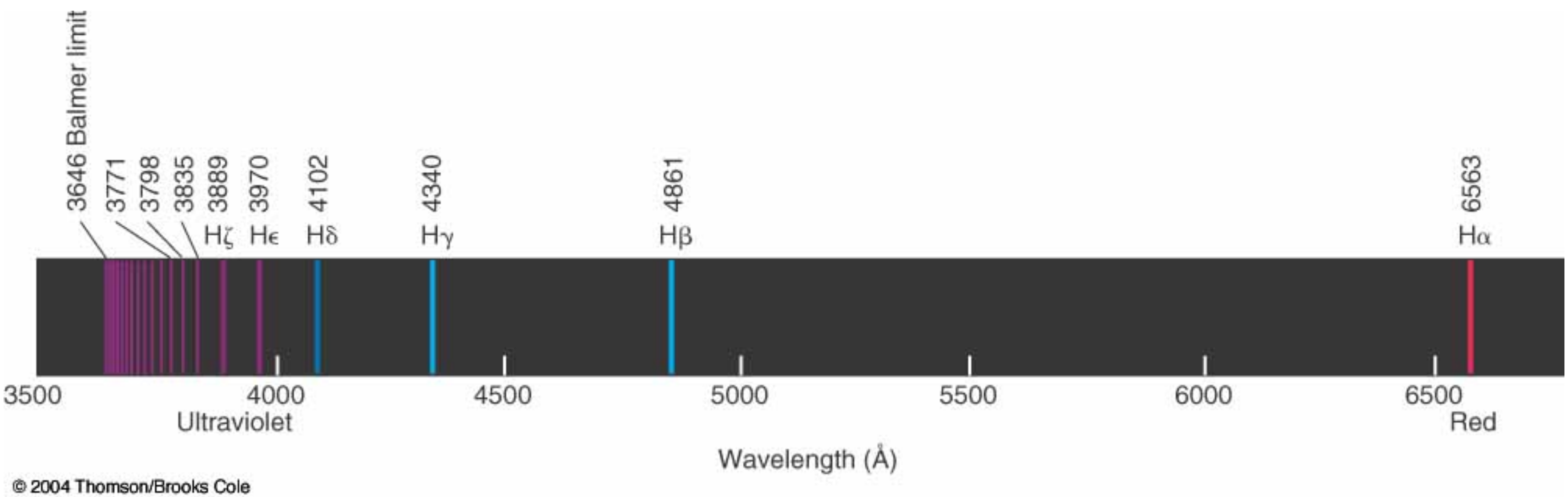
Chapter 5: Quantization of Atomic Energy Levels

- Chapter 4 was quantization of light
- This is now quantization of electron energies in atoms.
- Electrons cannot have any energy. Electrons have energy levels.
- By early 1900s it was understood that classical physics didn’t explain certain elements of atomic spectra.
- Atomic spectra were explained empirically by the Bohr model of the atom before a full theory of quantum mechanics was developed.

Emission spectrum of hydrogen

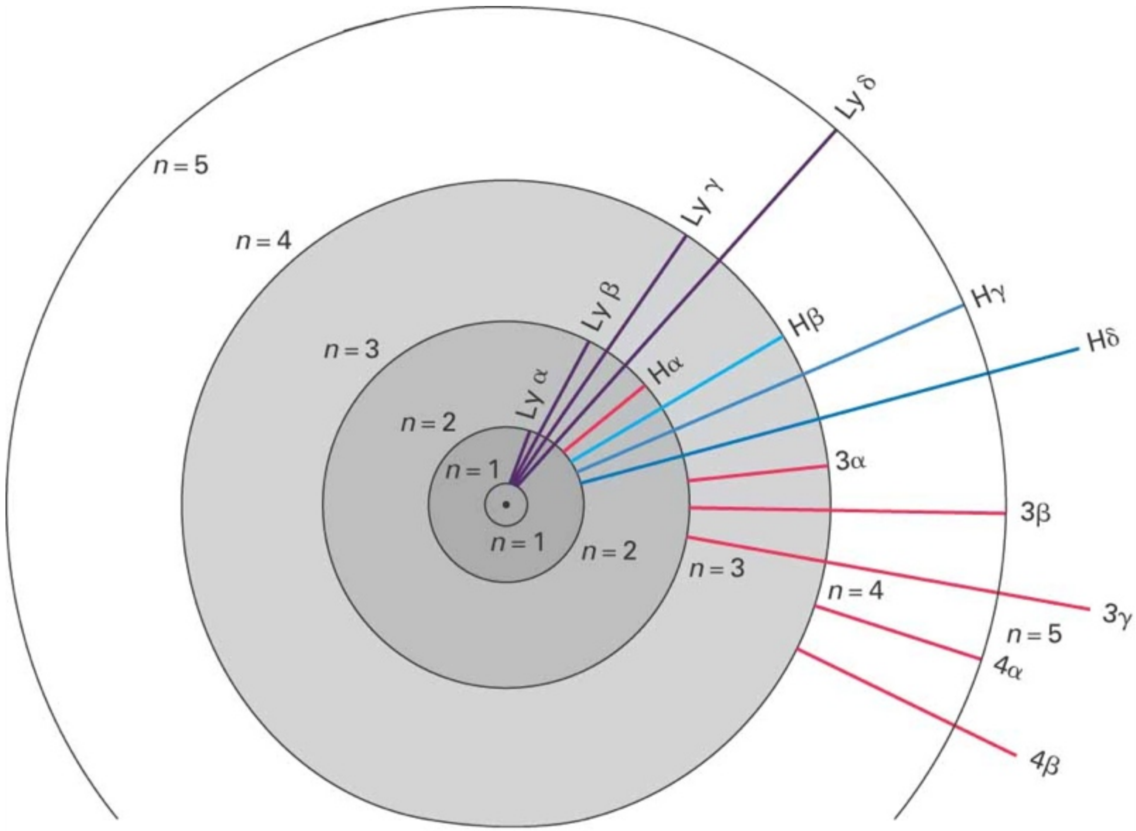
Each element, like hydrogen, has a characteristic spectrum. It can be seen in emission or absorption.



Discharge Tube That Produces Spectra

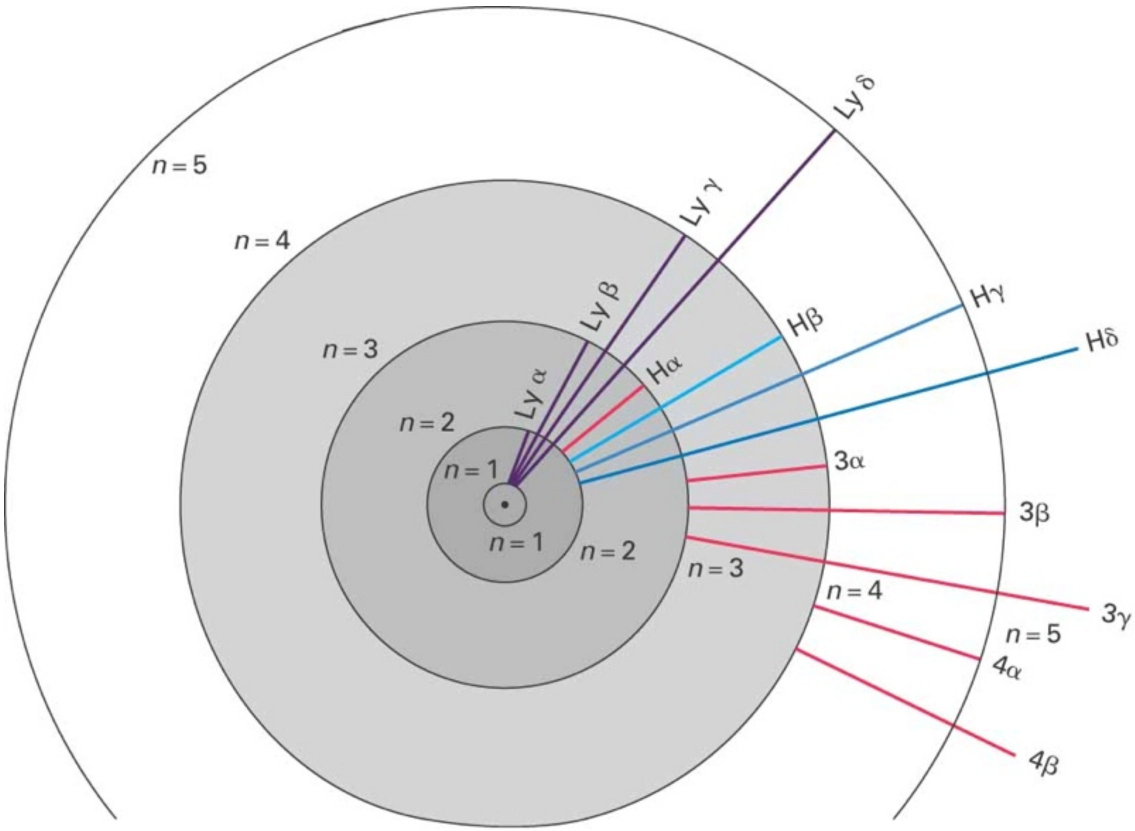
- http://phet.colorado.edu/en/simulation/discharge-lamps

The Bohr Model of the Atom



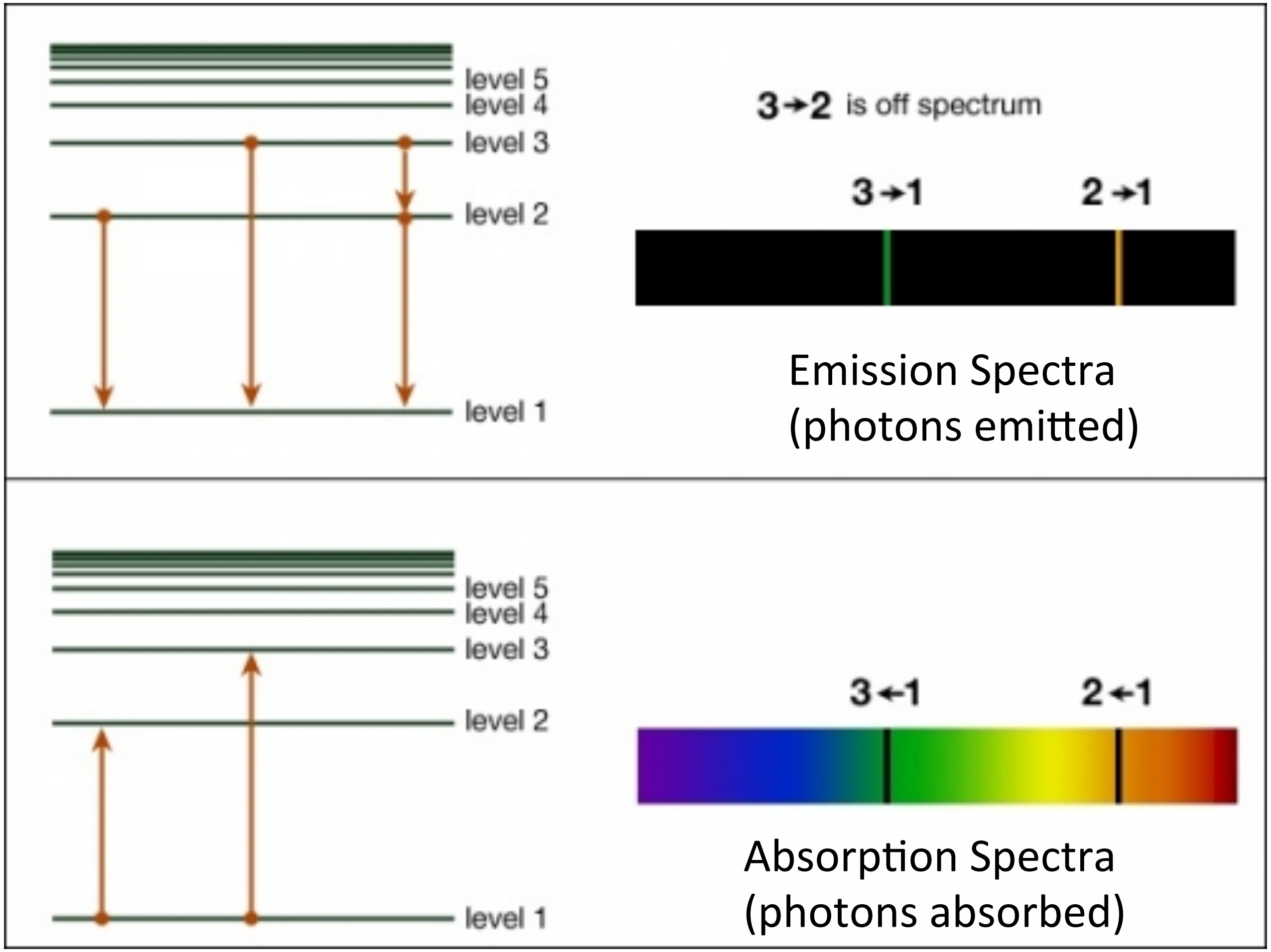
- Electron can only be in certain orbits around atom
- Larger radii = higher energy
- Must absorb photon to jump up, emit photon to jump down

The Bohr Model and the Atom

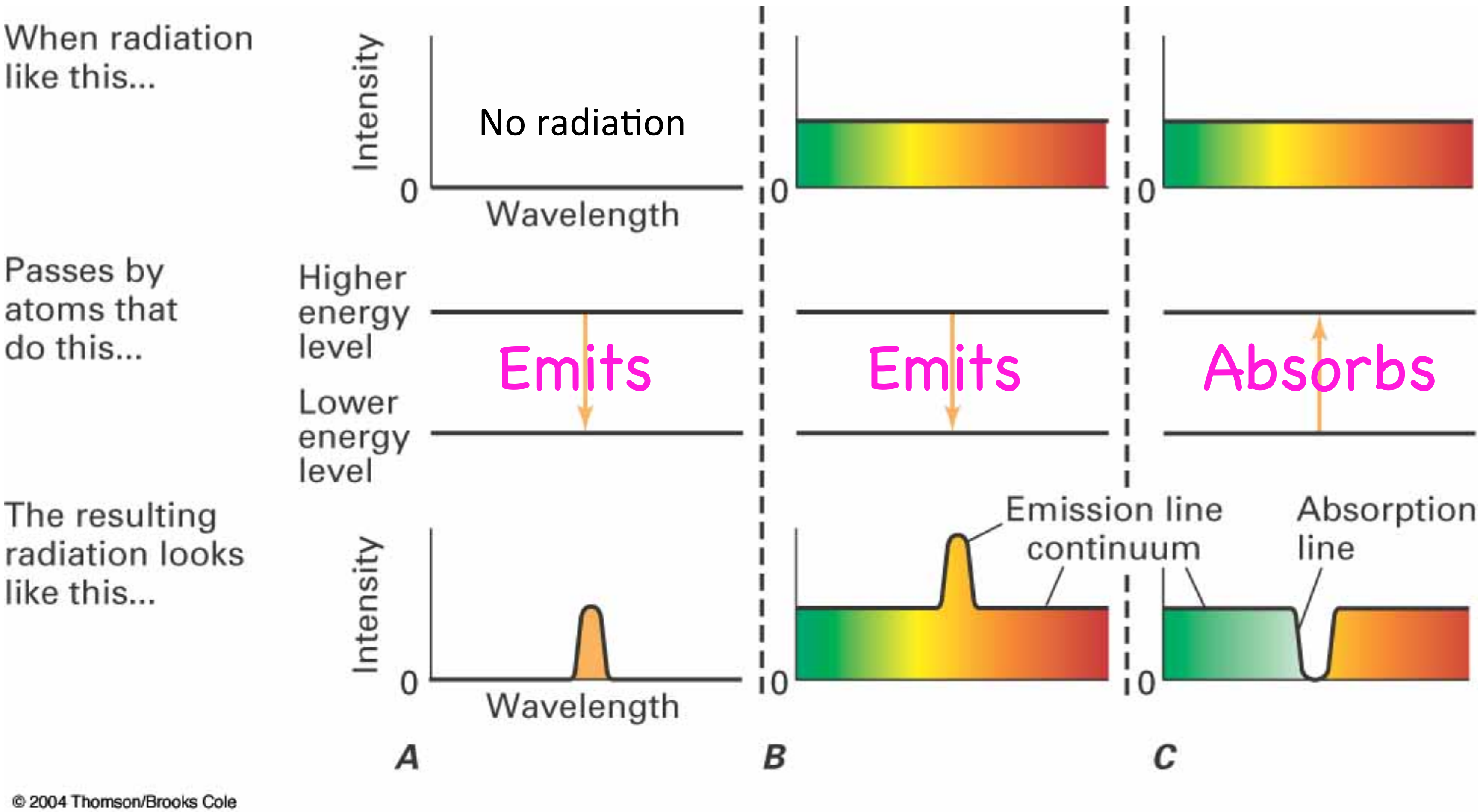


- Energy of emitted or absorbed photon corresponds to change of electron energy due to transition.
- Energy "quantized"
- Photon energy corresponds to certain frequency or wavelength. ($E=hf = hc/\lambda$) Certain wavelengths correspond to certain electron transitions.

Spectra of Hydrogen



Emission and absorption lines



Interaction of photons and matter

- Matter absorbs and emits photons at specific energies (and wavelengths), corresponding to specific electron transitions.

- The "spectrum of hydrogen" is a bunch of lines at specific frequencies corresponding to these transitions of electrons between energy levels.

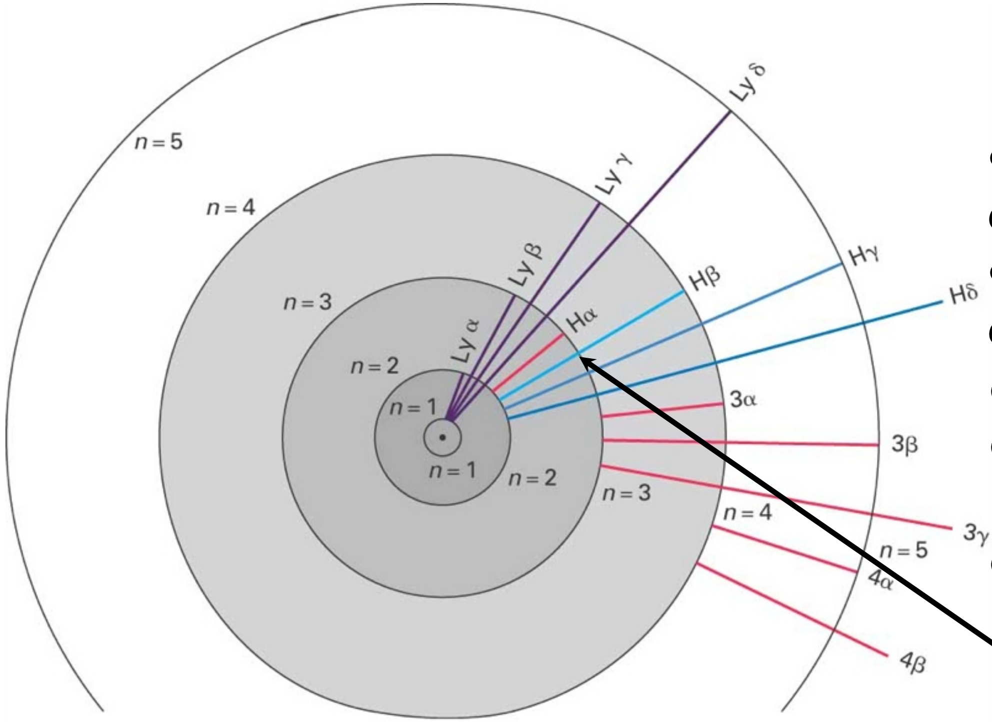
- True for atoms, ions, molecules: have specific spectral lines (simple or complex depending on range of energy transitions available)

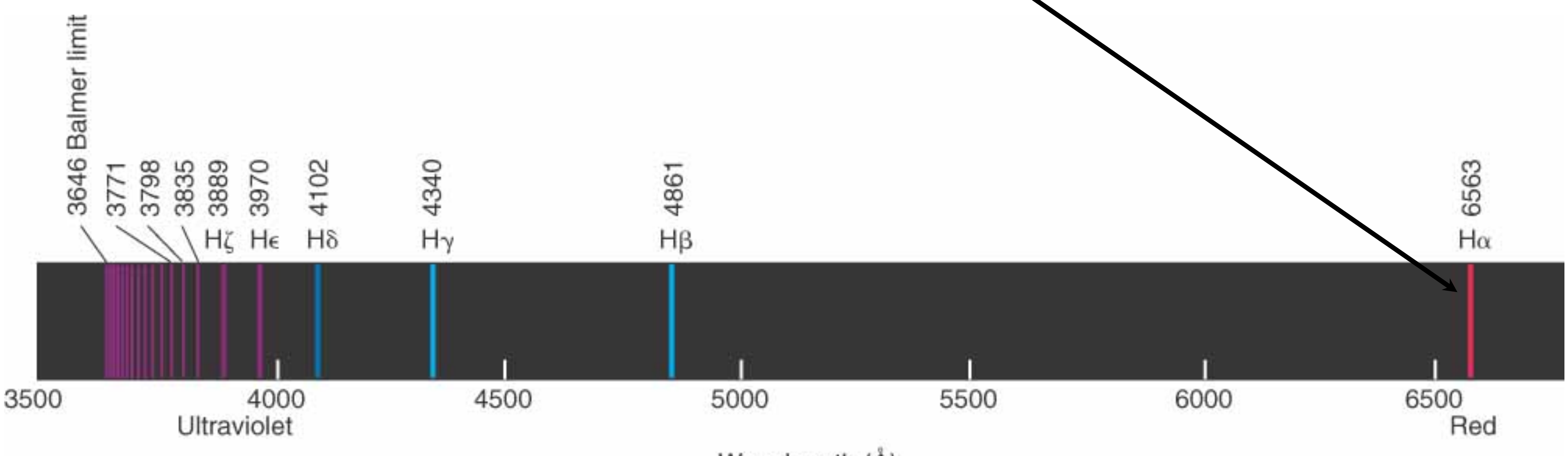
Bohr Model

- Empirical model: electrons occupy discrete energy levels in atoms.
- Electrons lose energy when they "jump down" or make a transition to a lower energy level, and emit photons with energy, freq.:

$$E=E_2-E_1=hf,$$

e.g., for a jump from 2 to 1





iquestion

- An atom at rest emits a photon at time $t=0$. Which of the following are true?
  - I. The atom has less internal energy at $t<0$ than at $t>0$.
  - II. The atom has more internal energy at $t<0$ than at $t>0$.
  - III. The atom recoils with momentum h/λ, where λ is the wavelength of the photon.

  a. I only.
  b. I and III only.
  c. II only.
  d. II and III only.
  e. I, II, and III.

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Balmer-Rydberg formula for hydrogen

- Hydrogen is first to be modeled and analyzed.

Wavelengths emitted by hydrogen observed by Balmer in optical region satisfy this formula:

$$1/\lambda = R(1/2^2 - 1/n^2)$$

where $n=3,4,....$ and $R=0.0110\ \mathrm{nm}^{-1}$

Later generalized by Rydberg to:

$$1/\lambda = R[(1/n'^2) - (1/n^2)],$$

Where $n' < n$, both integers

Balmer-Rydberg formula for hydrogen

- What is the wavelength of a photon **emitted** when an electron jumps from the n=3 state to the n=2 state?:

  $1/\lambda = (0.0110\ \text{nm}^{-1}) \times (1/2^2 - 1/3^2)$

  $\lambda = 654.5$ nm ("true" value 656.3 nm; close enough)

- What is the wavelength of a photon that is **absorbed** when an electron jumps from the ground (n=1) state to the n=4 state of hydrogen?

  $1/\lambda = (0.0110\ \text{nm}^{-1}) \times (1/1^2 - 1/4^2)$

  $\lambda = 97.0$ nm

- Note: the formula was only **empirical**, with no explanation.

Problem of Atomic Stability

- Rutherford’s “nuclear model” had electron circling nucleus like miniature solar system.
- Classical electrodynamics says a proton and an electron will be attracted and crash into each other. Not a good way to make an atom.
- Classical electrodynamics says that charged particle that is accelerating (velocity changes direction and/or magnitude) should radiate electromagnetic waves, lose energy, ultimately spiral into nucleus
- Approximate timescale is 10^{-11} s!!! Electron should crash into the nucleus in this short a time.
- Obviously, something is preventing this from happening...

Bohr Model

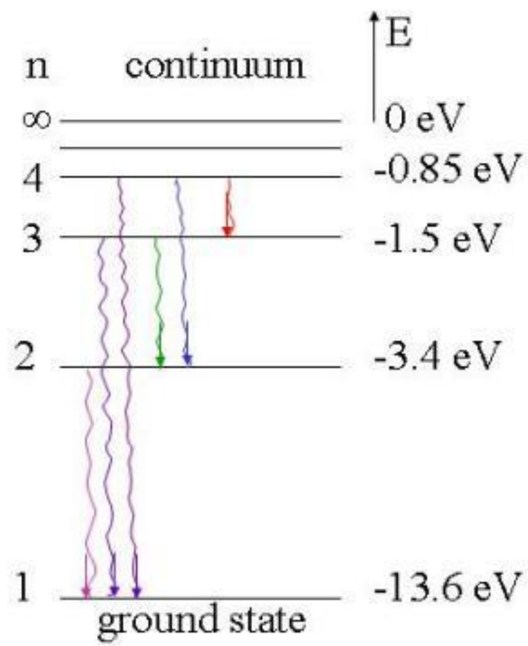
- Only certain discrete orbits are allowed for an electron; called “stationary orbits” or “stationary states”
- Energies are also discrete or quantized; only possible set is E_1, E_2, E_3, E_4, ...
- Bohr **postulated** that electrons in these stationary states remain there, without losing energy, until disturbed
- This postulate is pretty close to what modern quantum mechanics says, without the “orbit” (or with a slightly more complex version of orbiting)
- For transition from n down to n´, photons are emitted with energies $hf = E_n - E_{n'}$
- For transition from n´ up to n, photons are absorbed with energies $hf = E_n - E_{n'}$

The Bohr Model: quantitatively

- See board notes.

Hydrogen Energy Levels

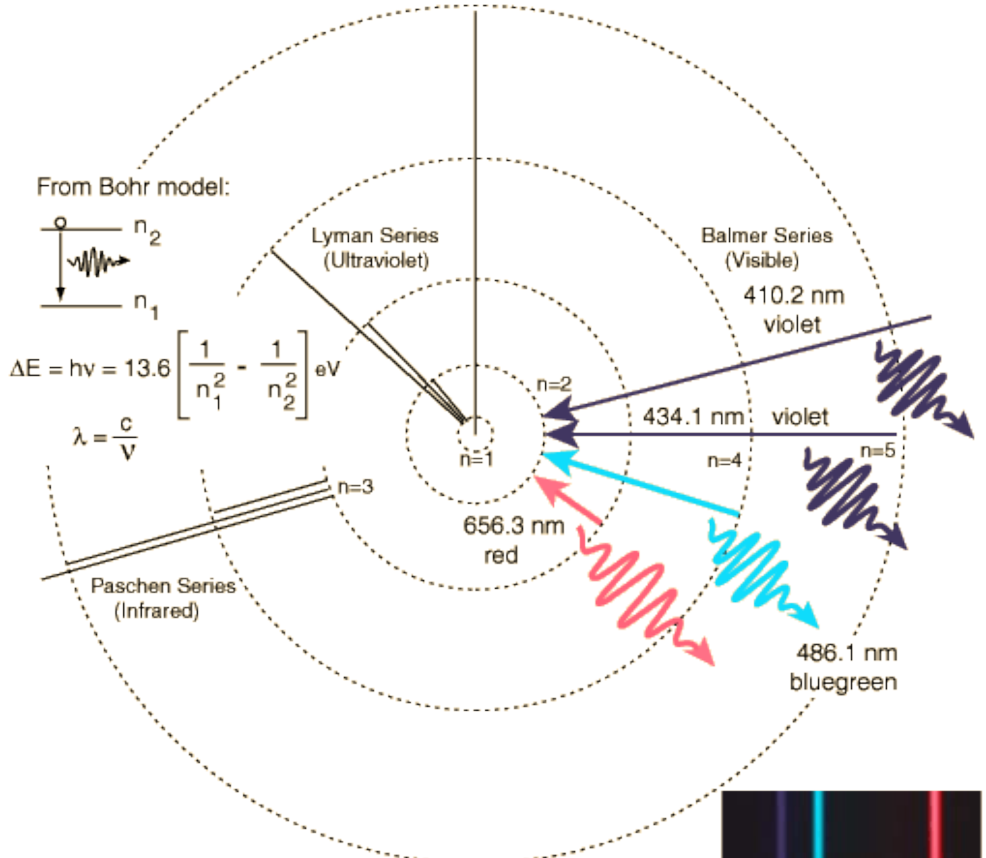
Energy levels of H



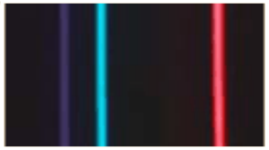
$$E_n = -\frac{E_R}{n^2}$$

$$E_R = \frac{ke^2}{2a_B} = 13.6 \text{ eV}$$

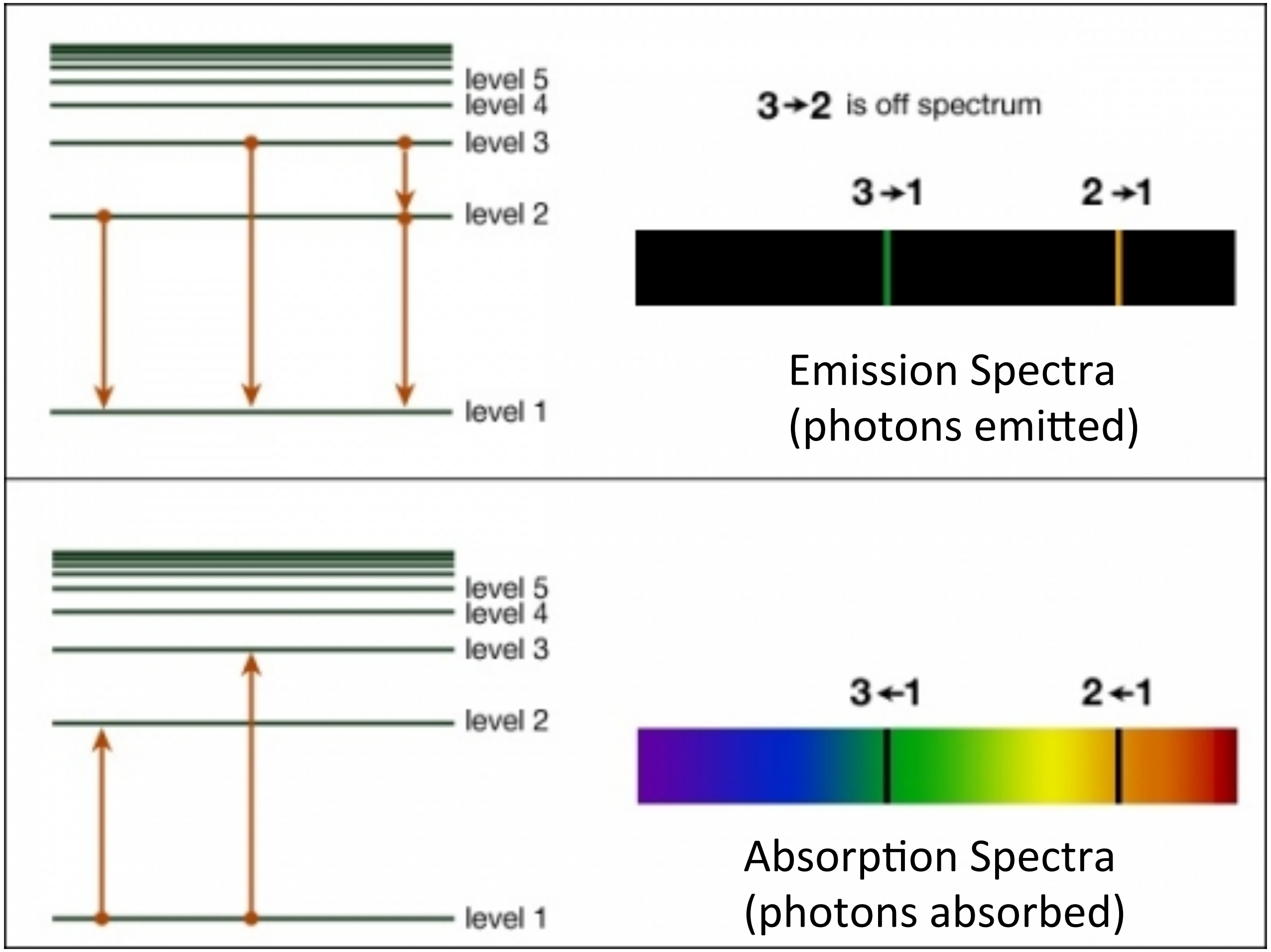
Bohr Model



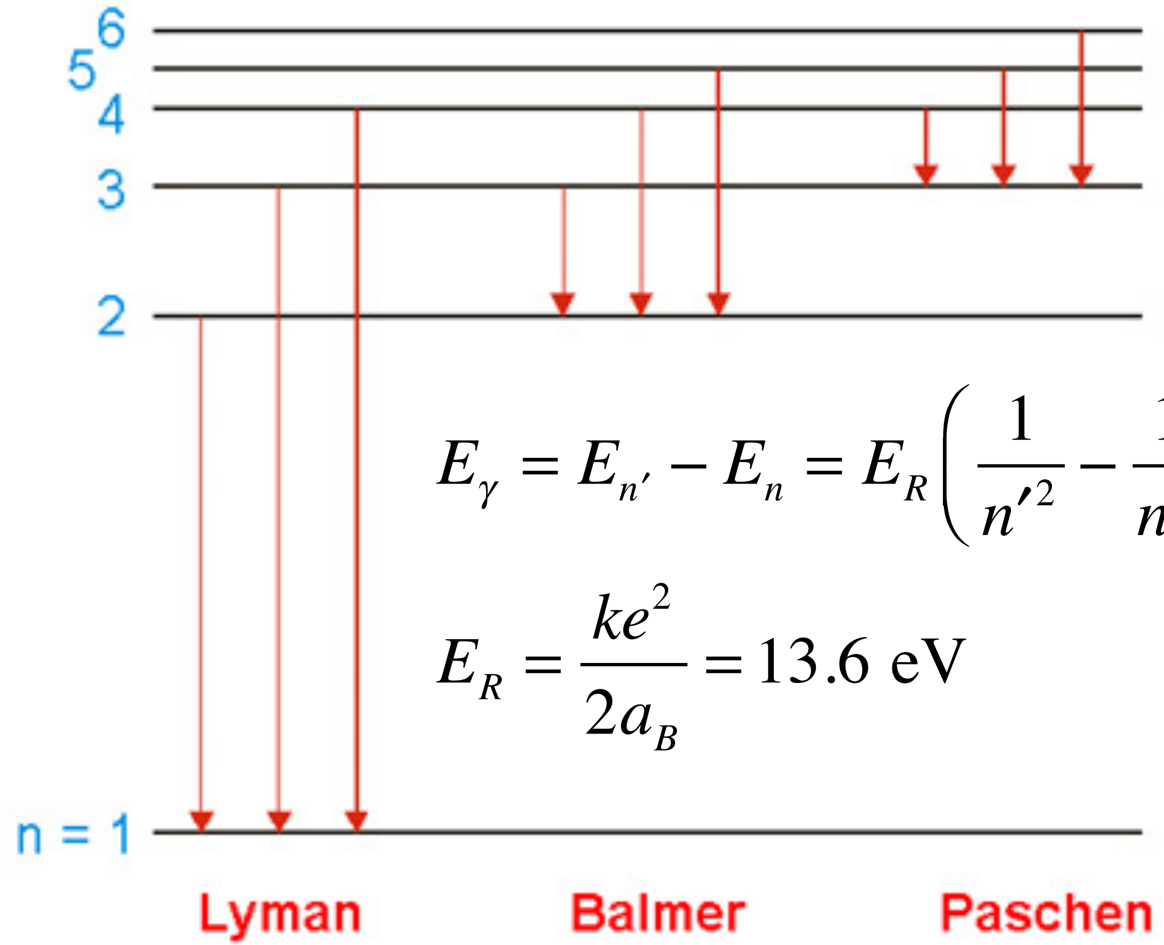
From Bohr model:
n_2
n_1
$\Delta E = h\nu = 13.6 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \text{eV}$
$\lambda = \frac{c}{\nu}$
Lyman Series
(Ultraviolet)
Balmer Series
(Visible)
410.2 nm
violet
434.1 nm violet
n=1
n=2
n=3
n=4
n=5
656.3 nm
red
486.1 nm
bluegreen
Paschen Series
(Infrared)



Spectra of Hydrogen



Hydrogen Spectra



$$E_\gamma = E_{n'} - E_n = E_R\left(\frac{1}{n'^2} - \frac{1}{n^2}\right)$$

$$E_R = \frac{ke^2}{2a_B} = 13.6 \text{ eV}$$

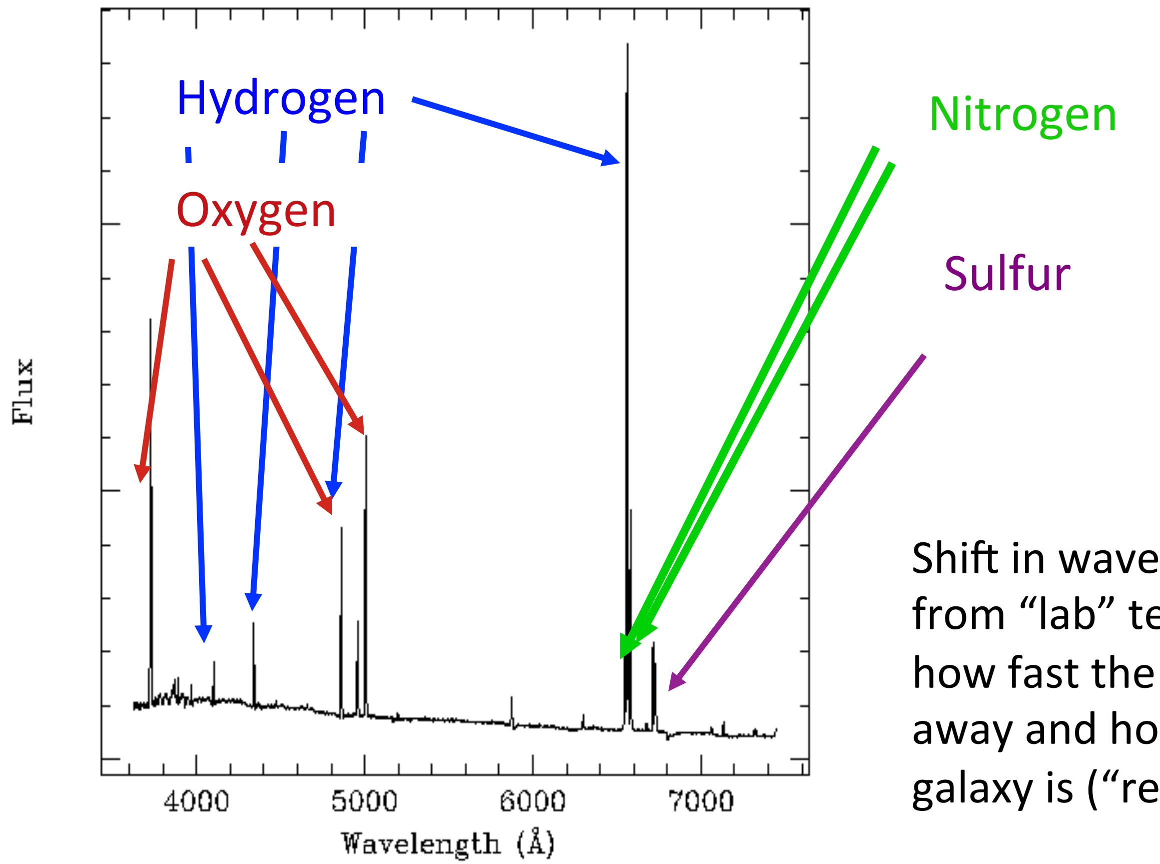
Spectra of Elements

- http://jersey.uoregon.edu/vlab/elements/Elements.html

Spectra in astrophysics

- These lines are one of our primary diagnostic tools
- They help us determine the compositions of galaxies that are many millions, or even billions, of light years away.

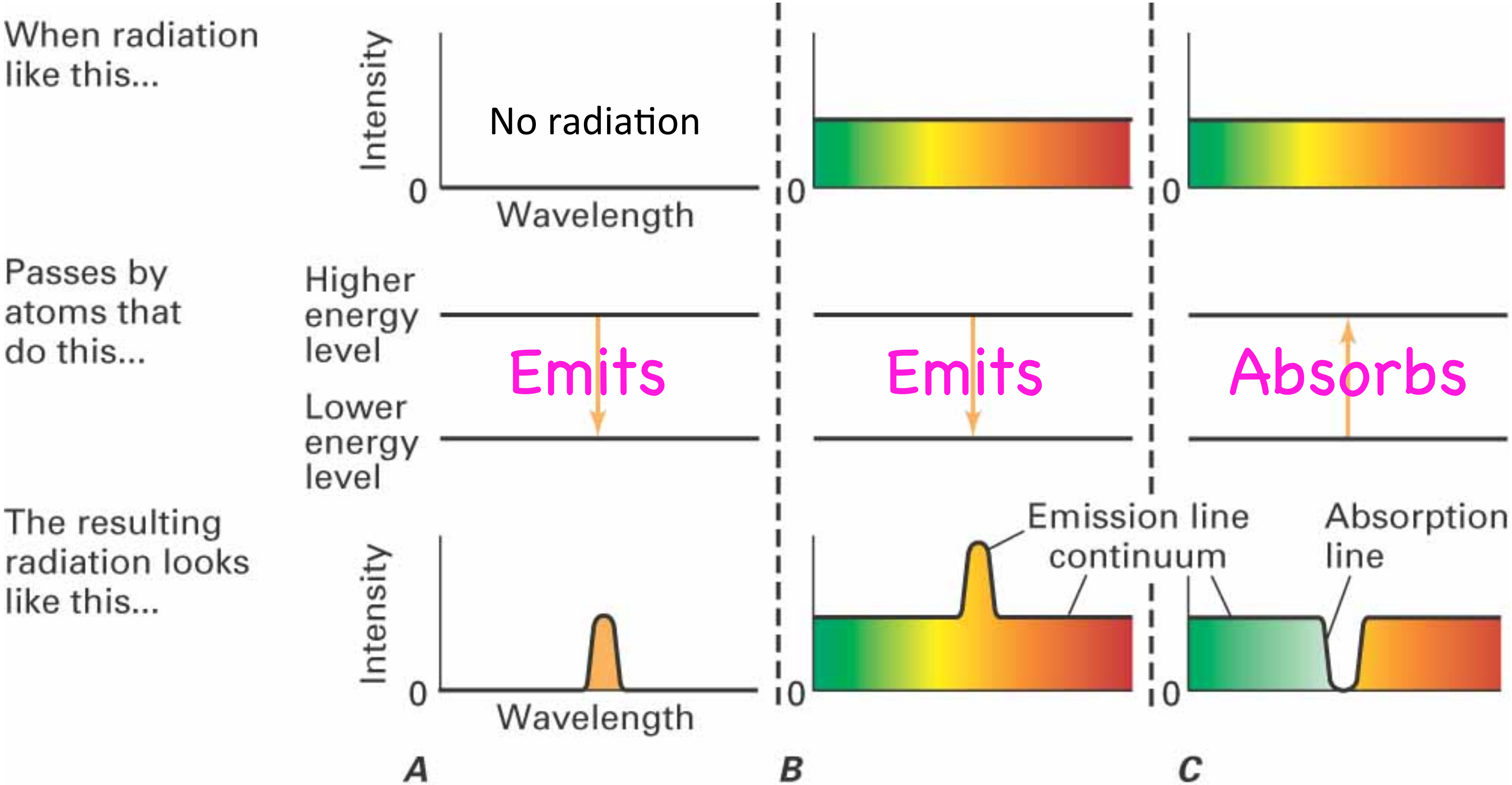
Analyzing the light from stars tells us what the stars are made of and how far away they are



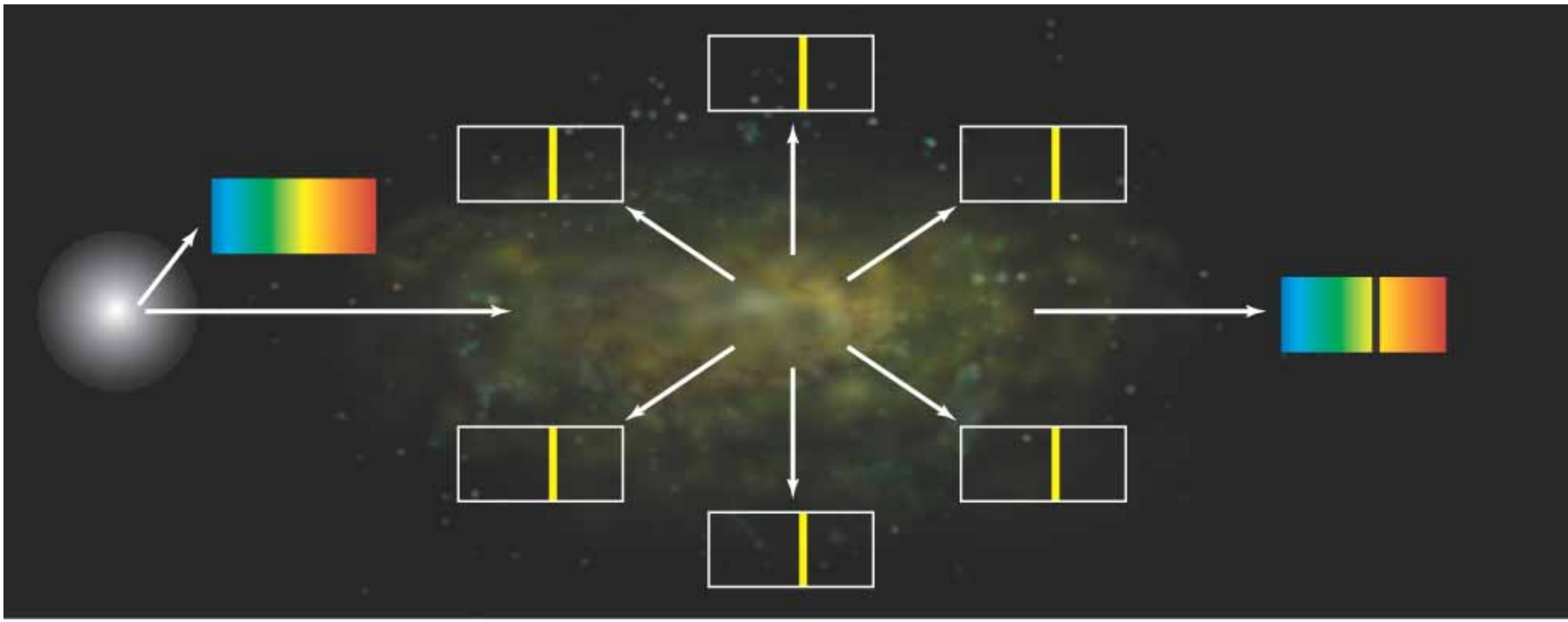
(Angstroms)

Shift in wavelength
from "lab" tells us
how fast the galaxy is moving
away and how far away the
galaxy is ("redshift")

Different spectra in different situations

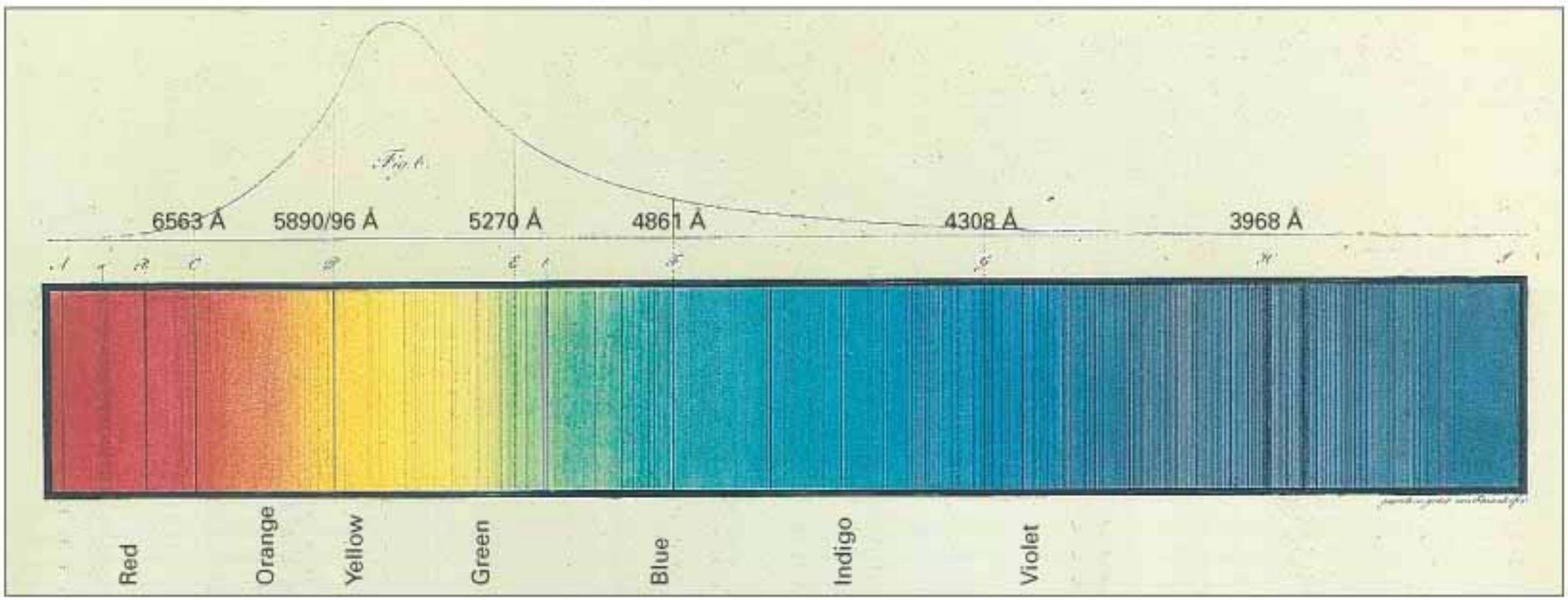


Gas absorbs and emits



- Gas absorbs photons at characteristic wavelengths; then re-emits these wavelengths in other directions

Gas in outskirts of Sun absorbs certain wavelengths



- gas in outskirts of sun absorbs
- pattern of lines tells you what gas is made of

X-Ray Spectra

- Innermost electron of an atom only feels nuclear force – hydrogen-like (Outer electrons ~spherical shell of charge exert no force on inner electrons)
- $E_n = -Z^2 E_R / n^2$
- For zinc, $Z=30$, $Z^2 E_R = (30)^2 \times (13.6\text{ eV}) \approx 12{,}000\text{ eV}$
- X-ray energies involved in transitions of innermost electrons

X-Ray Energies

- X-Rays are generated in a x-ray tube when electrons strike the anode, ejecting an inner electron, creating a vacancy.
- Outer electrons cascade down to fill the hole, emitting photons.
- Example: n=2 ➔ n=1

$$E_\gamma = E_2 - E_1 = Z^2 E_R\left(1 - \frac{1}{2^2}\right) = \frac{3}{4} Z^2 E_R$$

$$E_\gamma = \frac{3}{4}(30)^2\left(13.6 \text{ eV}\right) = 9200 \text{ eV} = 9 \text{ keV}$$

$$\mathrm{E}_\gamma = hf \sim Z^2 \Rightarrow Z \propto \sqrt{f}$$