Chapter 6: Wave Nature of Matter

- Classically, light is a wave. Quantum mechanics says it’s also particles (photons).
- Classically, an electron is a particle. Louis De Broglie proposed it’s also a wave.
- Wave-particle duality.

De Broglie's Hypothesis

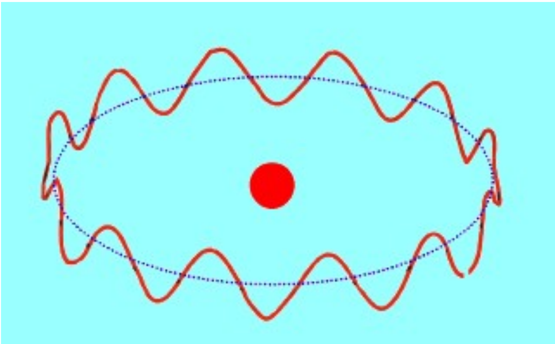
- De Broglie relations

$$E = hf \qquad \text{and} \qquad p = \frac{h}{\lambda}$$

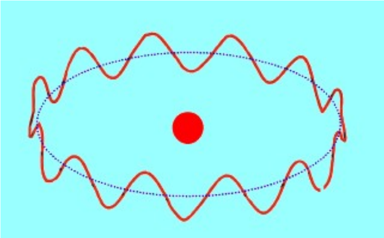
- These apply to light. De Broglie argued they applied to particles.
- Obviously frequency $f=c/\lambda$ applies to light but not to electrons since matter does not travel at velocity c.

Electron Standing Waves

- To explain quantization of electron orbits, De Broglie postulated electrons in orbits formed standing waves.
- http://www.youtube.com/watch?v=70OgGm2U9vU



Electron Standing Waves



- Circumference (2πr) of electron orbit is an integer number of wavelengths

$$2\pi r = n\lambda \quad \text{where} \quad n = 1,2,3\ldots$$

$$\lambda = \frac{h}{p} \quad (\text{De Broglie relation } p = \frac{h}{\lambda})$$

$$2\pi r = \frac{nh}{p}$$

$$rp = \frac{nh}{2\pi}$$

- Orbital angular momentum L = r x p, or |L|=rp, so we obtain Bohr's quantization of L

$$\boxed{L = \frac{nh}{2\pi} = n\hbar}$$

- Caveat: orbits are not 1D

Wavelength of Electrons

- Find the wavelengths of electrons with energies K=10, 100, 1000, and 10,000 eV.

$$\lambda = \frac{h}{p}$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{p^2}{m}$$

$$p = \sqrt{2mK}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{hc}{\sqrt{2mc^2K}}$$

Bigger m, shorter λ

- For K = 10 eV and mc^2 = 0.51 MeV:

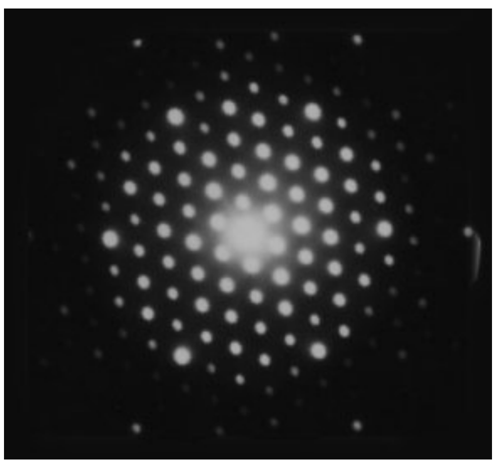
$$\lambda = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240 \text{ eV-nm}}{\sqrt{2\times(0.51\times10^6 \text{ eV})\times(10 \text{ eV})}} = 0.39 \text{ nm}$$

K (eV)	10	100	1000	10,000
λ (nm)	0.39	0.12	0.039	0.012

- Electron wavelengths << visible light (400 – 700 nm)

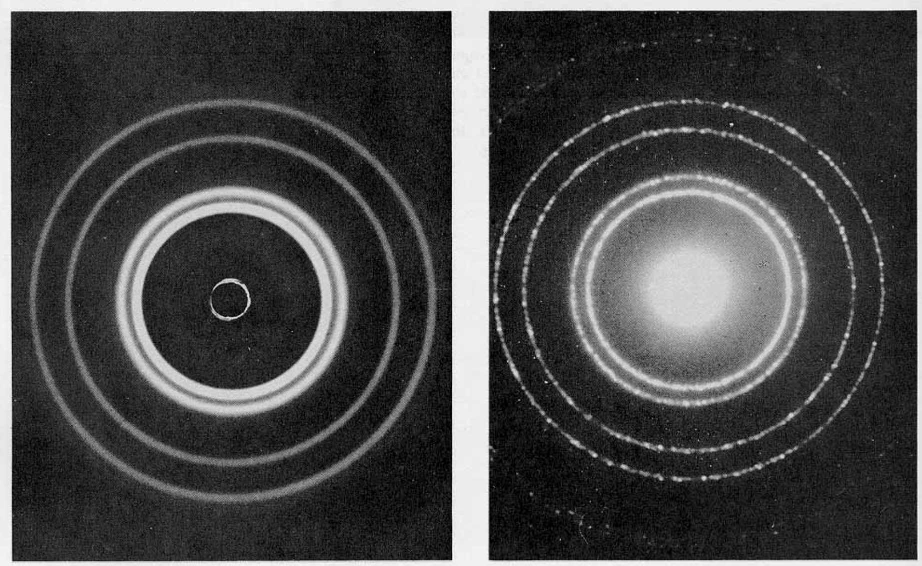
Experimental Evidence of Wave Nature of Electrons

- Electron diffraction by a crystal (Davisson and Germer; Thomson)
- <u>L</u>ow-<u>E</u>nergy <u>E</u>lectron <u>D</u>iffraction (LEED) used to study surface properties
- Neutron diffraction used to study structure of solids



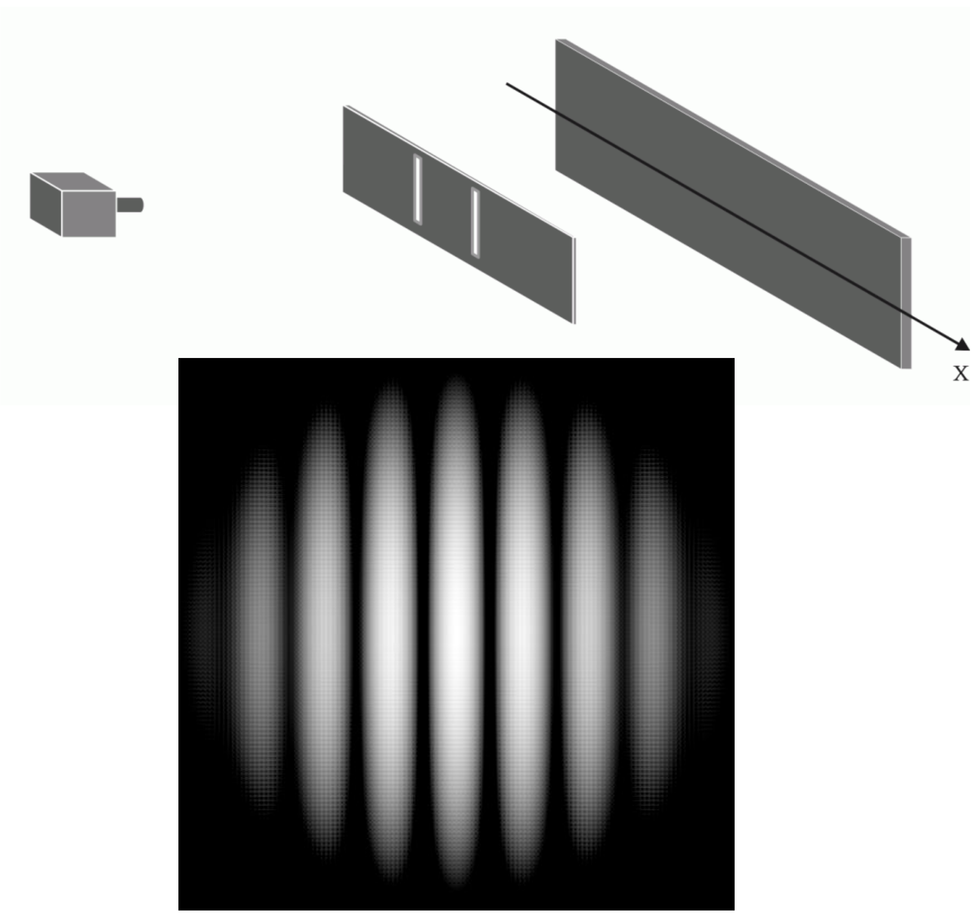
Electron Diffraction from Crystal

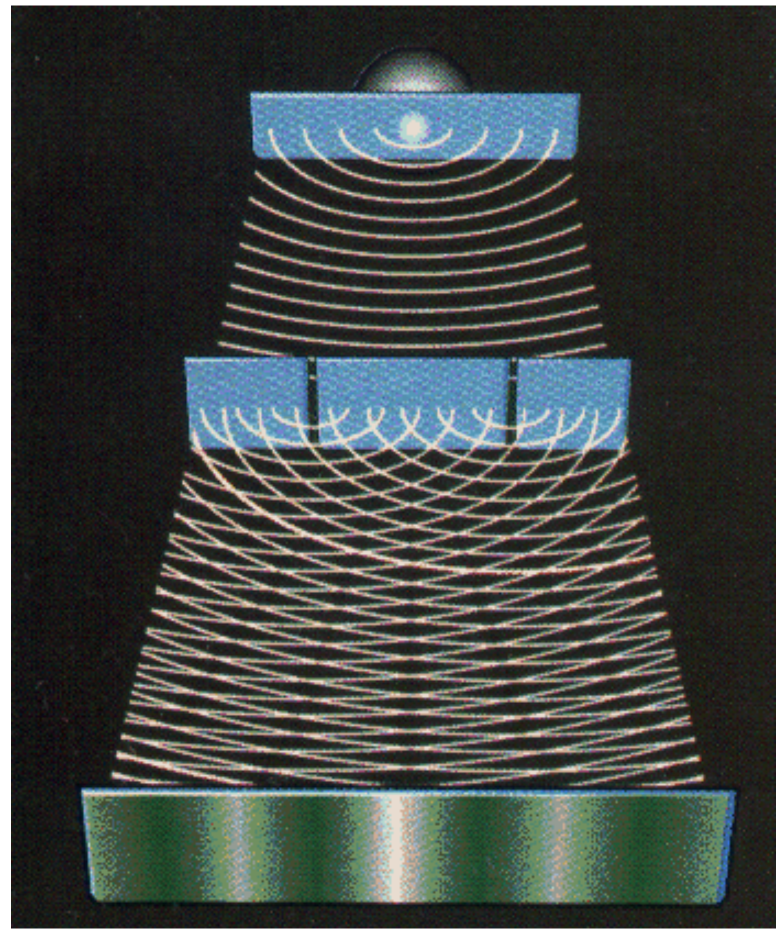
The diffraction pattern on the left was made by a beam of x rays passing through thin aluminum foil. The diffraction pattern on the right was made by a beam of electrons passing through the same foil.



Experimental Evidence of Wave Nature of Electrons

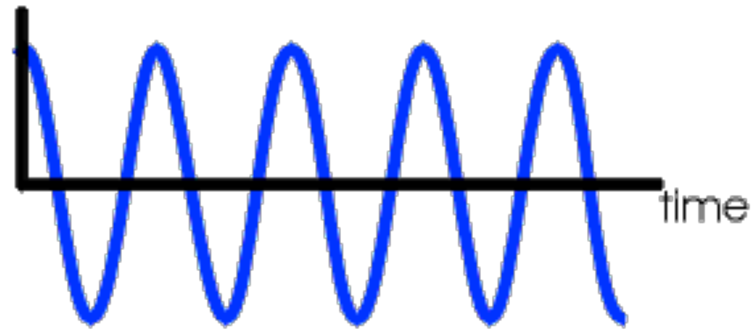
- 2-slit experiment with electron waves produces interference pattern.
- http://www.youtube.com/watch?v=DfPeprQ7oGc



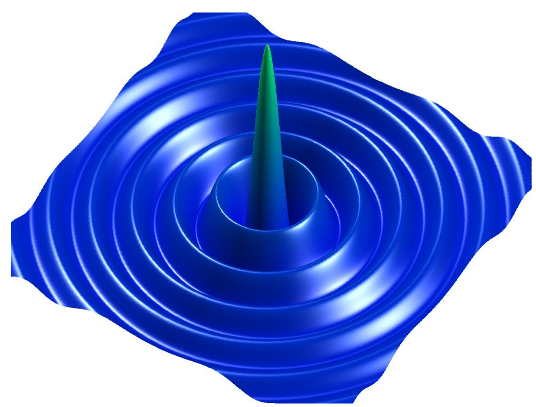


Waves

1D Wave:
Displacement is described by a function f(x,t)



2D Wave: Displacement is described by a function g(x,y, t)



Quantum Wave Function

- See board notes

Wave Packet is Sum of Waves

- Waves can add constructively or destructively.
- Waves with different wavelengths sum to give a wave packet with a finite extent in space.

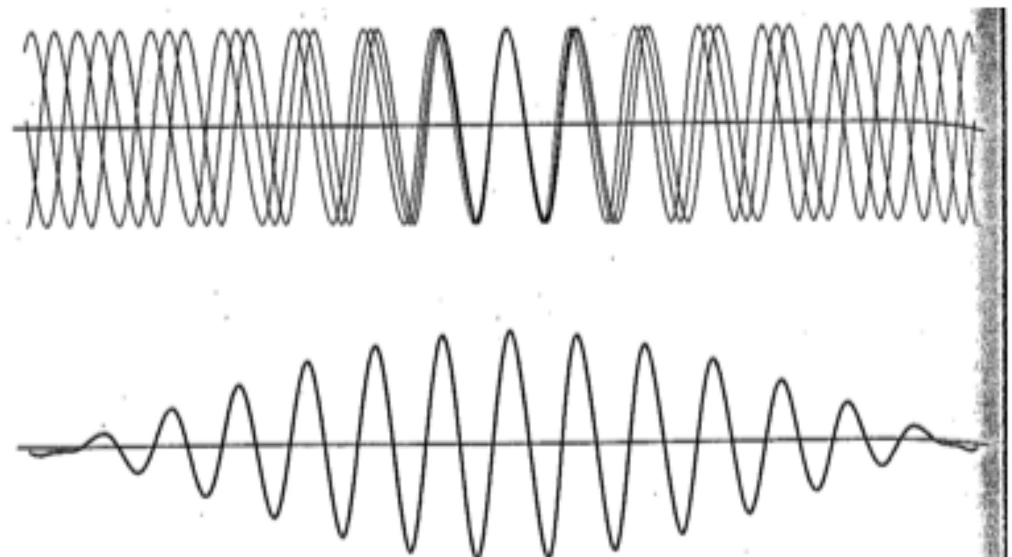


FIGURE 6.13
A wave packet (below) is made up of a series of sinusoidal waves of various wavelengths (above).

Wave Packet Spread

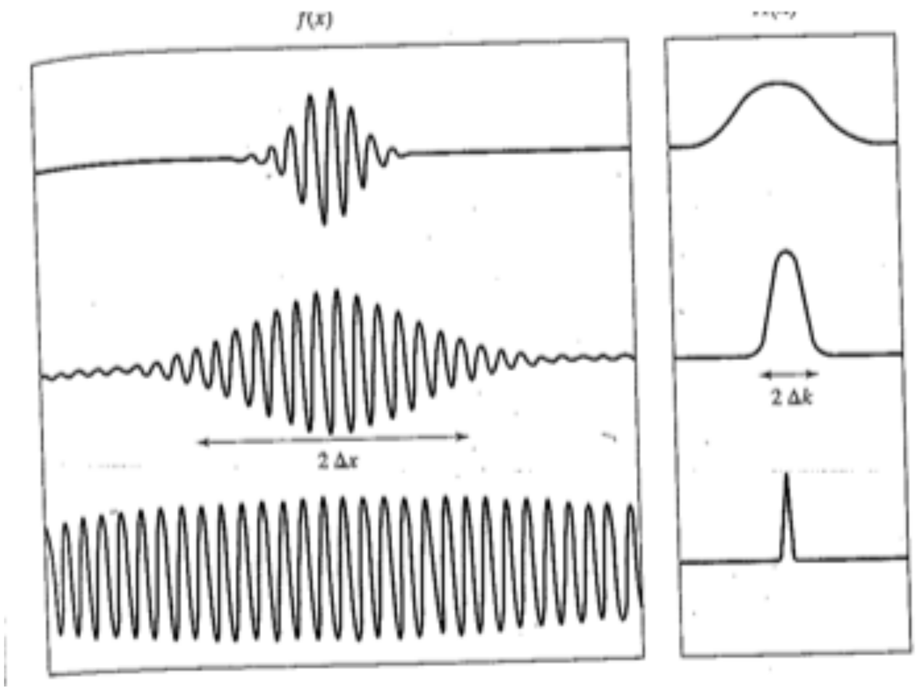


FIGURE 6.14
A narrow wave packet (small Δx) corresponds to a large spread of wavelengths (large Δk). A wide wave packet (large Δx) corresponds to a small spread of wavelengths (small Δk).

Δx Δk

2 Cosine Waves

- 2 Cosine waves with different wavelengths
- Wavelengths differ by $2\Delta\lambda$
- Waves sum near $x=0$ (interfere constructively)
- Waves cancel (interfere destructively: $2N\Delta\lambda=\lambda/2$) after N cycles around $x=N\lambda$

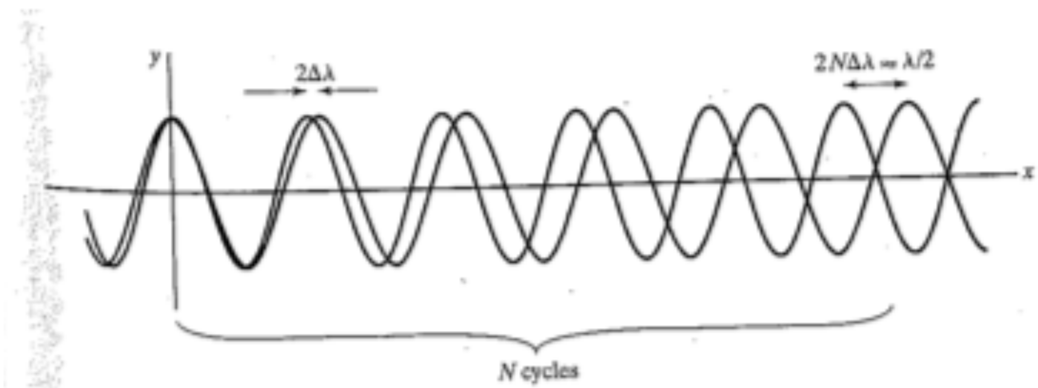


FIGURE 6.15
Two cosine functions with wavelengths differing by 2 Δλ.