VERSION A

1. (20 points) **PART I: SHORT ANSWER**

   (a) (4 points) What is the first law of thermodynamics? Be sure to define all variables.

   Answer: The change in the internal energy of a system equals the heat Q it absorbs plus the work W done *on* the system.

   $$\Delta E_{int} = Q + W \tag{1}$$

   $\Delta E_{int} = E_{int}(\text{final}) - E_{int}(\text{initial})$ = change in the internal energy of the system
   Q = heat absorbed by the system
   W = work done *on* the system. (Work done by the system would have the opposite sign.)
   Differential form: $dE_{int} = dQ + dW$

   (b) (4 points) A rock is thrown into a lake with water of uniform temperature. Which of the following is true:

   i. The buoyant force on the rock is zero as it sinks.
   ii. The buoyant force on the rock increases as it sinks.
   iii. The buoyant force on the rock decreases as it sinks.
   iv. The buoyant force on the rock is constant as it sinks.
   v. The buoyant force on the rock as it sinks is nonzero at first but becomes zero once the terminal velocity is reached.
   vi. Not enough information is given.
   vii. None of the above.

   Answer:

iv

   The buoyant force is equal to the weight of the water displaced by the rock. The amount of water displaced is constant as the rock sinks, so the buoyant force on the rock is constant as it sinks.

(c) (4 points) What is the relation between the gas constant R and Boltzmann's constant k_B? In other words, express R in terms of k_B. Be sure to define any new variables that you introduce.

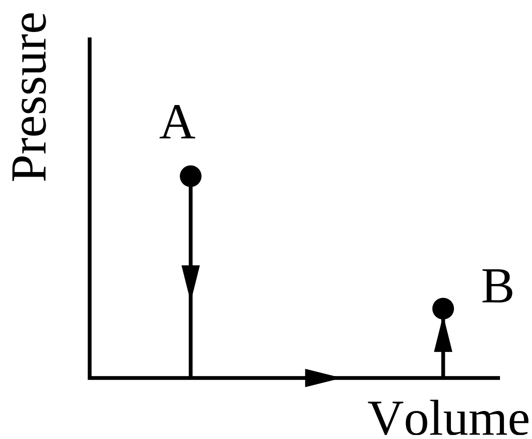
Answer:

$R = N_A k_B$

where N_A = Avogadro's number

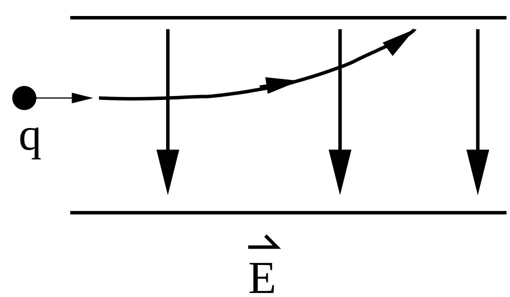
(d) (4 points) Draw a reversible path from point A to point B during which no work is done. Use arrows to indicate the direction of the path.

Answer: Since the work is proportional to the area under the path, we draw a path with no area under it.



(e) (4 points) A particle with velocity $\mathbf{v}$ and negative charge $q < 0$ enters a uniform electric field $\mathbf{E}$. Draw the trajectory of the particle as it passes through the field. Use an arrow to indicate the direction of the particle's path.

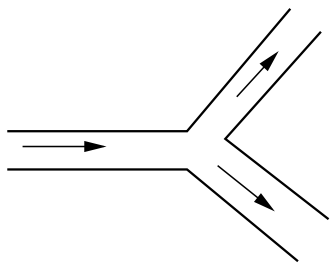
Answer: The electric field points in the direction of the force on a positive charge. A negative charge feels a force in the opposite direction.



PART II: WORK OUT PROBLEMS

2. (20 points)

(a) (18 points) Water flows through a large horizontal pipe of radius r_o at speed v_o. The pressure in the pipe is P_o. The pipe branches into two horizontal pipes of radius r_1 and r_2, respectively, both at the same height as the large pipe. Write the equations that would allow you to find the pressures P_1 and P_2 and the speeds v_1 and v_2 in the two pipes. Do *not* solve the equations.



Answer: Use the continuity equation for fluids and Bernoulli's equation. The cross sectional areas of the pipes are

$$\begin{aligned} A_o &= \pi r_o^2 \\ A_1 &= \pi r_1^2 \\ A_2 &= \pi r_2^2 \end{aligned}$$

The continuity equation for fluids gives:

$$\begin{aligned} A_o v_o &= A_1 v_1 + A_2 v_2 \\ \pi r_o^2 v_o &= \pi r_1^2 v_1 + \pi r_2^2 v_2 \\ r_o^2 v_o &= r_1^2 v_1 + r_2^2 v_2 \end{aligned}$$

Bernoulli's equation says that along a streamline

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant} \tag{2}$$

For horizontal pipes, $h = 0$.

$$\boxed{\begin{aligned} r_o^2 v_o &= r_1^2 v_1 + r_2^2 v_2 \\ P_o + \frac{1}{2}\rho v_o^2 &= P_1 + \frac{1}{2}\rho v_1^2 \\ P_o + \frac{1}{2}\rho v_o^2 &= P_2 + \frac{1}{2}\rho v_2^2 \end{aligned}}$$

(b) (2 points) How many of these 4 quantities (P_1, P_2, v_1, v_2) must be known to be able to solve for the others? Explain your answer.

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Reason: For 3 equations, you need 3 unknowns. So you need to be given 1 of the 4 quantities.

3. (20 points) The latent heat L_v of vaporization of water at 1 atm and 100 ^oC is 2.26×10^6 J/kg. What fraction of that latent heat Q is converted into the mechanical work W needed to change the volume of 2 g of water into 2 g of steam at 1 atm and 100 ^oC? In other words, find the ratio W/Q. The sign of the ratio should reflect the relative signs of W and Q. The molecular weight of water is 18 g/mole. (The density of water is 1000 kg/m^3.)

Answer: The heat Q is given by

$$\begin{aligned} Q &= L_v m \\ &= \left(2.26 \times 10^6 \text{J/kg}\right)\left(2 \times 10^{-3} \text{kg}\right) \\ &= 4.52 \times 10^3 \text{ J} \end{aligned}$$

The work W at a constant pressure of 1 atm is

$$W = -\int PdV = -P\left(V_{steam} - V_{water}\right) = -PV_{steam} \tag{3}$$

The volume of 2 g of water is 2 cm^3 which is negligible compared to the volume of the steam. So we ignored the volume of the water in the the last step.

From the ideal gas law,

$$W = -PV_{steam} = -nRT \tag{4}$$

The number of moles of water or steam is $n = 2/18 = 1/9$. The temperature in Kelvin is $T = 100 + 273$ K =373 K. So the work is

$$\begin{aligned} W &= -nRT \\ &= -\frac{1}{9}(8.314 \text{ J/K} - \text{mol})(373 \text{ K}) \\ &= -345 \text{ J} \end{aligned}$$

(Note that the volume of the steam $V_{steam} = nRT/P = 3.4 \times 10^{-3}\text{m}^3 = 3401 \text{ cm}^3 \gg V_{water} = 2 \text{ cm}^3$. This justifies our neglect of the volume of the water.)

So the ratio is

$$\frac{W}{Q} = -\frac{345 \text{ J}}{4.52 \times 10^3 \text{ J}} \tag{5}$$

$\frac{W}{Q} = -7.63 \times 10^{-2}$

4. (20 points) A 3 dimensional gas mixture consists of 2.0 moles of diatomic oxygen (O_2) and 1.0 mole of monoatomic argon (Ar). Find the molar specific heat C_p at constant pressure of the mixture. Do not ignore the vibrational degrees of freedom. Show your work.

Answer: First calculate the internal energy E_{int} by counting the number of degrees of freedom for each gas.

For argon, there are just 3 translational degrees of freedom since it's monatomic. Since the number n_{Ar} of moles of argon is 1,

$$E_{int,Ar} = \frac{3}{2} n_{Ar} RT = \frac{3}{2} RT \tag{6}$$

For O_2 which is diatomic, there are

- 3 translational degrees of freedom associated with the center of mass
- 2 rotational degrees of freedom
- 2 vibrational degrees of freedom (1 for potential energy and 1 for kinetic energy)

So there are a total of 7 degrees of freedom. The number of moles is $n_O = 2$. So

$$E_{int,O} = \frac{7}{2} n_O RT = \frac{7}{2} \cdot 2RT = 7RT \tag{7}$$

So the total internal energy is

$$\begin{aligned} E_{int,tot} &= E_{int,Ar} + E_{int,O} \\ &= \frac{3}{2} RT + 7RT \\ &= 8.5RT \end{aligned}$$

The total number of moles is $n_{tot} = n_O + n_{Ar} = 3$. The molar specific heat C_V at constant volume is

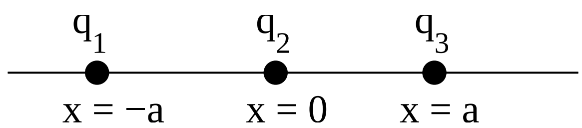
$$C_V = \frac{1}{n_{tot}} \left. \frac{dE_{int,tot}}{dT} \right|_V = 8.5R/3 = 2.83R \tag{8}$$

The specific heat C_p at constant pressure is

$$C_P = C_V + R = 3.83R \tag{9}$$

$C_p = 3.83R = 31.9$ J/mol-K

5. (20 points) Three charges are located along the x-axis as shown in the drawing. q_1 is at $x = -a$, q_2 is at $x = 0$, and q_3 is at $x = +a$. Note that $q_1 = q_3 = -e$, and that $q_2 = +e$. Calculate the electric force $\vec{F}$ on each charge.



Answer: First look at the diagram. Note by symmetry that the $\vec{F}_{2,tot} = 0$ since q_2 has identical charges on either side of it. q_1 and q_3 will be attracted to the positive charge at the origin, so $\vec{F}_{1,tot} \| \hat{i}$ and $\vec{F}_{3,tot} \| -\hat{i}$. By symmetry, $\vec{F}_{1,tot}$ and $\vec{F}_{3,tot}$ will be equal in magnitude but opposite in direction.

From Coulomb's Law and the principle of superposition,

$$\begin{aligned} \vec{F}_{1,tot} &= \vec{F}_{1\leftarrow 2} + \vec{F}_{1\leftarrow 3} \\ &= k_e \frac{q_1 q_2}{r_{12}^2} \hat{r_{12}} + k_e \frac{q_1 q_3}{r_{13}^2} \hat{r_{13}} \\ &= k_e \frac{-e^2}{a^2} \hat{r_{12}} + k_e \frac{e^2}{(2a)^2} \hat{r_{13}} \end{aligned}$$

Note that

$$\begin{aligned} \hat{r}_{12} &= \frac{\vec{r}_1 - \vec{r}_2}{|r_1 - r_2|} \\ &= \frac{\vec{r}_1}{a} \\ &= -\hat{i} \\ \hat{r}_{13} &= \frac{\vec{r}_1 - \vec{r}_3}{|r_1 - r_3|} \\ &= \frac{(-a\hat{i}) - (a\hat{i})}{2a} \\ &= -\hat{i} \end{aligned}$$

So

$$\begin{aligned} \vec{F}_{1,tot} &= k_e \frac{-e^2}{a^2} \left(-\hat{i}\right) + k_e \frac{e^2}{(2a)^2} \left(-\hat{i}\right) \\ &= k_e \frac{e^2}{a^2} \hat{i} - k_e \frac{e^2}{4a^2} \hat{i} \\ &= k_e \frac{e^2}{a^2} \left(1 - \frac{1}{4}\right) \hat{i} \\ &= k_e \frac{3e^2}{4a^2} \hat{i} \end{aligned}$$

and

$$\vec{F}_{3,tot} = -\vec{F}_{1,tot} = -k_e \frac{3e^2}{4a^2} \hat{i} \tag{10}$$

$$\vec{F}_{1,tot} = k_e \frac{3e^2}{4a^2}\hat{i} = \frac{1}{4\pi\varepsilon_o}\frac{3e^2}{4a^2}\hat{i}$$

$$\vec{F}_{2,tot} = 0$$

$$\vec{F}_{1,tot} = -k_e \frac{3e^2}{4a^2}\hat{i} = -\frac{1}{4\pi\varepsilon_o}\frac{3e^2}{4a^2}\hat{i}$$