noise $S_M(f)$ versus frequency f at various temperatures for $N = 8^3$ in three dimensions averaged over 200 runs. The noise goes as $1/f$, and the low frequency noise is a maximum at T_C. The frequencies are in units of MCS^{-1}.

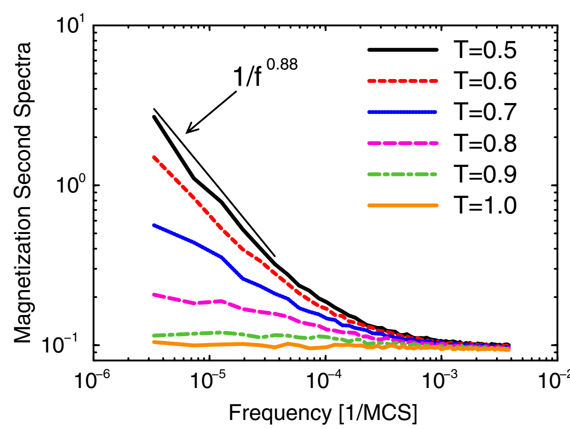

FIG. 2 (color online). Log-log plot of the second spectrum $S_M^{(2)}(f_2; f_1)$ versus frequency f_2 at various temperatures for $N = 16^2$ in 2D averaged over 200 runs. $f_a = 0.25$ MCS^{-1} and $f_b = 2f_a = 0.5$ MCS^{-1}. $S_M^{(2)}(f_2; f_1)$ has been normalized by the square of the average of the octave sums. The total unnormalized noise power equals the variance in the magnetization octave sums.

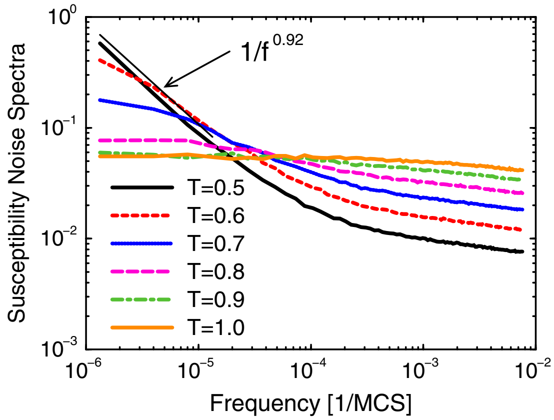

FIG. 3 (color online). Log-log plot of the noise in the real part of the susceptibility versus frequency f_2 at various temperatures for $N = 16^2$ in two dimensions averaged over 200 runs. The magnetization time series was divided into segments, each with a duration of 64 MCS. Curves decrease in temperature going from top to bottom at high frequencies.

Experiments see evidence for both correlation and anticorrelation. This indicates that there are slow fluctuators switching between correlation and anticorrelation, so that averaging the experimental cross spectra over long times would give zero, preserving time reversal symmetry [32].

To summarize, we have performed Monte Carlo simulations of the 2D and 3D Ising spin glass with nearest neighbor interactions. We find that the $1/T$ temperature dependence of the paramagnetic susceptibility above T_C is consistent with experiment. In three dimensions the low noise frequency magnetization noise is a maximum at T_C, implying that the low frequency flux noise should be a maximum at the spin glass transition temperature. In addition, in both two and three dimensions, we find the frequency and temperature dependence of the noise in the magnetization and susceptibility is consistent with SQUID measurements of the flux and inductance noise, respectively. This implies that the spins on metal surfaces produce noise like that of spins with random interactions.

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