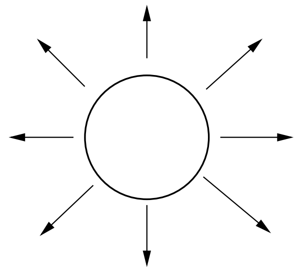
VERSION A

1. Short Answer Problems (5 points each)

   (a) (5 points) A hollow spherical conducting shell has a charge $q > 0$. Draw the electric field lines inside and outside the shell.



   (b) (5 points) Air in a thunder cloud expands as it rises. If its initial temperature is $T_i = 300$ K and no energy is lost by thermal conduction on expansion, what is its final temperature T_f when the initial volume V_i has doubled? Since air is diatomic gas, assume that $\gamma = 1.4$.

   Answer: This is an adiabatic process where no heat or energy is lost by thermal conduction. So we use $P_i V_i^\gamma = P_f V_f^\gamma$ and the ideal gas law.

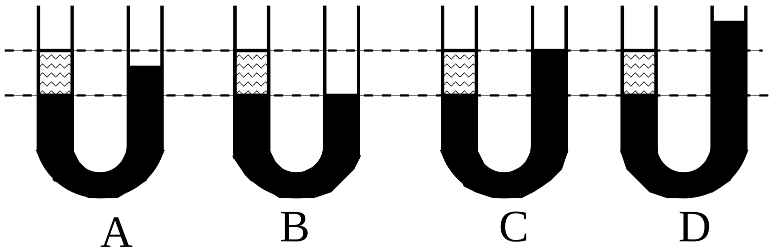
$$\begin{aligned}
PV &= nRT \\
P &= \frac{nRT}{V} \\
P_i V_i^\gamma &= P_f V_f^\gamma \\
\frac{nRT_i}{V_i} V_i^\gamma &= \frac{nRT_f}{V_f} V_f^\gamma \\
T_i V_i^{\gamma-1} &= T_f V_f^{\gamma-1} \\
T_f &= T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} \\
&= (300\ \text{K}) \left(\frac{1}{2}\right)^{1.4-1} \\
&= (300\ \text{K}) \left(\frac{1}{2}\right)^{0.4} \\
&= 227\ \text{K}
\end{aligned}$$

$T_f = 227$ K

(c) (5 points) What is the name of the unit of resistance? (Give the name, not the symbol.)

ohm

(d) (5 points) The figure shows four situations in which two different liquids are in a U-tube. Both liquids are heavier than air. In one situation the liquids cannot be in static equilibrium. Which situation is that? Explain your answer.



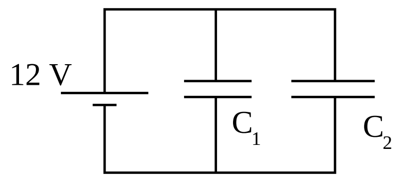
Answer: B is not in static equilibrium. In B along the lower dashed line, the pressure is equal on both sides of the tube. But that cannot be true in static equilibrium because the striped liquid is heavier than air.

B

2. Short Answer Problems II (5 points each)

(a) (5 points) Two capacitors C_1 and C_2 have been connected in parallel to a 12 V battery for a very long time. What is the total energy stored U in the capacitors if $C_1 = C_2 = 10\ \mu$F? Show your work.

Answer:



Add capacitors in parallel to get the equivalent capacitance C_{eq}.

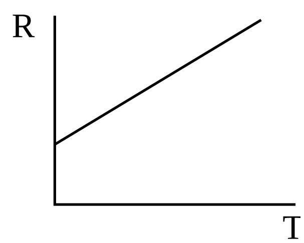
$$
\begin{aligned}
C_{eq} &= C_1 + C_2 \\
&= 20\ \mu\text{F} \\
U &= \frac{1}{2} C_{eq} V^2 \\
&= \frac{1}{2}(20\ \mu\text{F})(12\ \text{V})^2 \\
&= \frac{1}{2}(20 \times 10^{-6}\ \text{F})(12\ \text{V})^2 \\
&= 1.44 \times 10^{-3}\ \text{J}
\end{aligned}
$$

Note that 1 Joule = 1 Volt^2 $\times$ 1 Farad.

$U = 1.44 \times 10^{-3}$ J

(b) (5 points) Explain why an incandescent light bulb burns out when you first turn it on (as opposed to a bulb that has been on for a while). (An incandescent light bulb has a (tungsten) filament and is not a fluorescent light bulb.)

Answer: The filament in the bulb is cold before the light is turned on. Its resistance increases linearly as the temperature increases. So the resistance R is low.



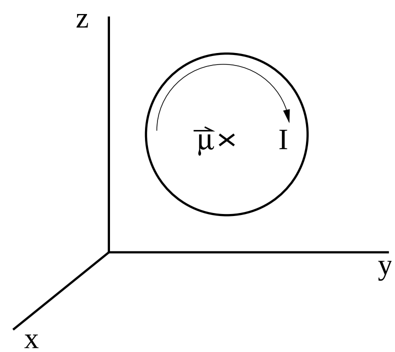
The power dissipated is

$$P = \frac{V^2}{R} \tag{1}$$

The voltage V = constant = 110 Volts. So a small R gives a large P, i.e., a large amount of power is dissipated. Hence, the chance of burning out the filament is greater when the bulb is first turned on.

(c) (5 points) A circular loop of wire encloses an area of 9 cm^2 and lies in the y-z plane. It carries a current $I = 2$ A in the clockwise direction. What is the magnetic dipole moment $\vec{\mu}$? Show your work.

Answer: The direction of the dipole moment is given by the right hand rule where your fingers curl in the direction of the current and your thumb points in the direction of the dipole moment. In this case $\vec{\mu}$ points into the page along $-\hat{\imath}$ direction.



To obtain the magnitude of μ:

$$
\begin{aligned}
\mu &= IA \\
&= \left(2\text{ A})(9\text{ cm}^2\right)\left(\frac{(1\text{ m})^2}{(100\text{ cm})^2}\right) \\
&= 1.8 \times 10^{-3}\text{ A m}^2
\end{aligned}
$$

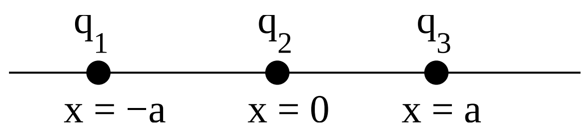
$\vec{\mu} = -1.8 \times 10^{-3}\,\hat{\imath}$ A m^2

(d) (5 points) Suppose you see two helium balloons floating next to each other at the ends of strings secured to a table. The facing surfaces of the balloons are separated by 1 or 2 cm. If you blow through the opening between the balloons, what happens to the balloons? Explain your answer.

Answer: The balloons move together because of Bernoulli's equation. The high-speed air between the balloons results in low pressure in this region. The higher pressure on the outer surfaces of the balloons pushes them toward each other.

3. Short Answer Problems III (5 points each)

(a) (5 points) Three charges are located along the x-axis as shown in the drawing. q_1 is at $x = -a$, q_2 is at $x = 0$, and q_3 is at $x = +a$. Note that $q_1 = q_3 = -e$, and that $q_2 = +e$. What is the potential energy U stored in this system?



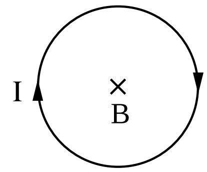
Answer: The electric potential energy stored in a pair of point charges represents the work required to bring the charges from an infinite separation to the separation r_{12}. In this case there are 3 pairs of charges.

$$
\begin{aligned}
U &= k_e\left[\frac{q_1q_2}{r_{12}} + \frac{q_2q_3}{r_{23}} + \frac{q_1q_3}{r_{13}}\right] \\
&= k_e\left[\frac{-e^2}{a} + \frac{-e^2}{a} + \frac{e^2}{2a}\right] = k_e\left[\frac{-2e^2}{a} + \frac{e^2}{2a}\right] \\
&= k_e\left[\frac{-4e^2+e^2}{2a}\right] = k_e\left[\frac{-3e^2}{2a}\right]
\end{aligned}
$$

where $k_e = 1/4\pi\varepsilon_o$.

$U = -\frac{3}{2}k_e\frac{e^2}{a}$

(b) (5 points) A metal ring lying in the plane of the page has an decreasing magnetic field pointing into the page. Indicate on the figure the direction of the induced current I? Explain your answer. **Answer:**



According to Lenz's law, the induced current produces magnetic flux that opposes the change in flux. Since the external flux is decreasing, the induced flux produces a magnetic field that points into the page. Using the right hand rule where your right thumb points along the magnetic field (into the page), your fingers curl in the direction of the current. So the current flows in the clockwise direction.

direction of I clockwise:

(c) (5 points) A loop of wire has a self inductance of 10 mH and carries a current of 0.1 A. What is the self-induced flux Φ through the loop?

Answer:

$$\begin{aligned}\Phi &= LI \\ &= (10\,\text{mH})(0.1\text{ A}) \\ &= (10\times 10^{-3}\text{ H})(0.1\text{ A}) \\ &= 10^{-3}\text{ A H} \\ &= 10^{-3}\text{ weber} = 10^{-3}\text{ Tm}^2 = 10^{-3}\text{ V s}\end{aligned}$$

$\Phi = 10^{-3}$ weber= 10^{-3} T m^2

(d) (5 points) The electric potential $V(\vec{r}) = ax^2 + by$. What is the electric field $\vec{E}(\vec{r})$?

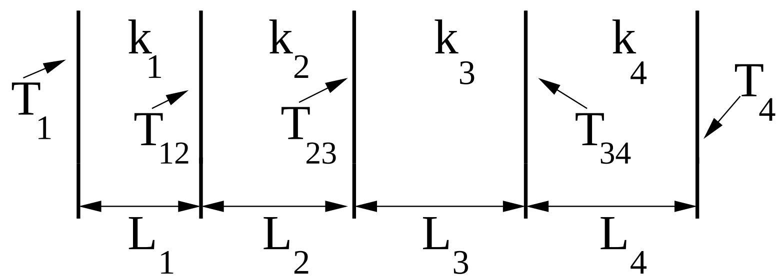
Answer:

$$\begin{aligned}E_x &= -\frac{\partial V}{\partial x} = -2ax \\ E_y &= -\frac{\partial V}{\partial y} = -b\end{aligned}$$

$\vec{E}(\vec{r}) = -2ax\ \hat{\imath} - b\ \hat{\jmath}$

WORK OUT PROBLEMS

4. (15 points) The figure show (in cross section) a wall consisting of four layers with thermal conductivities $k_1 = 0.060$ W/m-K, $k_3 = 0.040$ W/m-K, and $k_4 = 0.12$ W/m-K (k_2 is not known). The layer thicknesses are $L_1 = 1.5$ cm, $L_3 = 2.8$ cm, and $L_4 = 3.5$ cm (L_2 is not known). The known temperatures are $T_1 = 30^o$C, $T_{12} = 25^o$C, and $T_4 =$ -10^{o}C. Energy (heat) transfer through the wall is steady. What is the interface temperature T_{34}?



Answer: The fact that energy transfer through the wall is steady means that the rate of heat transfer dQ/dt is the same through every interface. So

$$\frac{dQ}{dT} = kA\left|\frac{dT}{dx}\right| = \text{constant} \qquad (2)$$

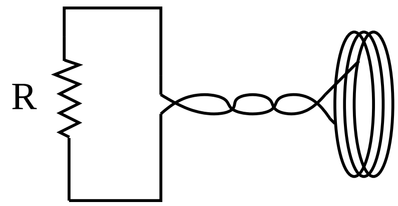
where k is thermal conductivity, A is the area of the interface, and Q is the heat.

So we equate the energy transfer through interface 12 with the energy transfer through interface 34.

$$\begin{aligned}
k_1 A\left|\frac{T_1 - T_{12}}{L_1}\right| &= k_4 A\left|\frac{T_{34} - T_4}{L_4}\right| \\
T_{34} &= T_4 + \left(\frac{k_1}{k_4}\right)\left(\frac{L_4}{L_1}\right)|T_1 - T_{12}| \\
&= -10^o\text{ C} + \left(\frac{0.06}{0.12}\right)\left(\frac{3.5\text{ cm}}{1.5\text{ cm}}\right)(30^o\text{ C} - 25^o\text{ C}) \\
&= -10^o\text{ C} + 5.83^o\text{ C} \\
&= -4.17^o\text{ C} \\
&= -4.17^o\text{ C} + 273.15\text{ K} \\
&= 269\text{ K}
\end{aligned}$$

$T_{34} = -4.17^o$ C $= 269$ K

5. (15 points) The magnetic flux through each turn of a coil with N turns changes from Φ_1 to Φ_2. What is the charge q that flows through the circuit of total resistance R? Just give the magnitude of the charge; do not worry about the sign of the charge.



Answer: From Ohm's law

$$\mathcal{E} = IR \qquad \text{or} \qquad I = \frac{\mathcal{E}}{R} \tag{3}$$

From the definition of current I:

$$I = \frac{dq}{dt} = \frac{\mathcal{E}}{R} \tag{4}$$

From Faraday's Law

$$\mathcal{E} = -N\frac{d\Phi}{dt} \tag{5}$$

Putting all this together gives

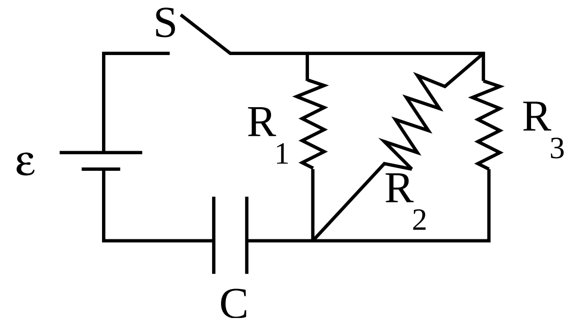
$$\begin{aligned} I &= \frac{dq}{dt} = \frac{\mathcal{E}}{R} = -\frac{N}{R}\frac{d\Phi}{dt} \\ q &= -\frac{N}{R}\int_{\Phi_1}^{\Phi_2} d\Phi = -\frac{N(\Phi_2 - \Phi_1)}{R} \end{aligned}$$

Since we don't have to worry about the sign, we can write

$$q = \frac{N\left|\Phi_1 - \Phi_2\right|}{R} \tag{6}$$

$q = N\left|\Phi_1 - \Phi_2\right|/R$

6. (15 points) What is the time constant τ of the circuit shown in the figure? τ is the time it takes for the capacitor to reach (1-1/e)=63% of its final charge after the switch S is closed. In the circuit shown $R_1 = 100\ \Omega$, $R_2 = 50\ \Omega$, $R_3 = 150\ \Omega$, $C = 10\ \mu$F and the battery has an emf $\mathcal{E} = 12$ V.



Answer: Note that the resistors are all in parallel. So the equivalent resistance R_{eq} is given by

$$\begin{aligned}
\frac{1}{R_{eq}} &= \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \\
R_{eq} &= \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \\
&= \frac{1}{\frac{1}{100\ \Omega} + \frac{1}{50\ \Omega} + \frac{1}{150\ \Omega}} \\
&= \frac{1}{\frac{3}{300\ \Omega} + \frac{6}{300\ \Omega} + \frac{2}{300\ \Omega}} \\
&= \frac{1}{\frac{11}{300\ \Omega}} \\
&= \frac{300}{11}\ \Omega
\end{aligned}$$

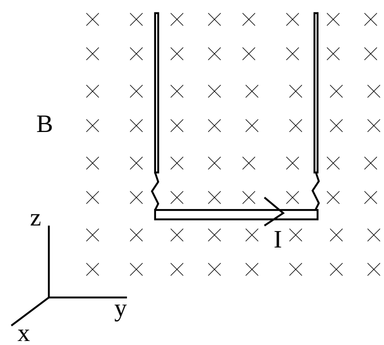
The time constant τ is given by

$$\begin{aligned}
\tau &= R_{eq} C \\
&= \left(\frac{300}{11}\ \Omega\right)\left(10 \times 10^{-6}\ \text{F}\right) \\
&= 2.73 \times 10^{-4}\ \text{sec}
\end{aligned}$$

Note that 1 FΩ = 1 sec.

$\tau = 2.73 \times 10^{-4}$ sec

7. (15 points) A wire of length 60 cm and mass 10 grams is suspended by a pair of flexible leads in a uniform magnetic field of 0.40 Tesla. The field points into the page. The wire is parallel to the y-axis. What are the magnitude and direction of the current I required to remove the tension in the supporting leads? (The gravitational acceleration $g = 9.8$ m/s^2.)



Answer: The force $\vec{F}_B$ on a current carrying wire in a magnetic field must point upward because it must be equal and opposite to the downward force of gravity. So the total force on the wire $\vec{F}_{tot} = 0$.

$$\begin{aligned}
\vec{F}_{tot} &= \vec{F}_B + \vec{F}_{grav} = 0 \\
\vec{F}_B &= -\vec{F}_{grav} \\
\vec{F}_B &= I\vec{\ell} \times \vec{B} = I\ell B(\hat{k}) \\
\vec{F}_{grav} &= -mg\hat{k} \\
I\ell B\hat{k} &= mg\hat{k} \\
I\ell B &= mg \\
I &= \frac{mg}{\ell B} \\
&= \frac{(10\text{ g})\left(\frac{1\text{ kg}}{10^3\text{ g}}\right)\left(9.8\text{ m/s}^2\right)}{(0.6\text{ m})(0.4\text{ T})} = \frac{(10^{-2}\text{ kg})\left(9.8\text{ m/s}^2\right)}{(0.6\text{ m})(0.4\text{ T})} \\
&= 0.408\text{ A} \simeq 0.41\text{ A}
\end{aligned}$$

Note that we can remove the cross product because the current is perpendicular to the direction of the current I.

direction of I: $\hat{j}$ or $\hat{\jmath}$

magnitude of $I = 0.41$ A