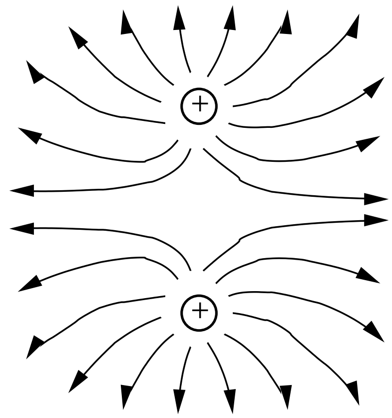
VERSION A

1. (20 points) **PART I: SHORT ANSWER**

   (a) (5 points) Sketch the electric field lines for 2 equal positive point charges.

   (b) (5 points) If hot air rises, why is it cooler at the top of a mountain than at sea level?

   Answer: As the air rises, it expands, pushing away the surrounding air and doing work. As a result, its internal energy decreases and it cools. Recall that for an ideal gas $E_{int} = nC_VT$ which shows that the internal energy is proportional to the temperatures.

(c) (5 points) The speeds of 5 molecules are 2.0, 3.0, 4.0, 5.0, and 6.0 km/s. What is the their root-mean-square speed v_{rms}? Show your work.

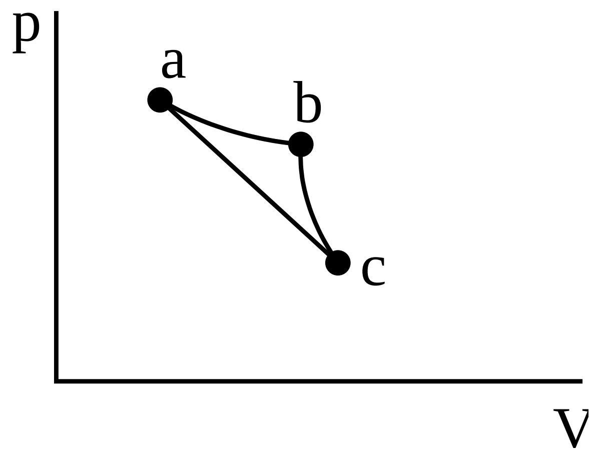
Answer:

$$\begin{aligned}\overline{v}^2 &= \frac{1}{5}\sum_i v_i^2 \\ &= \frac{1}{5}\left(2^2 + 3^2 + 4^2 + 5^2 + 6^2\right) \text{km}^2/\text{s}^2 \\ &= 18 \text{ km}^2/\text{s}^2\end{aligned}$$

$$v_{rms} = \sqrt{\overline{v}^2} = \sqrt{18} \text{ km/s} = 4.24 \text{ km/s} \tag{1}$$

$v_{rms} = 4.24$ km/s

(d) (5 points) In the PV (pressure-volume) diagram, the ideal gas does 5 J of work along the isotherm ab, and it does 4 J of work along the adiabat bc. (In other words, going from a to b is an isothermal process, and going from b to c is an adiabatic process.) What is the change ΔE_{int} in the internal energy of the gas if the gas traverses the straight path from a to c? Be sure to get the sign right. Explain your answer.



Answer: Along ab, $T =$ constant. For an ideal gas, $E_{int} = nC_V T$. So if the temperature is constant, E_{int} is constant and $\Delta E_{int} = 0$ along ab. Along bc, $Q = 0$. From the first law of thermodynamics, $\Delta E_{int} = Q + W = W = -4$ J where W is the work done *on* the system. We have a minus sign because work is done **by** the system. Since the change in internal energy is independent of the path taken,

$\Delta E_{int} = -4$ J.

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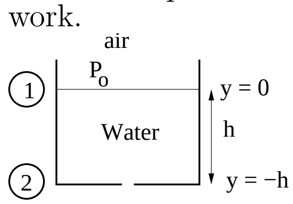
PART II: WORK OUT PROBLEMS

2. (20 points)

(a) (15 points) A cylindrical bucket hangs above the ground. The radius of the bucket is $R_1 = 25$ cm and it is filled with water to a depth of $h = 50$ cm. A hole of radius $R_2 = 0.5$ cm is poked in the center of the bottom of the bucket. What is the initial speed v at which the top of the water in the bucket falls? Show your work.



Answer: You need to use 2 equations and 2 unknowns. Notice that you can think of the bucket as a vertical pipe that is wide at the top and narrow at the bottom. So you can use the continuity equation for fluids.

$$\begin{aligned} A_1 v_1 &= A_2 v_2 \\ v_2 &= \frac{A_1}{A_2} v_1 \end{aligned} \tag{2}$$

Plug this into Bernoulli's equation. Air surrounds the bucket. So let P_o be the atmospheric pressure at the top and at the hole at the bottom. Define the coordinate system to be $y_1 = 0$ at the top and $y_2 = -h$ at the bottom of the bucket. So from Bernoulli's equation

$$\begin{aligned} \frac{1}{2}\rho v_1^2 + \rho g y_1 + P_o &= \frac{1}{2}\rho v_2^2 + \rho g y_2 + P_o \\ \frac{1}{2}\rho v_1^2 + \rho g * 0 &= \frac{1}{2}\rho v_2^2 + \rho g(-h) \\ \frac{1}{2}\rho v_1^2 &= \frac{1}{2}\rho v_2^2 - \rho g h \\ \frac{1}{2}\left(v_1^2 - v_2^2\right) &= -gh \\ \frac{1}{2}\left[v_1^2 - \left(\frac{A_1}{A_2}\right)^2 v_1^2\right] &= -gh \\ \frac{1}{2}v_1^2\left[1 - \left(\frac{A_1}{A_2}\right)^2\right] &= -gh \\ v_1^2 &= -\frac{2gh}{1-(A_1/A_2)^2} \\ &= \frac{2gh}{(A_1/A_2)^2 - 1} \end{aligned}$$

Recall that the area of a circle $A = \pi r^2$, so $A_1/A_2 = (R_1/R_2)^2 = (25/0.5)^2 = 50^2 = 2500$. So $(A_1/A_2)^2 = 2500^2 = 6.25 \times 10^6$.

$$\begin{aligned} v_1^2 &= \frac{2(9.8\text{ m/s}^2)(0.5\text{ m})}{6.25\times 10^6 - 1} = 1.568 \times 10^{-6}\text{ m}^2/\text{s}^2 \\ v_1 &= \sqrt{1.568\times 10^{-6}}\text{ m/s} = 1.252 \times 10^{-3}\text{ m/s} \end{aligned}$$

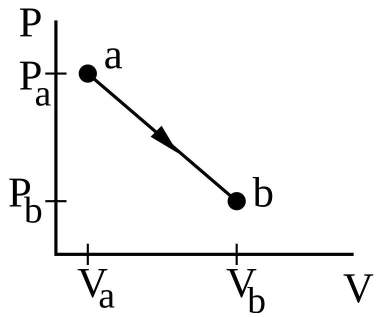
$v = 1.25 \times 10^{-3}$ m/s

(b) (5 points) Will the speed of the top of the water decrease, increase, or stay the same with time? Explain your reasoning.

Answer: The speed will decrease as time goes on. Notice that $v \sim \sqrt{h}$, so as the height of the water decreases, so does v. This makes sense since less water means less pressure on the water at the bottom of the bucket. So the water exits more slowly through the hole as the water level drops. This implies that the surface of the water will drop more slowly.

3. (20 points) One mole of an ideal diatomic gas goes from a to b along the diagonal path shown in the PV (pressure-volume) plot. $P_a = 5.0$ kN/m^2, $V_a = 2.0$ m^3, $P_b = 2.0$ kN/m^2, $V_b = 4.0$ m^3.

(a) (10 points) What is the change ΔE_{int} in internal energy of the gas? Include the intramolecular vibrational degrees of freedom. Show your work.



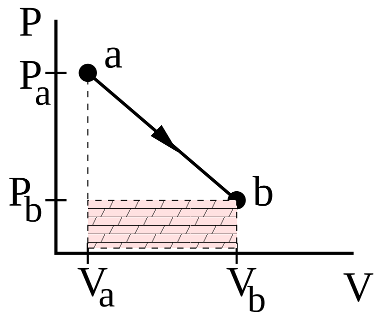
Answer: From the equipartition theorem, we know that the average internal energy of a diatomic molecule is the sum of the translational ($3k_BT/2$), rotational ($2 \times k_BT/2 = k_BT$), and vibrational (k_BT) degrees of freedom. (For the vibrational degrees of freedom, the kinetic energy is associated with $k_BT/2$ and the potential energy of the spring is associated with $k_BT/2$ to give a total of k_BT for the vibrations.) The total internal energy for one molecule is $7k_BT/2$. For n moles of the diatomic gas, the total internal energy is $E_{int} = 7nRT/2$. So the change in internal energy will be proportional to the change in the temperature. From the ideal gas law $PV = nRT$

$$E_{int} = \frac{7}{2}PV \tag{3}$$

$$\begin{aligned} \Delta E_{int} &= \frac{7}{2}\left(P_bV_b - P_aV_a\right) \\ &= \frac{7}{2}\left[(2)(4) - (5)(2)\right]\,\frac{\text{kN}}{\text{m}^2}\cdot \text{m}^3 \\ &= \frac{7}{2}[8 - 10]\ \text{kN} - \text{m} \\ &= -7\ \text{kJ} \end{aligned}$$

$\Delta E_{int} = -7$ kJ

(b) (10 points) How much energy is added to the gas as heat in going from a to b? Again, $P_a = 5.0$ kN/m^2, $V_a = 2.0$ m^3, $P_b = 2.0$ kN/m^2, $V_b = 4.0$ m^3. Show your work.



Answer: From the first law of thermodynamics

$$\begin{aligned} \Delta E_{int} &= Q + W \\ Q &= \Delta E_{int} - W \end{aligned}$$

The work W done on the system is $W = -\int P dV = -$(area under the curve). To find the area under the curve, we can use

$$\begin{aligned} \text{Area under diagonal} &= \left(\frac{P_a + P_b}{2}\right) \Delta V \\ &= \left(\frac{7}{2} \text{ kN/m}^2\right) \cdot 2 \text{ m}^3 \\ &= 7 \text{ kJ} \end{aligned}$$

So the work $W = -7$ kJ.
Another way to find the area is to split up the area into a rectangle and a triangle.

$$\begin{aligned} \text{Area of rectangle} &= P_b (V_b - V_a) \\ &= \left(2 \text{ kN/m}^2\right) (4 - 2) \text{ m}^3 \\ &= 4 \text{ kJ} \end{aligned}$$

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2}(\text{area of square}) \\ &= \frac{1}{2} (P_a - P_b) (V_b - V_a) \\ &= \frac{1}{2}(5 - 2) \text{ kN/m}^2 (4 - 2) \text{ m}^3 \\ &= \frac{1}{2} \cdot 3 \cdot 2 \text{ kJ} \\ &= 3 \text{ kJ} \end{aligned}$$

So the work $W = -(3 + 4)$ kJ $= -7$ kJ.
From part (a), $\Delta E_{int} = -7$ kJ. The heat $Q = \Delta E_{int} - W = [-7 - (-7)] = 0$.

$Q = 0$

4. (20 points) A flow calorimeter is used to measure the specific heat of a liquid. Heat is added at a known rate to a stream of the liquid as it passes through the calorimeter at a known rate. Then a measurement of the resulting temperature difference between the inflow and the outflow points of the liquid stream enables us to compute the specific heat of the liquid.

   A liquid of density $\rho = 0.85$ g/cm^3 flows through a calorimeter at the rate of 8 cm^3/sec. Heat is added by means of a 250-watt electric heating coil, and a temperature difference of $\Delta T = 15^o$C is established in steady-state conditions between the inflow and outflow points. Assume that all the heat from the heating coil goes into the liquid. Find the specific heat c of the liquid.

   Answer: Since the system is in steady state, the amount of heat that goes into the liquid in one second from the heating coil equals the amount of heat carried away by the liquid that leaves the calorimeter in one second. In 1 sec, the volume of liquid that leaves (or enters) the calorimeter is $V = 8$ cm^3. The mass of this liquid is

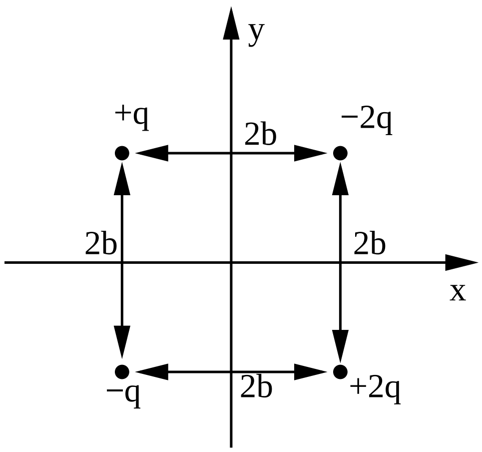
$$m = \rho V = \left(0.85\ \text{g/cm}^3\right)\left(8\ \text{cm}^3\right) = 6.8\ \text{g} \tag{4}$$

   In one second, the amount of heat that this mass takes away from the calorimeter is $Q = 250$ J.

$$\begin{aligned} Q &= mc\Delta T \\ c &= \frac{Q}{m\Delta T} \\ &= \frac{250\ \text{J}}{(6.8\ \text{g})(15^o\text{C})} \\ &= 2.45\ \frac{\text{J}}{\text{g}^o\text{C}} \cdot \left(\frac{1000\ \text{g}}{1\ \text{kg}}\right) \\ &= 2.45 \times 10^3\ \frac{\text{J}}{\text{kg}^o\text{C}} \end{aligned}$$

$c = 2.45 \times 10^3$ J/kg ^{o}C

5. (20 points) What are the components of the electric field **E** at the center of the square? Assume that $q = 1.0 \times 10^{-8}$ C and $b = 5$ cm. (Hint: It might be easier to find the magnitude and direction of the field.)



Answer: Method I Use symmetry. Look at the pair $-2q$ and $+2q$. Notice that at the center, $\mathbf{E_{-2q}} + \mathbf{E_{+2q}}$ points along $+\hat{\jmath}$. Now look at the pair $-q$ and $+q$. At the center, $\mathbf{E_{-q}} + \mathbf{E_{+q}}$ points along $-\hat{\jmath}$. So the total field points along $+\hat{\jmath}$. To find the magnitude, we need to add up the y components of the field. Notice that all the charges are a distance $r = \sqrt{2}b$ away from the center. To obtain the y component of the force, we need to multiply the magnitude by $\cos\theta = 1/\sqrt{2}$ where $\theta = \pi/2$.

$$\begin{aligned} E_y &= k_e \frac{2q + 2q - q - q}{r^2} \cos\theta \\ &= k_e \frac{2q}{2b^2} \frac{1}{\sqrt{2}} \\ &= k_e \frac{q}{\sqrt{2}b^2} \\ &= \left(8.99 \times 10^9 \; N - m^2/C^2\right) \frac{1 \times 10^{-8} \text{ C}}{\sqrt{2}(0.05 \text{ m})^2} \\ &= 2.5 \times 10^4 \text{ N/C} \end{aligned}$$

$E_x = 0$

$E_y = 2.5 \times 10^4$ N/C

Method II: Vectors This method uses vectors. Just sum the **E** field vectors due to each corner charge. Note that all the charges are a distance $r = \sqrt{2}b$ away from the center. The angle θ that each field $\mathbf{E_i}$ makes at the center with the x axis and y axis is $\theta = \pi/2$. So $\sin\theta = \cos\theta = 1/\sqrt{2}$.

$$\begin{aligned}
\mathbf{E_{+q}} &= k_e \frac{q}{2b^2}\left(\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}\right) \\
\mathbf{E_{-q}} &= k_e \frac{q}{2b^2}\left(-\frac{1}{\sqrt{2}}\hat{\imath} - \frac{1}{\sqrt{2}}\hat{\jmath}\right) \\
\mathbf{E_{+2q}} &= k_e \frac{2q}{2b^2}\left(-\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right) \\
\mathbf{E_{-2q}} &= k_e \frac{2q}{2b^2}\left(\frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{\sqrt{2}}\hat{\jmath}\right)
\end{aligned}$$

Summing the fields, we obtain

$$\begin{aligned}
\mathbf{E_{tot}} &= k_e \frac{2q}{2b^2}\left(\frac{1}{\sqrt{2}}\hat{\jmath}\right) \\
&= k_e \frac{q}{\sqrt{2}b^2}\hat{\jmath}
\end{aligned}$$

This is the same answer as with Method I.