$1/f$ noise in electron glasses

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We show that $1/f$ noise is produced in a three-dimensional electron glass by charge fluctuations due to electron hopping between isolated sites and a percolating network at low-temperatures. The low-frequency noise spectrum goes as $\omega^{-\alpha}$ with α slightly larger than 1. This result together with the temperature dependence of α and the noise amplitude are in good agreement with the recent experiments. These results hold true both with a flat, noninteracting density of states and with a density of states that includes the Coulomb interactions. In the latter case, the density of states has a Coulomb gap that fills in with increasing temperature. For a large Coulomb gap width, this density of states gives a dc conductivity with a hopping exponent of ≈ 0.75 which has been observed in recent experiments. For a small Coulomb gap width, the hopping exponent ≈ 0.5.

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I. INTRODUCTION

Low-frequency $1/f$ noise[1–3] is found in a wide variety of conducting systems such as metals, semiconductors, tunnel junctions,[4] and even superconducting quantum interference devices.[5,6] Yet the microscopic mechanisms are still not well understood. One example is an electron glass which is an insulator where electrons are localized by a strong random potential. A special case of this is a Coulomb glass in which the electrons interact with one another via a long-range Coulomb potential. Doped semiconductors and strongly disordered metals provide examples of electron glasses. Experimental studies on doped silicon inversion layers have shown that low-frequency $1/f$ noise is produced by hopping conduction.[7] Since the systems are glassy, electron hopping can occur on very long time scales which can produce low-frequency noise. In this paper, we show that the resulting noise spectrum goes as $f^{-\alpha}$, where f is frequency and the temperature-dependent exponent $\alpha > 1$.

Shklovskiĭ has suggested that $1/f$ noise is caused by fluctuations in the number of electrons in an infinite percolating cluster.[8] These fluctuations are caused by slow exchange of electrons between the infinite conducting cluster and the small isolated donor clusters. Subsequently, Kogan and Shklovskiĭ combined a more rigorous calculation with numerical simulations and found a noise spectrum where α was considerably lower than 1.[9] Furthermore, below a minimum frequency of order 1–100 Hz, the noise spectral density saturated and became a constant independent of frequency. Their calculations were valid only in the high-temperature regime where the impurity band was assumed to be occupied uniformly and the long-range Coulomb correlations were essentially neglected. Since then there have been attempts to include the effects of correlations.

In particular, Kozub suggested a model,[10] in which electron hops within isolated pairs of impurities produce fluctuations in the potential seen by other hopping electrons that contribute to the current. While leading to $1/f$-type noise within some frequency range, this model also shows low-frequency noise saturation due to the exponentially small probability of finding an *isolated* pair of sites with a long tunneling time. Moreover, the noise magnitude is predicted to increase as the temperature $T \to 0$ in contradiction with the recent experimental findings of Massey and Lee.[11] This, in part, led Massey and Lee to the conclusion that the single-particle picture is inconsistent with the observed noise behavior. A different approach was proposed by Kogan[12] who considered intervalley transitions as the source of the hopping conduction noise. Unfortunately, this approach does not seem to be analytically traceable and is not easily generalizable.

In this paper, we extend Kogan and Shklovskiĭ's approach[9] by including the energy dependence of the hopping as well as the effects of electron-electron interactions on the single-particle density of states $g(\varepsilon)$. This is essentially a mean-field approximation: we assume that charge is carried by electronlike quasiparticles whose interaction with the other charges is taken into account via the single-particle density of states. Later we will present some justification for why we believe this approach works for low-frequency noise. For comparison we also consider the case of noninteracting electrons with a flat density of states.

The paper is organized as follows. In Sec. II A, we describe our calculation of the noise spectral density. In Sec. II b, we present the density of states that includes the Coulomb gap and that models the decrease in the gap with increasing temperature. We show that this form of the density of states yields the usual value of the hopping exponent $\delta \approx 0.5$ for small values of the Coulomb gap width E_g. However, for large values of E_g, $\delta \approx 0.75$. Both values have been seen experimentally.[11,13–17] In Sec. III, we present our results.

II. CALCULATION

A. Noise spectral density

We start with a model of the Coulomb glass in which electrons occupy half of the impurity sites. Each site can have at most one electron due to a large on-site repulsion. The sites are randomly placed according to a uniform spatial distribution, and each has a random on-site energy ϕ_i chosen from a uniform distribution extending from $-W/2$ to $W/2$. Thus, g_o, the density of states without interactions, is flat. At $T=0$ such a system is a perfect insulator, while at low but

finite temperatures it will be able to conduct via variable range hopping.[18–20] In this picture, the dc conductivity is dominated by particles hopping along the percolating network, which is constructed as follows. The resistance R_{ij} associated with a transition between sites i and j grows exponentially with both their separation r_{ij} and energy difference ε_{ij}:

$$R_{ij}=R_{ij}^{o}\exp(x_{ij}), \tag{1}$$

where the prefactor $R_{ij}^{o}=kT/(e^2\gamma_{ij}^{o})$ with γ_{ij}^{o} being given by[18]

$$\gamma_{ij}^{o}=\frac{D^2|\Delta_i^j|}{\pi\rho s^5\hbar^4}\left[\frac{2e^2}{3\kappa\xi}\right]^2\frac{r_{ij}^2}{\xi^2}\left[1+\left(\frac{\Delta_i^j\xi}{2\hbar s}\right)^2\right]^{-4}, \tag{2}$$

where D is the deformation potential, s is the speed of sound, ρ is the mass density, ξ is the localization length, and κ is the dielectric constant. $\Delta_i^j=\varepsilon_j-\varepsilon_i-e^2/\kappa r_{ij}$ is the change in energy that results from hopping from i to j with $\varepsilon_i=\phi_i+\Sigma_j(e^2/\kappa r_{ij})n_j$ being a single-site energy. In Eq. (1), the exponent is given by

$$x_{ij}=\frac{2r_{ij}}{\xi}+\frac{\varepsilon_{ij}}{kT}. \tag{3}$$

The exponent reflects the thermally activated hopping rate between i and j as well as the wave-function overlap between the sites:

$$\varepsilon_{ij}=\begin{cases}|\varepsilon_j-\varepsilon_i|-\dfrac{e^2}{\kappa r_{ij}}, & (\varepsilon_i-\mu)(\varepsilon_j-\mu)<0\\ \max[|\varepsilon_i-\mu|,|\varepsilon_j-\mu|], & (\varepsilon_i-\mu)(\varepsilon_j-\mu)>0\end{cases} \tag{4}$$

(in what follows, we choose the Fermi level $\mu=0$).

A noninteracting picture of dc conduction is described in terms of electron hopping between sites in the cluster that spans the entire sample. In order to determine which sites are in a cluster, we introduce the ''acceptance'' parameter x such that any two sites i and j are considered ''connected'' if $x_{ij}\leqslant x$ and disconnected otherwise. For small values of x, only rare pairs of sites are connected. As we increase x, more such pairs appear and small clusters start coalescing into bigger ones until an infinite cluster—the critical percolating network—is formed at some x_c. At this point we can neglect the contribution of the remaining impurity sites to the dc conductivity since it is exponentially small as compared to that of the sites already in the percolating network (although the former sites are important for understanding both ac conductivity and noise). In a similar manner the resistance of the critical percolating network is dominated by a few pairs with $x_{ij}=x_c$—these are the pairs that bridge the gaps between large finite clusters enabling the formation of the infinite cluster. Hence, the resistance of the entire sample is well approximated by $R_{tot}\approx\overline{R^o}\exp(x_c)$, where $\overline{R^o}\equiv kT/(e^2\overline{\gamma^o})$ with $\overline{\gamma^o}$ being the average value of γ_{ij}^{o} given by Eq. (2).

In the presence of the Coulomb interactions, there is no exact mapping of transport onto a percolation picture. We, nevertheless, assume that upon diagonalizing the interacting Hamiltonian one finds that charge carrying excitations are of a local nature, and so they can be treated within the percolation picture as noninteracting quasiparticles. The Coulomb interactions renormalize the single-particle density of states which acquires a soft gap. We will discuss this in more detail in the section on the density of states. However, we will mention here that this approach appears to work well for dc conduction and leads to a temperature dependence of the conductivity,[18,21,22] which is distinctly different from the noninteracting case and which agrees with experiment (see, for example Ref. 23). However, the question about the validity of this approach is still far from being settled—see Ref. 24 for a different point of view.

In our treatment we will focus on the noise caused by quasiparticle hopping between isolated clusters and the percolating network, producing fluctuations of charge in the latter.[8,9] Let $N_{\mathcal{P}}$ be the average number of such particles in the critical percolating network and $\delta N_{\mathcal{P}}(t)$ be its time-dependent fluctuation. Assuming that only stationary processes are involved [i.e., $\langle\delta N_{\mathcal{P}}(t_2)\delta N_{\mathcal{P}}(t_1)\rangle=f(t_2-t_1)$], we can use the Wiener-Khintchine theorem[3] to relate the noise spectral density $S_I(\omega)$ of current fluctuations to the Fourier transform of the autocorrelation function:

$$\frac{S_I(\omega)}{I^2}=\frac{2\langle\delta N_{\mathcal{P}}(t_2)\delta N_{\mathcal{P}}(t_1)\rangle_\omega}{N_{\mathcal{P}}^2}, \tag{5}$$

where I is the average current. The charge fluctuation autocorrelation function can be expressed as a superposition of modes α, each of which relax exponentially with a characteristic time τ_α. Thus the Fourier transform $\langle\cdots\rangle_\omega$ of the autocorrelation function is a weighted sum over Lorentzians,[9]

$$\langle\delta N_{\mathcal{P}}(t_2)\delta N_{\mathcal{P}}(t_1)\rangle_\omega=\frac{2kT}{e^2}\sum_{\alpha\neq0}\frac{\tau_\alpha}{1+\omega^2\tau_\alpha^2}\left|\sum_{i\in\mathcal{P}}C_i\psi_\alpha(i)\right|^2. \tag{6}$$

Here, $C_i\equiv(e^2/kT)f_i(1-f_i)$ is the capacitance of site i (with $f_i=[\exp(\varepsilon_i/kT)+1]^{-1}$ being its equilibrium occupancy), while τ_α^{-1} and $\psi_\alpha(i)$ are the αth eigenvalue and eigenvector, respectively, of the following system of linear equations:

$$\sum_j R_{ij}^{-1}[\psi_\alpha(i)-\psi_\alpha(j)]=\tau_\alpha^{-1}C_i\psi_\alpha(j), \tag{7}$$

with R_{ij} being the intersite resistances given by Eq. (1). Since R_{ij}^{-1} is proportional to the hopping rate $\tau_{ij}^{-1}=\gamma_{ij}^{o}\exp(-x_{ij})$ from site i to site j, Eq. (7) relates τ_{ij}^{-1} to the relaxation rates τ_α^{-1} of the entire percolating network. The sum over sites i in Eq. (6) runs only over those sites that belong to the critical percolating network since only their occupancies affect the current through the sample. The physical meaning of the quantity $C_i\psi_\alpha(i)$ is that it is proportional to the fluctuation δf_i of the occupation of site i and decays exponentially with the associated time constant τ_α. The eigenvectors satisfy the following conditions:

$$\sum_i C_i \psi_\alpha(i) \psi_\beta^*(i) = \delta_{\alpha\beta}, \tag{8}$$

$$\sum_\alpha C_i \psi_\alpha(i) \psi_\alpha^*(j) = \delta_{ij}, \tag{9}$$

$$\sum_i C_i \psi_\alpha(i) = 0, \quad \forall \alpha \neq 0. \tag{10}$$

The first condition states that the eigenfunctions are orthonormal; the second states that the functions form a complete set. One of the eigenfunctions is a constant, which we take to be the one corresponding to $\alpha=0$. This has the eigenvalue $\tau_0^{-1}=0$. Equation (10) is the orthonormalization condition between this eigenstate and the others. It represents the fact that the fluctuations in occupation represented by the $\alpha\neq 0$ modes do not affect the total number of electrons on the impuritiy sites. Thus, the last equation is just the statement of overall charge conservation. We remark here that Eqs. (7) are *linear* only within the assumption made earlier about noninteracting quasiparticles. Otherwise, R_{ij} are *not* constant coefficients; they depend on the on-site energies, which in turn depend on the occupancies of other sites.

Since we are interested in the modes that affect the charge in the conducting network, we can replace the sum over α by a sum over all finite clusters that coalesce with the infinite cluster as the acceptance parameter increases above x_c. In particular, we can replace the sum over α by an integral over x and a sum over all finite clusters merging with the infinite cluster at a given value of x. With this in mind, we can evaluate Eq. (6) using Eqs. (8) and (10). For a single mode α, the sum over sites i can be split into a sum over finite clusters (FC) and a sum over the infinite cluster (IC). So we can write the normalization condition, Eq. (8), and the charge conservation condition, Eq. (10), as

$$\sum_{m\in \mathrm{FC}} C_m \psi_\alpha^2(m) + \sum_{n\in \mathrm{IC}} C_n \psi_\alpha^2(n) = 1, \tag{11}$$

$$\sum_{m\in \mathrm{FC}} C_m \psi_\alpha(m) + \sum_{n\in \mathrm{IC}} C_n \psi_\alpha(n) = 0, \tag{12}$$

respectively. Since the fast modes equilibrate the occupations of sites within each cluster, the eigenfunctions do not depend on their site indices within each cluster, i.e., $\psi_\alpha(m) = \psi_{\alpha,\mathrm{FC}}$, $\forall m \in$ FC and $\psi_\alpha(n)=\psi_{\alpha,\mathrm{IC}}$, $\forall m \in$ IC. As a result, we can take $\psi_{\alpha,\mathrm{FC}}$ and $\psi_{\alpha,\mathrm{IC}}$ out of the sums in Eqs. (11) and (12). The sum over capacitances in the finite clusters, will be much smaller than the sum over the infinite cluster which implies that $(\psi_{\alpha,\mathrm{IC}})^2$ is negligible in Eq. (11). This leads to

$$\psi_{\alpha,\mathrm{FC}} = \left(\sum_{m\in \mathrm{FC}} C_m \right)^{-1/2}. \tag{13}$$

Plugging this into Eq. (12) yields

$$\psi_{\alpha,\mathrm{IC}} \sum_{m\in \mathrm{IC}} C_m = -\left(\sum_{m\in \mathrm{FC}} C_m \right)^{1/2}. \tag{14}$$

We can use these results to evaluate the sum over sites in Eq. (6) by noting that all the sites in the critical network are also in the infinite cluster by definition. Thus,

$$\sum_{i\in \mathcal{P}} C_i \psi_\alpha(i) = \frac{N_{\mathcal{P}}}{N_{\mathrm{IC}}(x)} \sum_{i\in \mathrm{IC}} C_i \psi_{\alpha,\mathrm{IC}} = -\frac{N_{\mathcal{P}}}{N_{\mathrm{IC}}(x)} \left(\sum_{i\in \mathrm{FC}} C_i \right)^{1/2}, \tag{15}$$

where $N_{\mathrm{IC}}(x)$ is the number of sites in the infinite cluster at a given value of x.

In evaluating Eq. (6), we make the following approximation for τ_α. Since we are interested in the modes α that affect the charge of the percolating network, we only consider particle exchange between the isolated clusters and the infinite cluster. This involves hopping times that are longer than those within the percolating network itself by definition. Due to the exponentially wide distribution of hopping times τ_{ij}, such exchange is likely to be dominated by the single closest pair of sites in which one belongs to the finite and the other to the infinite cluster. The relaxation times within each cluster are much faster, and therefore the above-mentioned pair serves as a ''bottleneck'' for intercluster relaxation. A simple diagonalization of the system of Eqs. (7) for two clusters $\mathcal{A}_1$ and $\mathcal{A}_2$ with the bottleneck hopping resistance $R = \min(R_{ij}; i\in\mathcal{A}_1, j\in\mathcal{A}_2)$ between them (and with the assumption that all other intercluster resistances are much higher and all intracluster resistances are much lower than R) leads to the following expression for the intercluster relaxation time:

$$\tau = R\left(\left[\sum_{i\in\mathcal{A}_1} C_i \right]^{-1} + \left[\sum_{j\in\mathcal{A}_2} C_j \right]^{-1} \right)^{-1}. \tag{16}$$

Since we are interested only in the situation where one of the clusters is infinite, this simplifies Eq. (16) to $\tau = R\Sigma_{i\in\mathcal{A}} C_i$, where $\mathcal{A}$ is the finite cluster.

We can substitute this value of τ into Eq. (6) by replacing the sum over all modes α by a sum over all finite clusters that coalesce with the infinite cluster as the acceptance parameter x is increased above x_c. Each such finite cluster contributes one new term to the sum over α in Eq. (6) with the corresponding $\tau_\alpha = R(x)\Sigma_{i\in\mathcal{A}} C_i$, where $R(x)=\overline{R^0}e^x$. Then we can write the spectral density of the noise as follows:

$$\frac{S_I(\omega)}{I^2} = \frac{16kT}{e^2} \int_{\lambda x_c}^{\infty} dx \sum_{\mathcal{A}}{}' \frac{N_{\mathrm{IC}}^{-2}(x) R(x) \left(\sum_{i\in\mathcal{A}} C_i \right)^2}{1+\omega^2 R^2(x) \left(\sum_{i\in\mathcal{A}} C_i \right)^2}, \tag{17}$$

where $\Sigma'_{\mathcal{A}}$ stands for the sum over all finite clusters that coalesce with the infinite cluster as x increases by dx. The parameter $\lambda \geqslant 1$ and sets the distance in x space from the percolation threshold.

This equation is difficult to evaluate mathematically. Fortunately, however, we can extract the low-frequency asymptotic behavior of Eq. (17) where the above approximations are well justified. The lowest-frequency contributions

come from large values of x where the infinite cluster has already absorbed almost all the sites (i.e., $N_{\rm IC}\approx N$, the total number of sites). What is left are the small clusters, which are mostly isolated sites in the increasingly rare voids of the infinite cluster. The probability of having two such sites in the same void is negligibly small. Since low-frequency noise will be dominated by the hops between such isolated sites and the infinite cluster, we only consider such hops in obtaining the spectral density of current fluctuations. In Eq. (17), we can set λ to correspond to this situation at large x, and we can replace the sum over all finite clusters that are merging with the infinite cluster with a sum over all sites multiplied by the probability $\tilde{P}_1(x,\varepsilon)dx$ that a single site with energy ε has its nearest neighbor between x and $x+dx$.

We can write down an expression for $\tilde{P}_1(x,\varepsilon)dx$. We begin by defining $P_1(x,\varepsilon)$ to be the probability that a given site with on-site energy ε has no neighbors nearer than x. Let $\exp[-\rho(x,\varepsilon)dx]$ be the probability that a site with energy ε has no neighbors between x and $x+dx$. Then

$$P_1(x,\varepsilon)=\exp\left(-\int_0^x \rho(x',\varepsilon)dx'\right). \tag{18}$$

We can use this to express $\tilde{P}_1(x,\varepsilon)dx$ as the product of $P_1(x,\varepsilon)$, the probability of no neighbors within x, multiplied by the probability of having a neighbor between x and $x+dx$:

$$\tilde{P}_1(x,\varepsilon)dx=P_1(x,\varepsilon)[1-e^{-\rho(x,\varepsilon)dx}] \tag{19}$$

$$=-\left[\frac{\partial}{\partial x}P_1(x,\varepsilon)\right]dx. \tag{20}$$

Thus, $(-\partial P_1/\partial x)$ is the probability density for a site to have its nearest neighbor between x and $x+dx$. We can now write the spectral density of current fluctuations as

$$\frac{S_I(\omega)}{I^2}=\frac{16kTV}{e^2N^2}\int_{\lambda x_c}^{\infty}dx\int_{-W/2}^{W/2}d\varepsilon g(\varepsilon,T) \times\left(-\frac{\partial P_1(x,\varepsilon)}{\partial x}\right)\frac{R(x)C^2(\varepsilon)}{1+\omega^2R^2(x)C^2(\varepsilon)}, \tag{21}$$

where V is the volume, W is the bandwidth, and $f(\varepsilon)$ is the Fermi occupation number. To obtain an expression for $P_1(x,\varepsilon)$, we note that the average number dN of impurity sites found in a phase volume element $d\Omega=d^dr\,d\varepsilon'$ within a distance x of a site with energy ε is given by

$$dN=g(\varepsilon')\,\theta\left(x-\frac{2r}{\xi}-\frac{|\varepsilon|+|\varepsilon'|+|\varepsilon-\varepsilon'|}{2kT}\right)d\varepsilon'd^dr. \tag{22}$$

The probability that no sites are in $d\Omega$ is given by

$$\lim_{N\to\infty}\left[1-\frac{dN}{N}\right]^N=e^{-dN}. \tag{23}$$

Thus, the probability $P_1(x,\varepsilon)$ that a given site with on-site energy ε has no neighbors nearer than x is given by

$$P_1(x,\varepsilon)=\exp\left\{-\int d^dr\int_{-W/2}^{W/2}d\varepsilon' g(\varepsilon',T) \times\theta\left(x-\frac{2r}{\xi}-\frac{|\varepsilon|+|\varepsilon'|+|\varepsilon-\varepsilon'|}{2kT}\right)\right\}. \tag{24}$$

Notice the absence of the Coulomb energy in the argument of the θ function in Eq. (24), in accordance with our quasiparticle picture. Our quasiparticle picture is likely to work best for hops between isolated sites and the infinite cluster. Although one such hop may result in a sequence of other hops, these will mostly happen within the infinite cluster on a much shorter time scale, effectively renormalizing the properties of the "slow" particle. As was mentioned earlier, these renormalizations can be included in the single-particle density of states $g(\varepsilon,T)$.

To facilitate evaluating the integral in Eq. (21) numerically for the case where we include a Coulomb gap in the density of states, we define the dimensionless variables $\tilde{r}=r/\xi$, $\tilde{\varepsilon}=\varepsilon/E_g$, $\tilde{\omega}=\omega/\overline{\gamma^o}$, $\tilde{T}=kT/E_g$, $\tilde{\tau}=\overline{\gamma^o}R(x)C(\varepsilon)=f(\varepsilon)[1-f(\varepsilon)]e^x$, and $\tilde{g}(\tilde{\varepsilon},\tilde{T})=g(\varepsilon,T)/g_o$. g_o is the noninteracting density of states and $E_g\approx e^3\sqrt{\pi g_o/3\kappa^3}$ is the characteristic width of the Coulomb gap. Evaluating the integral over x in Eq. (21) leads us to define

$$\tilde{x}=2\tilde{r}+\frac{|\tilde{\varepsilon}|+|\tilde{\varepsilon}'|+|\tilde{\varepsilon}-\tilde{\varepsilon}'|}{2\tilde{T}}. \tag{25}$$

Then we can rewrite Eq. (21) as

$$\frac{S_I(\omega)}{I^2}=A\int_{-\tilde{W}/2}^{\tilde{W}/2}d\tilde{\varepsilon}\tilde{g}(\tilde{\varepsilon},\tilde{T})\int_{-\tilde{W}/2}^{\tilde{W}/2}d\tilde{\varepsilon}'\tilde{g}(\tilde{\varepsilon}',\tilde{T})\int_0^{\tilde{R}_V}\tilde{r}^2d\tilde{r} \times\theta(\tilde{x}-\lambda x_c)\frac{P_1(\tilde{x},\tilde{\varepsilon})\tilde{\tau}(\tilde{x},\tilde{\varepsilon})f(\tilde{\varepsilon})[1-f(\tilde{\varepsilon})]}{1+\tilde{\omega}^2\tilde{\tau}^2(\tilde{x},\tilde{\varepsilon})}, \tag{26}$$

where $A=64\pi g_o^2E_g^2V\xi^3/(N^2\overline{\gamma^o})$, $\tilde{R}_V=(3V/4\pi)^{1/3}/\xi$, $\tilde{W}=W/E_g$, $\eta=4\pi g_oE_g\xi^3$, and

$$P_1(\tilde{x},\tilde{\varepsilon})=\exp\left[-\eta\int_0^{\tilde{R}_V}\tilde{r}'^2d\tilde{r}'\int_{-\tilde{W}/2}^{\tilde{W}/2}d\tilde{\varepsilon}''\tilde{g}(\tilde{\varepsilon}'',\tilde{T}) \times\theta\left(\tilde{x}-2\tilde{r}'-\frac{|\tilde{\varepsilon}|+|\tilde{\varepsilon}''|+|\tilde{\varepsilon}-\tilde{\varepsilon}''|}{2\tilde{T}}\right)\right]. \tag{27}$$

For comparison we also consider the case with no Coulomb gap by setting $g(\varepsilon,T)=g_o$ in Eqs. (21) and (24). Since there is no natural energy scale, we do not rescale the energies. However, we can define $\tilde{r}$, $\tilde{\tau}$, and $\tilde{\omega}$ as before. As a result, the definition of $\tilde{x}$ in Eq. (25) becomes $\tilde{x}=2\tilde{r}+(|\varepsilon|+|\varepsilon'|+|\varepsilon-\varepsilon'|)/(2T)$. In Eq. (26), A is replaced by $A_o=64\pi Vg_o^2\xi^3/N^2\overline{\gamma^o}$ and $\tilde{W}$ is replaced by simply W. In Eq. (27), η is replaced by $\eta_o=4\pi\xi^3g_o$.

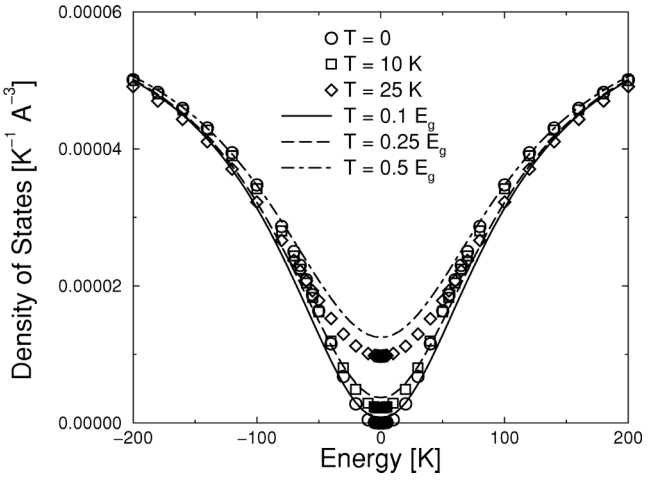

FIG. 1. The density of states $g(\varepsilon,T)$ versus ε at various temperatures. The symbols are calculated using Eq. (28) with $W_o/2 = 2.3\times10^4$ K. The density of states is measured from the Fermi energy $E_F=0$. The lines are the result of evaluating Eq. (29) with $E_g=100$ K. $g_o=6.25\times10^{-5}$ states/K Å^3.

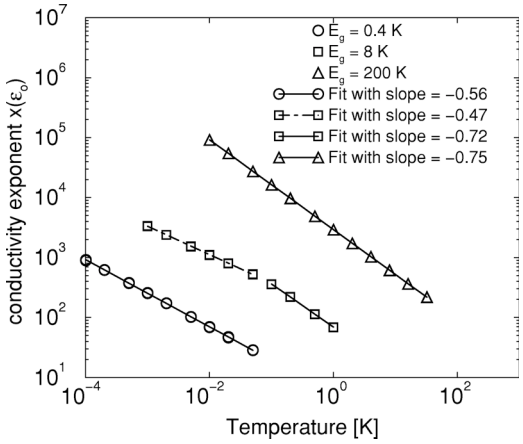

FIG. 2. $x(\varepsilon_o,T)$ versus temperature with $g_o=6.25\times10^{-5}$ states/K Å^3 and $\xi=10a_o=5.29177$ Å, where a_o is the Bohr radius and 10 is an estimate of the dielectric constant. We show plots for $E_g=0.4$ K (○), $E_g=8$ K (□), and $E_g=200$ K (△). $E_g=0.4$ K corresponds to the value of the Coulomb gap deduced from transport measurements, while $E_g=8$ K value from tunneling measurements on Si:B (Refs. 22 and 37). The lines are fits to the numerical data with the indicated slopes. The fit to the $E_g=0.4$ K data yields $\delta=0.56$ and $T_o=19$ K. The fit to the $E_g=8$ K data at low temperatures yields $\delta=0.47$ and $T_o=27\,206$ K, while the fit to the high-temperature data yields $\delta=0.72$ and $\delta=0.357$ K. The fit to the $E_g=200$ K data yields $\delta=0.75$ and $T_o=42\,068$ K. δ is virtually independent of g_o but T_o does depend on g_o. For example, changing g_o by 10 orders of magnitude to $6.25\times10^{+5}$ states/K Å^3 results in $\delta=0.75$ and $T_o=19$ K for $E_g=200$ K.

III. DENSITY OF STATES

At zero temperature, long-range interactions produce a Coulomb gap centered at the Fermi energy in $g(\varepsilon,T)$.[18,21,25,26] This gap arises because the stability of the ground state with respect to single-electron hopping from an occupied site i to an unoccupied site j requires that the energy difference $\Delta_i^j>0$. At finite temperatures, the Coulomb gap is partially filled and the density of states no longer vanishes at the Fermi energy.[27–32] The exact form of $g(\varepsilon,T)$ is not known, but some have argued[30–32] that its low-temperature asymptotic behavior is described by $g(\varepsilon=0,T) \sim T^{d-1}$. We have done Monte Carlo simulations of a three-dimensional Coulomb glass with off-diagonal disorder and we find that $g(\varepsilon=0,T)$ cannot be described by a simple power law.[28,33] The results of such simulations do not produce a density of states that is suitable for use in our noise integrals due to finite-size effects. In particular, $g(\varepsilon,T)$ goes to zero at energies far away from the Fermi energy because of the finite size of the system.

Another way to approximate the density of states is to use the Bethe-Peierls-Weiss (BPW) approximation.[31] The idea is to treat the interactions between one ''central'' site and all other sites (boundary sites) exactly, but to include the interactions between these boundary sites by means of effective fields. The density of states can then be written as a convolution

$$g(\varepsilon,T)=\int_{-W_o/2}^{W_o/2} d\varepsilon' g(\varepsilon-\varepsilon')\frac{1}{kT}h\left(\frac{\varepsilon'}{kT}\right), \qquad (28)$$

where $g(\varepsilon)$ is the zero-temperature density of states and W_o is the bandwidth. The function $h(\varepsilon/kT)$ takes into account thermal fluctuations in the occupation of the central site and the boundary sites. At low temperatures, it has a sharp peak with a width of the order kT at $\varepsilon=0$. We can make the approximation $(1/kT)h(\varepsilon/kT)\approx -f'(\varepsilon)$, where $f'(\varepsilon)$ is the derivative of the Fermi function. The zero-temperature density of states can be determined numerically by solving a self-consistent equation based on the ground-state stability condition that a single-electron hopping from an occupied site i to an unoccupied site j requires $\Delta_i^j>0$.[34,35] The result of evaluating Eq. (28) is shown in Fig. 1.

Since using the BPW approximation to evaluate Eqs. (21) and (24) is rather awkward, we model the finite-temperature-density of states by

$$g(\varepsilon,T)=g_o\frac{\varepsilon^2+(kT)^2}{E_g^2+\varepsilon^2+(kT)^2}. \qquad (29)$$

Notice that for $T=0$, $g(\varepsilon,T=0)\sim\varepsilon^2$ for $\varepsilon\ll E_g$ as is expected for a Coulomb gap in three dimensions. For large energies ($\varepsilon\gg E_g$ and $\varepsilon\gg kT$), $g(\varepsilon,T)$ approaches the noninteracting value g_o. A comparison of Eq. (29) with the BPW approximation at various temperatures is shown in Fig. 1. Equation (29) is the expression we use for the density of states of a Coulomb glass in Eqs. (21) and (24).

We can calculate the dc conductivity resulting from this density of states by following Mott's argument for variable range hopping.[18] We start with the hopping resistance R_{ij} given by Eq. (1). Mott pointed out that hopping conduction at low temperatures comes from states near the Fermi energy. If we consider states within ε_o of the Fermi energy ($E_F=0$), then the concentration of states in this band is

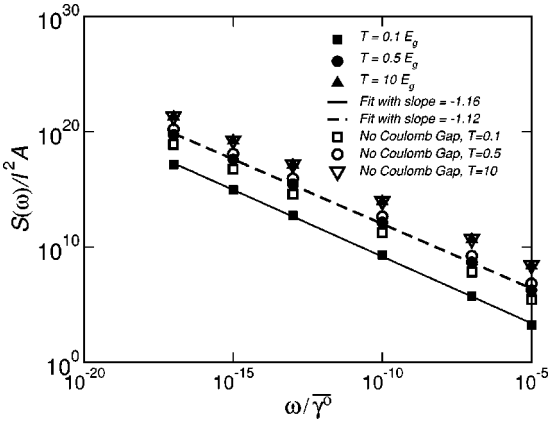

FIG. 3. The noise power spectrum as a function of frequency. The frequency is measured in the units of $\overline{\gamma^o}$ which is estimated to be of the order of 10^{13} Hz for values appropriate for insulating Si:B. Unless otherwise noted, all curves in this and the following figures which were obtained for the case with a Coulomb gap used $\eta = 4\pi E_g \xi^3 g_o \sim 12[E_g/(e^2/\kappa\xi)]^3 = 4.8\times 10^{-6}$, which in our estimates corresponds to the experimental dopant concentration of roughly $n = 0.8\, n_c$ for Si:B (Refs. 11 and 23). We set $\tilde{W} = 20$, $\tilde{R}_V = 100$, and $\lambda x_c = 1$ (the precise value of λ has no effect on the low-frequency noise that is governed by $x \gg x_c$). The parameter $A \equiv 64\pi V E_g^2 g_o^2 \xi^3 / N^2 \overline{\gamma^o}$. For comparison, we show the noise spectrum in the absence of a Coulomb gap with $g(\varepsilon, T) = g_o$ in Eqs. (21) and (24). In the absence of a Coulomb gap, A is replaced by $A_o \equiv 64\pi V g_o^2 \xi^3 / N^2 \overline{\gamma^o}$ and η is replaced by $\eta_o = 4\pi\xi^3 g_o = 4.8 \times 10^{-6}$. The energy is measured in arbitrary units and we set $W = 20$. The other variables are the same as in the case of a finite Coulomb gap.

$$N(\varepsilon_o, T) = \int_{-\varepsilon_o}^{\varepsilon_o} g(\varepsilon, T)\, d\varepsilon, \tag{30}$$

where $g(\varepsilon, T)$ is given by Eq. (29). So the typical separation between sites is $R_o = [N(\varepsilon_o, T)]^{-1/3}$. To estimate the resistance corresponding to hopping between two typical states in the band, we replace r_{ij} with R_o and ε_{ij} with ε_o in Eq. (3) to obtain $x(\varepsilon_o)$. Minimizing $x(\varepsilon_o)$ numerically yields ε_o. A plot of $x(\varepsilon_o)$ versus temperature is shown in Fig. 2. The dc conductivity is then given by $\sigma(T) = \sigma_o \exp[-x(\varepsilon_o)]$. We find that at low temperatures ($T \ll E_g$),

$$\sigma(T) = \sigma_o \exp\left[-\left(\frac{T_o}{T}\right)^{\delta}\right], \tag{31}$$

where δ is the hopping exponent. The value of δ depends on E_g. For large values of the Coulomb gap ($E_g \gtrsim 50$ K) $\delta \approx 0.75$, while for small values of the Coulomb gap ($E_g \lesssim 1$ K) $\delta \approx 0.5$. When we tried intermediate values of $E_g = 8$, 10, and 20 K, we found that $\ln[x(\varepsilon_o)]$ versus $\ln(T)$ had a break in slope with $\delta \approx 0.5$ at low temperatures and with $\delta \approx 0.72$-0.75 at high temperatures. Examples are shown in Fig. 2. $\delta = 0.75$ is higher than the Mott value of $\delta = 0.25$ associated with a flat density of states and the value of $\delta = 0.5$ derived by Efros and Shklovskii[21] for the zero-temperature Coulomb gap. However, experiments on materials such as ultrathin metal films find values for $\delta = 0.75 \pm 0.05$[13–17] in agreement with our value of δ for large E_g. The mechanism behind this exponent has been a puzzle[13,36]. Here we see that a possible simple explanation for the experimental observation of an anomalous hopping exponent is that the Coulomb gap in the single particle density of states is filling in with increasing temperature. If one takes this into account in the variable range hopping calculations, then the observed exponent of 0.75 can be obtained naturally. However, we should be cautious that our calculation applies to three dimensions while a two dimensional calculation may be more appropriate for ultrathin films. In fact, we find that the analogous two-dimensional calculation with a density of states $g(\varepsilon, T) = g_o(|\varepsilon| + kT)/(E_g + |\varepsilon| + kT)$ yields $\delta \approx 0.5$.

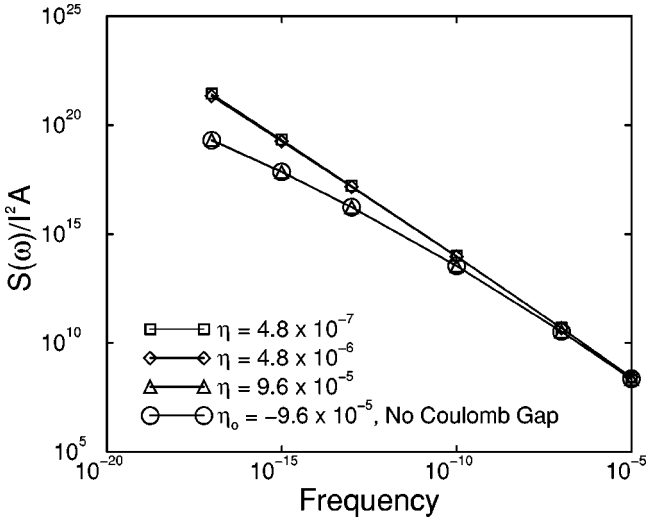

FIG. 4. The noise power spectrum as a function of frequency at $T = 10\ E_g$ for various values of $\eta = 4\pi E_g \xi^3 g_o$. The rest of the parameters are the same as in Fig. 3. Notice the saturation at low frequencies for large η. For comparison, we show the case with no Coulomb gap at $T = 10$ with a large value of $\eta_o = 4\pi\xi^3 g_o$. Large values of η_o lead to saturation but small values do not.

IV. RESULTS

We evaluate Eqs. (26) and (27) numerically and display the results in Figs. 3–6. In Fig. 3, we show the spectral density of the noise as a function of frequency. We find that for a wide range of parameters the noise spectral density is given by $S(\omega) \sim \omega^{-\alpha}$ with the spectral exponent α between 1.07 and 1.16 (see Figs. 3 and 5) which is $1/f$ noise. For comparison, we show in Fig. 3 the noise spectrum in the absence of a Coulomb gap with $g(\varepsilon, T) = g_o$ in Eqs. (21) and (24). The slope of a line through the open squares is -1.12 which is very close to the values obtained with a Coulomb gap. Notice that the presence of a Coulomb gap reduces the noise amplitude at low temperatures.

In Fig. 3, we use the *transport* value of $E_g \approx 0.4$ K, not the tunneling one ~ 8 K; the two were found to be different by an order of magnitude.[22,23] We find that increasing E_g by a factor of 20 does not produce a noticeable change in the results at low temperatures ($T = 0.1\ E_g$), but at high temperatures ($T = 10\ E_g$) it does lead to saturation of the noise

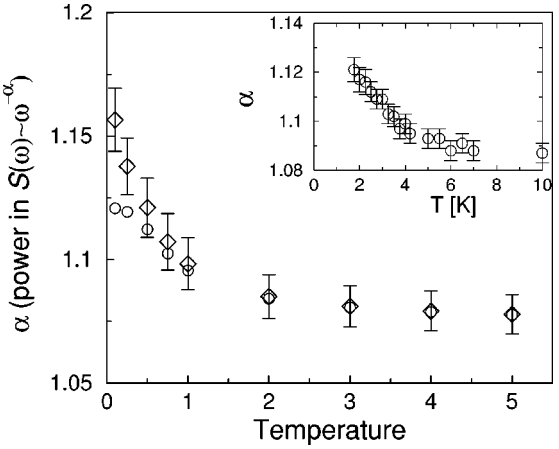

FIG. 5. The spectral exponent α as a function of temperature with a Coulomb gap in the density of states ($\diamond$) and with a flat density of states ($\bigcirc$). We have suppressed the error bars for the case with no Coulomb gap to avoid cluttering the graph. The suppressed error bars are comparable to those for the exponent with a Coulomb gap at high temperatures. The temperature is measured in units of the Coulomb gap E_g for the case where there is a Coulomb gap, and in arbitrary units for the case without a Coulomb gap. The inset shows the experimental data obtained for Si:B (Ref. 11).

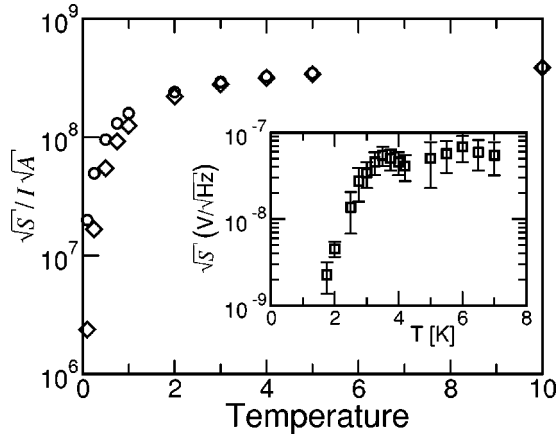

FIG. 6. Noise amplitude $\sqrt{S}$ at $\omega = 10^{-13}\overline{\gamma^0}$ (or $f \sim 1$ Hz) as a function of temperature for the cases with a Coulomb gap ($\diamond$) and without a Coulomb gap ($\bigcirc$). The temperature is measured in units of E_g for the case of a finite Coulomb gap and in arbitrary units in the case of no Coulomb gap. The inset shows the experimental data for $f = 1$ Hz (Ref. 11).

power at low frequencies. This is shown in Fig. 4 that also shows that saturation occurs in the absence of a Coulomb gap when η_o is increased by a factor of 20. This saturation of the noise power occurs because the probability $P_1(x,\varepsilon)$ of finding a site with no neighbors closer than x [see Eq. (24)] decreases exponentially with increasing temperature and with increasing η or η_o. In addition, $P_1(x,\varepsilon)$ becomes exponentially small as x becomes large, and it is the large values of x that contribute to the low-frequency noise. Finally we note that decreasing E_g by a factor of 10 does not produce a noticeable change in the results for either low temperatures ($T = 0.1\ E_g$) or high temperatures ($T = 10\ E_g$). We plot the spectral exponent α in Fig. 5 versus temperature for the cases with and without a Coulomb gap in the density of states. In both cases, we see that it decreases slightly with increasing temperature and eventually saturates in qualitative agreement with the experiment.[11] Figure 6 shows that the noise amplitude $\sqrt{S}$ grows with temperature and eventually saturates, both in good qualitative agreement with the experimental results of Massey and Lee.[11] The data of Massey and Lee span two decades in frequency, while our calculations are able to cover a much broader range. Again we see from Fig. 6 that the presence of a Coulomb gap reduces the noise amplitude at low temperatures. We obtain qualitatively the same results both with and without a Coulomb gap in the density of states which implies that the behavior of the noise spectral density with respect to temperature and frequency is not strongly tied to the hopping exponent δ or to the particular form of the density of states.

We will now discuss some of the physical reasons behind our results. The fact that we obtain $1/f$ noise is perhaps to be expected, since weighted sums over Lorentizians [see Eq. (6)] often result in $1/f$ noise.[1] The subtlety lies in the temperature dependence of the noise amplitude. For simplicity, let us consider the case of density of states with no Coulomb gap, which gives qualitatively the same results as the case with a Coulomb gap. The decrease in the noise amplitude $\sqrt{S}$ with decreasing temperature is due to the presence of activated hopping processes which decrease with decreasing temperature. However, this is not at all obvious from Eq. (26). The integral for the noise power at low frequencies is dominated by large $\tilde{x}$ which corresponds to long relaxation times $\tilde{\tau} \sim \exp(\tilde{x})$. In this case, the factor of $f(\varepsilon)[1-f(\varepsilon)]$ cancels between the numerator and denominator leaving the temperature dependence of the integrand dominated by $P_1(x,\varepsilon)\exp(-\tilde{x})$. $P_1(x,\varepsilon)$ increases while $\exp(-\tilde{x})$ decreases with decreasing temperature. The fact that our calculations yield a decrease in the noise amplitude with decreasing temperature implies that the activated hopping processes associated with $\exp(-\tilde{x})$ dominate. We should mention that experimentally the noise power does not always decrease with decreasing temperature. In some cases, it increases with decreasing temperature,[38,39] but we do not know the differences in the samples which can account for this difference in behavior.

To summarize, recent experiments on $1/f$ noise[11] are consistent with a quasiparticle percolation picture of transport in electron glasses, though this does not exclude multiparticle correlations.

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