

Why Study Noise due to Two Level Systems: A Suggestion for Experimentalists

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Noise is often considered to be a nuisance. Here we argue that it can be a useful probe of fluctuating two level systems in glasses. It can be used to: (1) shed light on whether the fluctuations are correlated or independent events; (2) determine if there is a low temperature glass or phase transition among interacting two level systems, and if the hierarchical or droplet model can be used to describe the glassy phase; and (3) find the lower bound of the two level system relaxation rate without going to ultralow temperatures. Finally we point out that understanding noise due to two level systems is important for technological applications such as quantum qubits that use Josephson junctions.

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1. INTRODUCTION

It is a great pleasure to contribute a paper in honor of Professor Sigfried Hunklinger. I first met Professor Hunklinger in 1986 when I was a postdoc at the University of Illinois in Urbana. Andy Anderson and Jim Wolfe had organized a Phonon Scattering Conference, and due to a last minute cancellation, I was scheduled to give the first contributed talk. Andy made it quite clear that I should do a good job so that the conference would start off on the right foot, so I was a little nervous. Sigfried was the chair of the session. Before the session started, he came up to me, smiled, and introduced himself, saying that he wanted to make sure to pronounce my name correctly. The talk went fine.

The next Phonon Scattering Conference was in Heidelberg in 1989 and Sigfried was one of the organizers. In spite of being busy with running the conference, he managed to stop by my poster where I was presenting some work I had done with Tony Leggett proposing that interactions between

two level systems were important. This was not the prevailing view at the time. As he left, he smiled, waved the paper I had given him, and said, "I believe it." I would like to thank Professor Hunklinger for his encouragement through the years. His enthusiasm has helped to make the field of glasses at low temperatures a fun field.

I have found Professor Hunklinger's papers to be a source of learning and inspiration through the years. I still use his review article with Raychaudhuri¹ as a basic reference to two level systems. His recent work on the effect of magnetic fields provoked a great deal of interest and thoughtful discussion in the field. The following is based on a talk I gave in June 2003 in Dresden at the International Workshop on Collective Phenomena in the Low Temperature Physics of Glasses that Professor Hunklinger helped to organize.

To an experimentalist, noise is a nuisance at best and a serious problem hindering measurements at worst. However noise comes from the fluctuations of microscopic entities and it can act as a probe of what is happening physically at the microscopic scale. Utilizing noise has been done in electronic systems where conductivity can be measured²⁻⁴. In this paper I would like to argue that this should also be done in measurements of the dielectric function in glasses at low temperatures. Let us assume that fluctuations in the dielectric function are due to fluctuations in two level systems making transitions from one state to another. Measuring the noise could help to determine if the fluctuations are correlated or independent. It could also be used to find the lower bound on the two level system (TLS) relaxation rate due to phonon emission, or equivalently, the lower bound Δ_o^{min} on the TLS tunneling matrix element, without going to ultralow temperatures that are of order Δ_o^{min} . There has been speculation that interacting two level systems should undergo a phase transition as the temperature is lowered. Professor Hunklinger was involved in finding experimental evidence suggesting this⁵. Studying the noise may help to determine if there is a glass transition or phase transition of interacting two level systems. If such a transition does occur, the second spectrum of the noise can be used to determine if this phase can be described by the hierarchical or droplet model^{6,7}. (We will explain what a second spectrum is later.) Finally noise due to two level systems could be technologically important. For example, two level systems in the oxide layer that acts as a tunnel barrier in a Josephson junction can have deleterious effects on quantum qubits that use Josephson junctions⁸.

1.1. Introduction to Noise

Let us set up our notation and define what we mean by noise. Let $p(t)$ be a quantity that fluctuates in time. Let $\delta p(t)$ be the deviation from its average value of some quantity p at time t . If the processes producing the fluctuations are stationary in time, i.e., translationally invariant in time, then the autocorrelation function of the fluctuations $\langle \delta p(t_2) \delta p(t_1) \rangle$ will be a function $\psi(t_2 - t_1)$ of the time difference. In this case the Wiener-Khinchine theorem can be used to relate the noise spectral density $S_p(\omega)$ to the Fourier transform $\psi(\omega)$ of the autocorrelation function²: $S_p(\omega) = 2\psi_p(\omega)$ where ω is the angular frequency. In practice $S_p(\omega)$ typically is calculated by multiplying the time series $\delta p(t)$ by a windowing or envelope function so that the time series goes smoothly to zero, Fourier transforming the result, taking the modulus squared, and multiplying by two to obtain the noise power⁹.

1/f noise, which is ubiquitous and dominates at low frequencies, corresponds to $S_p(\omega) \sim 1/\omega$. A simple way to obtain 1/f noise was given by Dutta and Horn³. We can use the relaxation time approximation to write the equation of motion for δp :

$$\frac{d\delta p}{dt} = -\frac{\delta p}{\tau} \quad (1)$$

where τ^{-1} is the relaxation rate. The autocorrelation function $\psi_p(t)$ is given by

$$\psi_p(t) = \langle \delta p(t) \delta p(t=0) \rangle \quad (2)$$

The Fourier transform is a sum of Lorentzians:

$$\psi_p(\omega) = A \int_{-D}^D \frac{\tau(\varepsilon, T)}{1 + \omega^2 \tau^2(\varepsilon, T)} g(\varepsilon, T) d\varepsilon \quad (3)$$

where we assume that the relaxation time τ is a function of energy ε . The range of energies has a bandwidth of $2D$. A is an overall scale factor. If we use $g(\varepsilon, T) = g_o$ for the density of states and $\tau = \tau_o \exp(\varepsilon/k_B T)$ where g_o and τ_o are constants, T is temperature and k_B is Boltzmann's constant, the noise spectral density is given by

$$S(\omega) = 2\psi_p(\omega) \sim \frac{1}{\omega} \quad (4)$$

Thus we obtain 1/f noise from independent fluctuators with a distribution of relaxation times.

1.2. Two Level System Model

The standard model of noninteracting two level systems^{1,10} was introduced by Anderson, Halperin, and Varma¹¹, and independently by W. A. Phillips¹² in 1972. Let us briefly review this model in order to set up the notation that we will use. The standard Hamiltonian for a two level system is

$$H = \frac{1}{2} \begin{pmatrix} \Delta & \Delta_o \\ -\Delta_o & -\Delta \end{pmatrix} \quad (5)$$

Here we are using the left well – right well basis. Δ is the asymmetry energy, i.e., Δ is the energy difference between the right well and the left well. Δ_o is the tunneling matrix element and is given by

$$\Delta_o = \hbar\omega_o e^{-\lambda} \quad (6)$$

where ω_o is the attempt frequency and is typically of order the Debye frequency. The exponent λ is given by

$$\lambda = \sqrt{\frac{2mV}{\hbar^2} - d} \quad (7)$$

where m is the mass of the tunneling entity, V is the barrier height and d is the distance between wells. In the standard model of noninteracting two level systems Δ and λ have flat distributions:

$$P(\Delta, \lambda) d\Delta d\lambda = P_o d\Delta d\lambda \quad (8)$$

where P_o is a constant. The distribution $P(\Delta, \Delta_o)$ is given by

$$P(\Delta, \Delta_o) d\Delta d\Delta_o = \frac{P_o}{\Delta_o} d\Delta d\Delta_o \quad (9)$$

We can diagonalize the Hamiltonian to get the energy eigenvalues that are given by $\pm\varepsilon/2$ where

$$\varepsilon = \sqrt{\Delta^2 + \Delta_o^2} \quad (10)$$

The relaxation rate for an excited two level system to emit a phonon and return to its ground state is given by^{10,1}

$$\tau_1^{-1} = \frac{\gamma^2}{\rho} \left[\frac{1}{c_\ell^5} + \frac{2}{c_t^5} \right] \frac{\varepsilon^3}{2\pi\hbar^4} \left[\frac{\Delta_o}{\varepsilon} \right]^2 \coth \left[\frac{\beta\varepsilon}{2} \right] \quad (11)$$

where γ is the deformation potential, ρ is the mass density, c_ℓ is the longitudinal speed of sound, c_t is the transverse speed of sound, and $\beta = 1/k_B T$.

2. 1/f NOISE DUE TO TWO LEVEL SYSTEMS

2.1. Noninteracting Two Level Systems

Many two level systems have electric dipole moments associated with them. Fluctuating two level systems produce noise in the dielectric polarization. To find the noise power produced by noninteracting two level systems, we can plug the relaxation time from Eq. (11) into Eq. (3):

$$S(\omega) = AP_o \int_{-\Delta_o^{max}}^{\Delta_o^{max}} d\Delta \int_{\Delta_o^{min}}^{\Delta_o^{max}} \frac{d\Delta_o}{\Delta_o} \frac{\tau(\Delta, \Delta_o, T)}{1 + \omega^2 \tau^2(\Delta, \Delta_o, T)} \quad (12)$$

where Δ_o^{max} is the maximum value of the asymmetry energy, and Δ_o^{min} and Δ_o^{max} are the minimum and maximum values of the tunneling matrix element, respectively. The result of evaluating Eq. (12) is shown in Figure 1. At high frequencies, the noise power is 1/f because it can be fit quite well with $S(\omega) \sim \omega^{-0.96}$.

The noise saturates at low frequencies. The saturation frequency where the plot rolls over is given by the inverse of the longest relaxation time τ_{min}^{-1} , and hence depends on Δ_o^{min} and the temperature. In Figure 2 we show a plot of the saturation frequency as a function of Δ_o^{min} . Although we do not have a precise definition of the saturation frequency, we identify it to be the frequency below which the noise power saturates. Figure 2 shows that the saturation frequency increases as Δ_o^{min} increases. Figure 2 also shows the minimum relaxation rate τ_{min}^{-1} calculated using Eq. (11) with $\varepsilon = \Delta_o^{min}$. We see that saturation frequency matches τ_{min}^{-1} as expected.

When averaging over the distribution of parameters for two level systems, Δ_o^{min} is needed as a lower limit to prevent the Δ_o integral from diverging. In addition interactions between two level systems should produce a hole in the distribution of energy splittings¹⁴ which could be interpreted as evidence of the existence of Δ_o^{min} . There has been a long standing question of whether Δ_o^{min} exists and if so, what a reasonable value of it is. Finding whether or not 1/f noise saturates at low frequencies would provide a way to determine Δ_o^{min} without going to ultralow temperatures of order Δ_o^{min} . Of course a very small value of Δ_o^{min} would correspond to extremely low frequencies that may not be experimentally accessible. Typical values are shown in Figure 2.

2.2. Interacting Two Level Systems

Two level systems can interact with one another via strain fields and via dipole-dipole interactions if they have electric dipole moments^{15,14,16}.

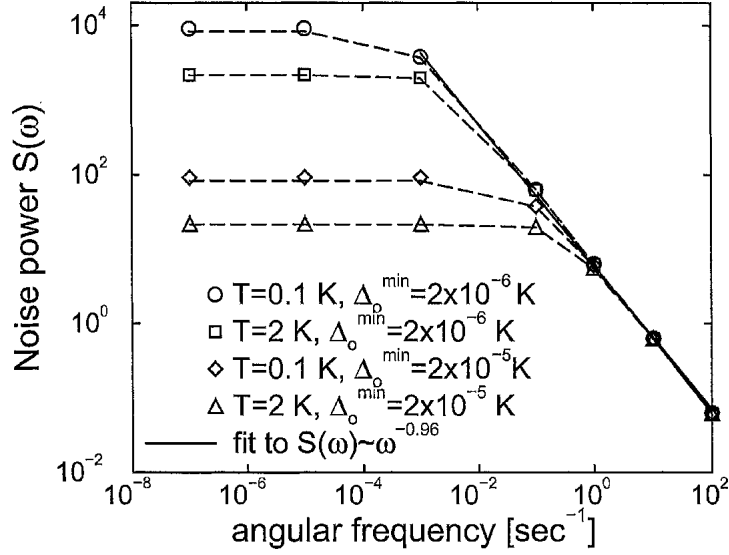


Fig. 1. Log-log plot of noise power vs. frequency ω from two level systems for $T = 0.1$ K and $T = 2$ K, and for $\Delta_o^{\min} = 2 \times 10^{-5}$ K and 2×10^{-6} K. The open symbols represent the spectral density of the noise due to noninteracting two level systems relaxing via phonons. $S(\omega)$ is calculated using Eqs. (11) and (12). The dashed lines show the noise power due to two level systems relaxing via phonons and via interactions with other two level systems. The dashed lines are calculated using Eqs. (12) and (13). The solid line is a fit to $S(\omega) \sim \omega^{-0.96}$ and shows that two level systems produce $1/f$ noise at high frequencies. We have used the values for SiO_2 : $\gamma = 1$ eV, $\rho = 2200$ kg/m³, $c_\ell = 5800$ m/s, $c_t = 3800$ m/s, $\Delta_o^{\max} = 4$ K, $\Delta_o^{\min} = 4$ K. We set the overall scale factor $AP_o = 1$. The values of Δ_o^{\min} are indicated in the figure. For the case of interacting two level systems, we set $T_C = 55$ mK which is the value for Mylar¹³.

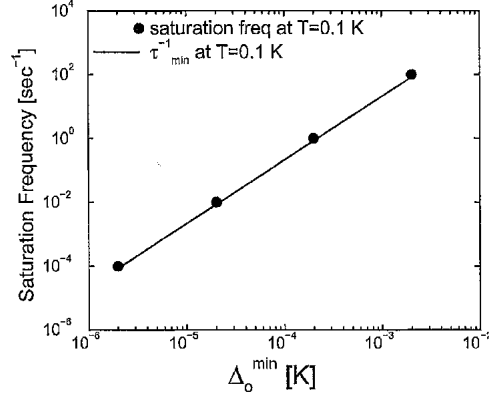


Fig. 2. Log-log plot of saturation frequency vs. Δ_0^{\min} at $T=0.1$ K for noninteracting two level systems. τ_{\min}^{-1} is calculated from Eq. (11) with $\varepsilon = \Delta_0^{\min}$. The values of the parameters are the same as in Figure 1.

So we can ask about the noise spectrum produced by interacting two level systems. There are several ways to approach this. One way would be to allow an excited two level system to relax to its ground state by emitting a phonon or by transferring its excitation energy to other two level systems. In this case the relaxation rate is given by

$$\tau_1^{-1} = \tau_{1,phonons}^{-1} + B_o \left(\frac{\Delta_o}{\varepsilon} \right)^2 k_B T \quad (13)$$

where the second term represents the relaxation rate via energy transfer to other two level systems¹⁷. B_o is a constant given by

$$B_o = \frac{\gamma^2}{\rho} \left[\frac{1}{c_\ell^5} + \frac{2}{c_t^5} \right] \frac{(k_B T_C)^2}{2\pi \hbar^4} \coth(E/k_B T) \Big|_{E=k_B T} \quad (14)$$

where we set $E = k_B T$ in Eq. (14) because the two level systems that obey $E = k_B T$ are the dominant contribution to the dielectric function¹⁸. The reason for this is given by Nalbach *et al.*¹⁸. The resonant dielectric response of two level systems has a thermal occupation factor $\tanh(E/k_B T)$. This implies that at high temperatures ($k_B T > E$) both states are equally populated and there is no net polarization. Thus two level systems with $E \gtrsim k_B T$ dominate. On the other hand, only two level systems with $E \lesssim k_B T$ are accessible. So only two level systems with $E \sim k_B T$ are relevant for changes of the dielectric constant.

T_C is the temperature where the rate of two level system relaxation via phonon emission equals the rate of relaxation via interactions with other two level systems, i.e., where the two terms in Eq. (13) are equal. Plugging Eq. (13) into Eq. (12) by replacing $\tau(\Delta, \Delta_o, T)$ with τ_1^{-1} again yields $1/f$ noise at high frequencies which saturates at low frequencies as shown in Figure 1. From the figure we see that including interactions between two level systems has a negligible effect on the noise. One might expect that at lower temperatures, interactions would have a more pronounced effect, but even when we set $T = 0.02$ K, the spectral density of the noise follows almost the same curve as for $T = 0.1$ K. However, simply substituting Eq. (13) into Eq. (12) is not a correct way to calculate the noise power. The relaxation time in Eq. (3) refers to the relaxation time of a coherent mode, not to the relaxation time of a single two level system interacting with other two level systems. So the correct calculation would involve diagonalizing the system of interacting two level systems to find the relaxation times of different normal modes to the ground state. It is these relaxation times that one should use in Eq. (3).

3. SECOND SPECTRUM OF THE NOISE

We can also look at the so-called second spectrum of the noise. To understand the second spectrum, consider the following. Suppose we take a time series that has 500,000 points in it. We divide the time series into 50 segments, each with 10,000 points in it. Then we calculate the first spectrum $S(\omega)$ of each segment so that we obtain 50 first spectra $S(\omega, t_i)$ where $i = 1, \dots, 50$ and t_i is the starting time of the i th time series. So for a given value of the frequency $\omega = \omega_1$, we have a time series $S(\omega_1, t_i)$ with 50 points. We can Fourier transform this time series to get the second spectrum $S_2(\omega_1, \omega_2)$. The second spectrum is the power spectrum of the fluctuations of $S(\omega)$ with time, i.e., the Fourier transform of the autocorrelation function of the time series of $S(\omega)$ ^{19,6,7}. What we described above is not exactly how the second spectrum is usually calculated. To calculate the second spectrum, we can divide each first spectrum into octaves. An octave is a range of frequencies from ω_L to ω_H where typically $\omega_H = 2\omega_L$. We can discretize the first spectrum by associating each octave with the total noise power in that octave. The total power is the sum of the values $S(\omega)$ with $\omega_L < \omega < \omega_H$. We do this for each first spectrum. For each octave this gives us a time series with one number from each first spectrum labeled by t_i . This time series represents the fluctuations in the noise power in a given octave labeled by frequency ω_L , say. We can calculate the autocorrelation function of these

fluctuations, Fourier transform it and obtain the noise power $S_2(\omega_1, \omega_2)$ that is the second spectrum. Equivalently, by Parseval's theorem, we can Fourier transform the time series of noise power fluctuations, then take the modulus squared and multiply by two to obtain the second spectrum.

Rather than doing a Fourier transform for the first spectrum $S(\omega)$, one can do a simple wavelet transform which is known as a Haar transform²⁰; this is more efficient computationally. Doing a Haar transform effectively multiplies the time series by a square wave. To do a Haar transform, start with a time series. For the lowest frequency square wave, subtract the sum of the first half of the data from the sum of the second half of the data. For the next higher frequency subtract the sum of the first quarter of the data from the sum of the second quarter of the data. And subtract the sum of the third quarter of the data from the sum of the fourth quarter of the data. Keep going until you are subtracting the first point from the second point. Now you have a set of Haar transforms. Square the value of each Haar point to obtain the Haar power. For a given square wave, there are values of the Haar power at different times. This square wave can be associated with a given frequency ω_1 . Fourier transform the values of the Haar power corresponding to a given square wave. The frequency of the Fourier transform is ω_2 . Square the Fourier transform and this gives the second spectrum $S_2(\omega_1, \omega_2)$. It is customary to normalize the second spectrum by dividing by the square of the average of the first spectrum, i.e., the Haar power.

The second spectrum can tell us if the fluctuators are correlated or independent^{4,6,7}. If the second spectrum is white (independent of ω_2) the fluctuators are not correlated. If the fluctuators are correlated, then the second spectrum will be frequency dependent. The frequency dependence of the second spectrum could be used to detect a phase transition among interacting two level systems. If interacting two level systems undergo a glass transition or phase transition, the second spectrum will change from being frequency independent in the high temperature phase to being frequency dependent in the low temperature glassy phase.

We may be able to draw a useful analogy between interacting two level systems in glasses and spin glasses if we identify two level systems with (Ising) spins. Measurements of the second spectrum have been used to differentiate between the hierarchical model and the droplet model of spin glasses because these two models assume different correlations between the fluctuators^{6,7}.

In the droplet model, entire clusters or droplets of spins coherently flip and produce fluctuations in the magnetization²¹⁻²³. The energy for a cluster to flip scales as L^θ where L is the linear size of the droplet and the power θ is small. There are fewer large droplets than small droplets, and the big

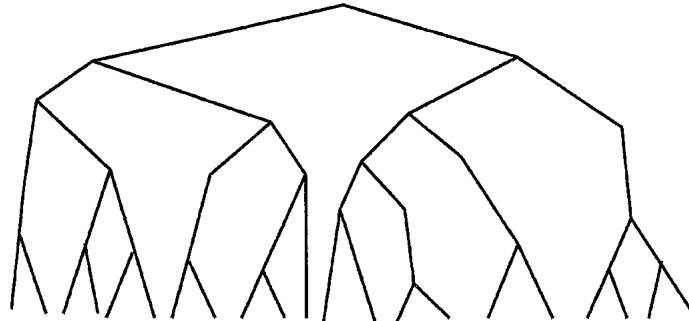


Fig. 3. Bifurcating tree structure of the hierarchical model.

droplets flip less frequently than the small droplets. So large clusters contribute to the low frequency noise and small fast clusters contribute to the high frequency noise. In the simplest version of this model the droplets are noninteracting. If this were the case, the second spectrum would be frequency independent. A more sophisticated version has interacting droplets. Large droplets are more likely to interact than small droplets so the second spectrum will be larger at low frequencies ω_1 .

In the hierarchical model^{24-29,6,7} the states (or spin arrangements) of the spin glass lie at the endpoints of a bifurcating hierarchical tree as shown in Figure 3. The tree structure is self-similar. The Hamming distance D between two states is the fraction of spins that must be reoriented to convert one state into another. It turns out that D corresponds to the highest vertex on the tree along the shortest path connecting the states. The farther two states are, the longer the time to go between them.

It is often useful to plot the second spectrum $S_2(\omega_2, \omega_1)$ as a function of the ratio ω_2/ω_1 . The hierarchical model predicts that the second spectrum should be scale invariant and only depend on the ratio ω_2/ω_1 and not on the frequency ω_1 , while the interacting droplet model predicts that for fixed ω_2/ω_1 , S_2 will be a decreasing function of ω_1 ^{6,7}. A sketch of this is shown in Figure 4. Measurements of the second spectrum of resistance fluctuations in the spin glass CuMn find that its behavior is consistent with the hierarchical model^{6,7}. Measurements have also been done of the second spectrum of the resistance noise in silicon MOSFETs near the metal-insulator transition³⁰. The electron system freezes into a glassy phase that is consistent with the hierarchical picture according to the second spectrum.

It may be possible to use a similar approach with interacting two level systems that are in a glassy phase. By looking at the second spectrum of fluctuations in the dielectric function, it may be possible to distinguish

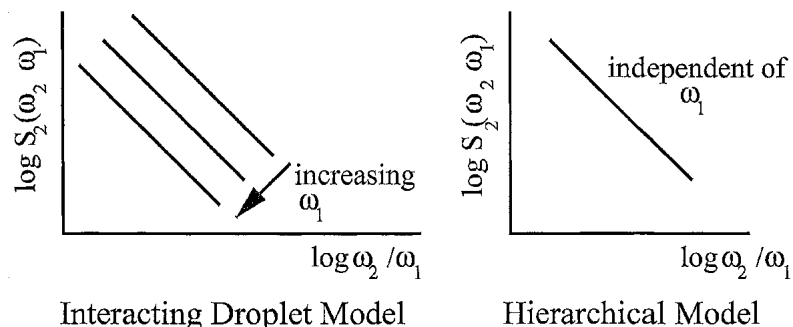


Fig. 4. Sketch of the second spectrum for the interacting droplet model and the hierarchical model as a function of the ratio ω_2/ω_1 for different values of ω_1 .

between the interacting droplet model and the hierarchical model as ways of describing the glassy state.

Experimental measurements of conductance fluctuations in the temperature range from 4–30 K in nanometer-scale samples of amorphous conductors (C-Cu and Si-Au) find second spectra that are non-white, indicating interactions between the fluctuators^{31,32}. Low frequency modulations of the higher frequency noise power indicate interactions between slow and fast fluctuators. The density of fluctuators ($1 \times 10^{17}/\text{cm}^3\text{K}$) is consistent with the concentration of two level systems ($3 \times 10^{17}/\text{cm}^3\text{K}$) estimated from the heat capacity of amorphous insulators³³ and superconducting amorphous metals³⁴.

A different approach to noise and the second spectrum due to interacting two level systems has been given by Nguyen and Girvin³⁵. They consider the dynamics of a model with infinite range spin-spin interactions. In addition, in their model the tunneling barrier height of a two level system is modulated by interactions with other fluctuating two level systems. There are both correlated fluctuations as well as uncorrelated fluctuations in the barrier heights at different sites. The correlated fluctuations cause all the barriers to collectively and simultaneously increase or decrease in a correlated manner. They speculate that such correlated fluctuations could be produced by the elastic interactions between two level systems. In the thermodynamic limit only the correlated fluctuations survive the central limit theorem and produce a non-white, non-Gaussian second spectrum. The other terms produce only Gaussian fluctuations corresponding to a frequency independent second spectrum.

4. JOSEPHSON JUNCTION QUBITS

Noise due to fluctuating two level systems has attracted the attention of those working to make quantum computing qubits, especially those researchers who are using Josephson junctions in their qubit design. The Josephson junction qubit is a leading candidate in the design of a quantum computer, with several experiments recently demonstrating single qubit preparation, manipulation, and measurement³⁶⁻³⁹, as well as the coupling of qubits to each other^{40,41}. A significant advantage of this approach is scalability, as these qubits may be readily fabricated in large numbers using integrated-circuit technology. A major obstacle to the realization of quantum computers with Josephson junction qubits is decoherence.

There are a number of ways to design a qubit involving Josephson junctions. Perhaps the simplest one is a phase qubit³⁸ which is a current biased Josephson junction. The $|0\rangle$ and $|1\rangle$ states of the qubit are simply the lowest 2 states in one of the potential wells of the washboard potential associated with the current biased Josephson junction. Another design is that of the flux qubit⁴². The simplest model of a flux qubit is an rf SQUID which has one Josephson junction. In this case the $|0\rangle$ and $|1\rangle$ states of the qubit are simply the lowest 2 states in the shallower potential wells of the double well potential of the flux biased SQUID. The third type of JJ qubit is a charge qubit in which a tiny superconducting island, the "Cooper pair box", is coupled to a superconducting reservoir via a Josephson junction. The qubit states correspond to charge states in the Cooper pair box that differ by one electron pair⁴³. For any of these designs the wavefunction ψ of the qubit is a coherent superposition of these two states:

$$\psi = \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} e^{i\phi} |1\rangle \quad (15)$$

Anything that leads to decoherence of this superposition is an anathema to operation of the qubit.

One source of decoherence are two level systems sitting in the insulating oxide tunnel barrier of the Josephson junction. Fluctuations in a two level system produces fluctuations in the barrier height which in turn produce fluctuations in the tunneling matrix element through the oxide barrier and the critical current I_o ⁸. The energy splitting of a phase qubit depends on the size of the critical current, and so noise in the critical current leads to fluctuations in the energy splitting of the qubit as well to dephasing. Since fluctuations in a two level system can lead to fluctuations in the qubit energy splitting, the two level system and qubit are coupled. This coupled system has four energy levels. If the qubit and two level system have the same energy splittings when they are uncoupled, then the coupling between the

qubit and two level system will split the degeneracy. Such an energy splitting has been observed experimentally⁸.

5. CONCLUSIONS

The point of this paper is to urge experimentalists to measure the noise in their measurements of the low temperature properties of glasses. Noise coming from fluctuating two level systems can further our understanding of two level systems in a number of ways. These include determining if the fluctuations are correlated or independent, determining if the two level systems undergo a glass transition or a phase transition with decreasing temperature, and if so, what model best describes the glassy state, and determining the lower bound of the two level system relaxation rate (or Δ_o^{min}). In addition understanding the noise produced by two level systems is important for technological applications such as Josephson junction devices and qubits that are made from them.

Most experimental measurements of noise involve electrical measurements of resistance or current or voltage. Measurements of conductance fluctuations in amorphous metals indicate that two level systems interact^{31,32}. For insulating glasses one would measure noise in the dielectric polarization. Measurements of the noise in the dielectric function are quite challenging experimentally, but I think they would be well worth the effort. Measurements of dielectric fluctuations have been done near the molecular glass transition of polyvinylacetate using atomic force microscopy techniques that are sensitive to the local dielectric polarization of the sample⁴⁴.

Finally let me wish Professor Hunklinger "A Very Happy 65th Birthday!".

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