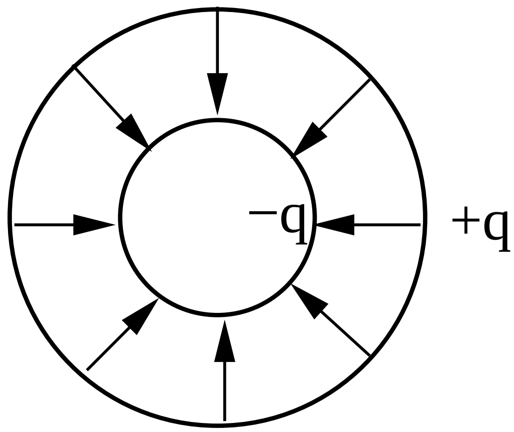
1. Short Answer Problems (5 points each)

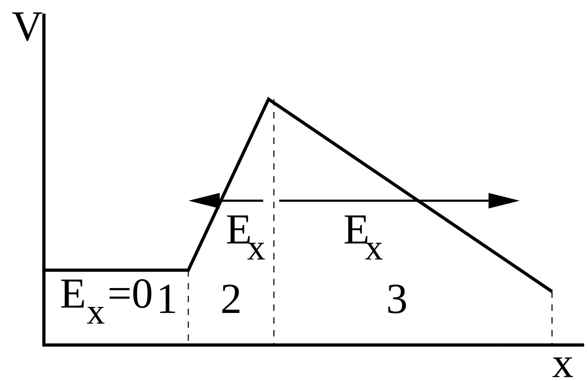
   (a) Draw the electric field lines of a charged spherical capacitor that consists of two concentric spherical conducting shells. The inner sphere has charge $-q$ and the outer sphere has charge $+q$.



   (b) What is the name of the unit of inductance?

   Answer: Henry

(c) The figure gives the electric potential V as a function of x. Rank the three regions according to the magnitude of the x component of the electric field within them, greatest first. Explain your reasoning. Draw an arrow indicating the direction of the electric field each region.



Answer: $E_2 > E_3 > E_1$. Note that $\vec{E} = -\nabla V$, i.e., $E_x = -\frac{\partial V}{\partial x}$, so the larger the slope, the larger the electric field. Note that positive charges want to go to lower voltages, and the electric field points in the direction that positive charges want to go.

(d) What is the first law of thermodynamics? Be sure to define all variables.

Answer: The change in the internal energy of a system equals the heat Q it absorbs plus the work W done *on* the system.

$$\Delta E_{int} = Q + W \tag{1}$$

$\Delta E_{int} = E_{int}(\text{final}) - E_{int}(\text{initial})$ = change in the internal energy of the system
Q = heat absorbed by the system
W = work done *on* the system. (Work done by the system would have the opposite sign.)
Differential form: $dE_{int} = dQ + dW$

2. Short Answer Problems II (5 points each)

(a) A 100 Ω resistor is connected to a 12 V battery. What is the current I in the resistor?
Answer: Ohm's law $V = IR$ or $\varepsilon = IR$. So

$$I = \frac{I}{R} = \frac{12\text{ V}}{100\Omega} = 0.12\text{ A} \tag{2}$$

$I = 0.12$ Amps

(b) You are to connect two identical capacitors, each with capacitance C, to a battery, first in series, and then in parallel. Which arrangement stores more charge? Explain your reasoning.

Answer: In parallel. The equivalent capacitance for capacitors in parallel is $C_{eq,p} = C_1 + C_2 = 2C$. The equivalent capacitance for capacitors in series is $1/C_{eq,s} = 1/C_1 + 1/C_2 = 2/C$ or $C_{eq,s} = C/2$. Since $Q = CV$, where V equals the voltage of the battery, the amount of charge stored is proportional to the capacitance. So arranging the capacitors in parallel stores more charge.

(c) If you stretch a cylindrical wire and it remains cylindrical, does the resistance of the wire (measured end to end along its length) increase, decrease, or remain the same? Explain your reasoning.

Answer: R increases. As you stretch the wire, it gets longer and thinner, but the volume V stays the same. Assume that the diameter of the wire remains uniform along its entire length L. Let A be the cross sectional area of the wire. Then $V = LA$. The resistance $R = \rho L/A$ where ρ is the resistivity. Since $A = V/L$,

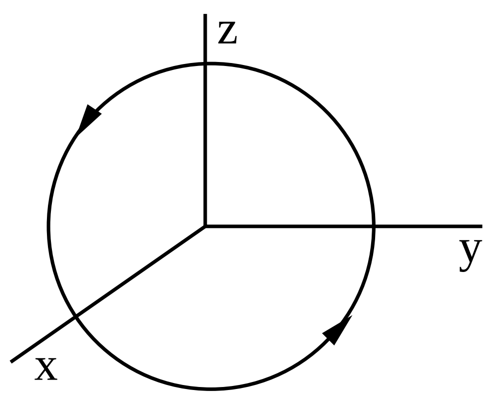
$$R = \rho \frac{L^2}{V} \tag{3}$$

so as L increases, $R \sim L^2$ increases. Alternatively, $L = V/A$, so

$$R = \rho \frac{V}{A^2} \tag{4}$$

As you stretch the wire, the cross sectional area A decreases because the wire gets thinner. So $R \sim 1/A^2$ increases.

(d) To an observer standing on the positive x axis, a positively charged particle goes counterclockwise in a circle in the $y - z$ plane due to the force from an external uniform magnetic field. What is the direction of the magnetic field $\mathbf{B}$?



Answer: Use the Lorentz Force Law: $\vec{F} = q\vec{v} \times \vec{B}$

direction of B $= -\hat{x}$

3. Short Answer Problems III (5 points each)

(a) We fully submerge an irregular 3 kg lump of material in a certain fluid. The fluid that would have been in the space now occupied by the lump has a mass of 2 kg. When we release the lump, does it move upward, move downward, or remain in place? Explain your reasoning.

Answer: The lump sinks downward. The gravitational force is mg downward, where $m = 3$ kg. The buoyant force is upwards. The magnitude of the buoyant force B is equal to the weight of the displaced fluid. So $B = (2 \text{ kg})g$ which is less than mg. So the lump sinks.

(b) A heat-gun paint stripper can easily bubble old paint applied on wooden surfaces but is unable to blister or bubble the same paint applied to metal surfaces. Explain why.

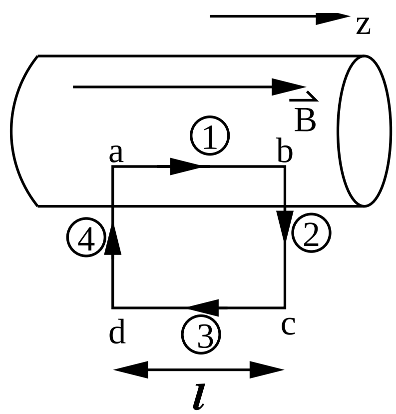
Answer: The paint blisters and bubbles when it gets hot and melts. This is what happens to the paint on the wood. Metal is a much better thermal conductor than wood. So the metal conducts the heat away before it can heat and melt the paint.

(c) When a furnace increases the temperature inside a house, the volume of the gas inside the house does not change, nor does the pressure change. Does this violate the ideal gas law? Explain your reasoning.

Answer: No, this does not violate the ideal gas law ($PV = nRT$) because the house is not air tight. When the furnace heats the air inside the house, the air expands and some of it escapes from the house. So the number of moles n of air decreases and compensates for the increase in the temperature T such that PV (or equivalently nRT) is the same before and after the furnace heated the house.

(d) An infinite solenoid carries a current I and has n turns of wire per unit length. What is the energy density u of the magnetic field inside the solenoid?

Answer: From Ampere's law, we can find the field inside the solenoid. Draw an Amperian loop as shown.



$I_{\text{enc}} = nI\ell$

$$\begin{aligned} \oint \vec{B} \cdot d\vec{s} &= \mu_0 I_{\text{enc}} \\ B\ell &= \mu_0 n I \ell \\ B &= \mu_0 n I \quad \text{inside the solenoid} \\ \vec{B} &= \mu_0 n I \hat{z} \end{aligned}$$

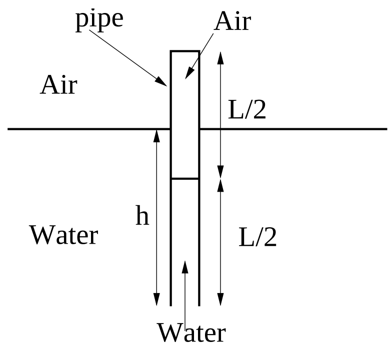
The energy density u is given by

$$u = \frac{B^2}{2\mu_o} = \frac{(\mu_o n I)^2}{2\mu_o} = \frac{\mu_o n^2 I^2}{2} \tag{5}$$

$u = \mu_o n^2 I^2 / 2$

WORK OUT PROBLEMS

4. (15 points) A pipe of length $L = 25$ m is open at one end and has a cap at the other end. The pipe contains air at atmospheric pressure. It is turned upside down so that the open end points down. Then it is thrust vertically into a freshwater lake until the water rises halfway up in the pipe. What is the depth h of the lower end of the pipe? In other words, how much of the pipe is below the water? Assume that the temperature is the same everywhere and does not change. Show your work. (Hint: Assume the air is an ideal gas.) The density ρ of water is 1000 kg/m^3.



Answer: The ideal gas law says $PV = nRT$. Since nRT is constant, PV is constant. The volume of the gas in the pipe gets reduced by half, so the pressure must be doubled to $2P_o$, where P_o is the atmospheric pressure. Use the equation that describes the variation of the pressure with depth. The pressure at the open end of the pipe at a depth h equals the pressure in the lake at a depth h. This gives

$$\begin{aligned}
P_o + \rho g h &= 2P_o + \rho g\left(\frac{L}{2}\right) \\
\rho g h &= P_o + \rho g\left(\frac{L}{2}\right) \\
h &= \frac{P_o}{\rho g} + \frac{L}{2} \\
&= \frac{1.013 \times 10^5 \text{ N/m}^2}{1000 \text{ kg/m}^3 \cdot 9.8 \text{ m/s}^2} + \frac{25 \text{ m}}{2} \\
&= 22.8 \text{ m}
\end{aligned}$$

Alternative solution: Solve for the distance x between the top of the water in the lake and the top of the water in the pipe. This gives

$$\begin{aligned}
2P_o &= P_o + \rho g x \\
x &= \frac{P_o}{\rho g} \\
h &= x + \frac{L}{2} \\
&= \frac{P_o}{\rho g} + \frac{L}{2} \\
&= 22.8 \text{ m}
\end{aligned}$$

$h = 22.8$ m

5. (15 points total) An elastic conducting material is stretched into a circular loop of 12.0 cm radius. It is placed with its plane perpendicular to a uniform 0.800 T magnetic field. When released, the radius of the loop starts to shrink at an instantaneous rate of 75.0 cm/s. (a) (12 points) What is the magnitude of the emf ε induced in the loop at that instant? Show your work.

(b) (3 points) If the loop lies in the plane of the page and the uniform external magnetic field points into the page, is the direction of the induced current in the loop clockwise or counterclockwise? Explain your reasoning.

Answer: (a) The area A of the loop is $A = \pi R^2$. The flux ϕ is given by

$$\phi = \int \vec{B} \cdot d\vec{a} = BA = B\pi R^2 \tag{6}$$

So from Faraday's Law:

$$\begin{aligned} \varepsilon &= -\frac{d\phi}{dt} \\ &= -B\pi\frac{dR^2}{dt} \\ &= -2\pi RB\frac{dR}{dt} \\ &= -2\pi(0.12 \text{ m})(0.8 \text{ T})(-0.75 \text{ m/s}) \\ &= 0.45 \text{ Volts} \end{aligned}$$

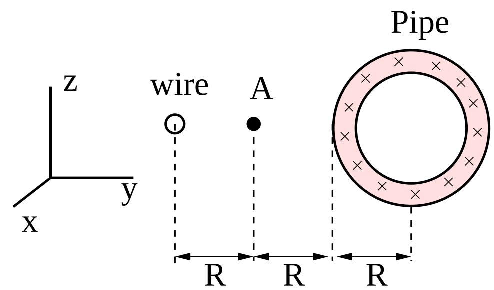
Note that since the loop is shrinking, the radius is decreasing and the rate that it shrinks is negative, i.e., $dR/dt = -$ 75 cm/s.

$\varepsilon = 0.45$ Volts

(b) The induced current will flow clockwise. Lenz' law says that the induced current produces magnetic flux that opposes the change in flux. The external flux is decreasing due to the shrinking loop, so the induced current will produce a supplemental magnetic field that points in the same direction as the external field, namely into the page.

Current direction is CLOCKWISE

6. (15 points) A long circular pipe with outside radius R carries a (uniformly distributed) current I_p into the page as shown. A wire runs parallel to the pipe at a distance of $3R$ from center to center. Find the magnitude and direction of the current I_w in the wire such that the net magnetic field at point A has the same magnitude as the net magnetic field at the center of the pipe but is in the opposite direction. Explain your reasoning and show your work.



Solution: The current in the pipe does not produce any magnetic field inside the pipe. You can see this by using Ampere's Law. The field inside the pipe is produced by the current in the wire.

We want the total magnetic field at A to be $\vec{B}_{tot}(A) = -\vec{B}_w(C)$ where $\vec{B}_{tot}(A)$ is the total field at point A and $\vec{B}_w(C)$ is the field at the center (C) of the pipe due to the wire. First determine the direction of the current in the wire. The field $\vec{B}_p(A)$ at A due to the pipe points up parallel to $\hat{z}$. If the current in the wire flows out of the page, then the field $\vec{B}_w(A)$ produced by the wire at A would also point up and add to $\vec{B}_p(A)$. If this were the case, the field $\vec{B}_w(C)$ at the center of the pipe would point up, parallel to $\vec{B}_{tot}(A)$, and be weaker than $B_{tot}(A)$. This is not what we want, so the current in the wire must flow into the page in the $-\hat{x}$ direction. In this case $\vec{B}_w(A)$ and $\vec{B}_w(C)$ point down, opposite to $\vec{B}_p(A)$.

Use Ampere's law to find the magnitude of the field $B_w(C)$ at the center (C) of the pipe. Let $\vec{B}_w$ be the field produced by the wire. Draw a circular Amperian loop centered at the wire that goes through the center of the pipe. It has radius $3R$.

$$\begin{aligned}
\oint \vec{B} \cdot d\vec{s} &= \mu_o I_{enc} \\
B_w(C) \cdot 2\pi(3R) &= \mu_o I_w \\
B_w(C) &= \frac{\mu_o I_w}{2\pi \cdot 3R} \\
&= \frac{\mu_o I_w}{6\pi R} \\
\vec{B}_w(C) &= -\frac{\mu_o I_w}{6\pi R}\hat{z}
\end{aligned}$$

Now find the field at point A due to the pipe. Denote the field due to the pipe by $\vec{B}_p$. At A, $\vec{B}_p(A)$ will point up. Use Ampere's law to find the magnitude of $B_p(A)$ at A. Draw an Amperian circle centered at the pipe and going through A. The radius of the circle is $2R$.

$$\begin{aligned}
\oint \vec{B} \cdot d\vec{s} &= \mu_o I_{enc} \\
B_p(A) \cdot 2\pi(2R) &= \mu_o I_p \\
B_p(A) &= \frac{\mu_o I_p}{2\pi \cdot 2R} \\
&= \frac{\mu_o I_p}{4\pi R} \\
\vec{B}_p(A) &= \frac{\mu_o I_p}{4\pi R}\hat{z}
\end{aligned}$$

From Ampere's law the field due to the wire at A is

$$\vec{B}_w(A) = -\frac{\mu_o I_w}{2\pi R}\hat{z} \tag{7}$$

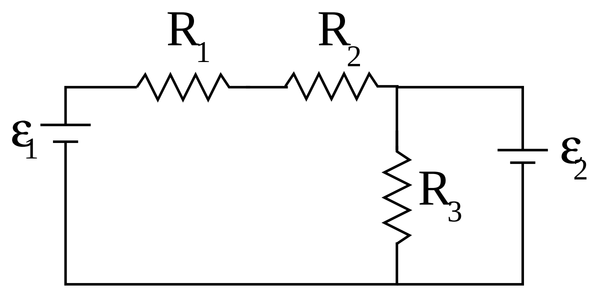
So the total field at A is

$$\begin{aligned}
\vec{B}_{tot}(A) &= \vec{B}_w(A) + \vec{B}_p(A) \\
&= -\frac{\mu_o I_w}{2\pi R}\hat{z} + \frac{\mu_o I_p}{4\pi R}\hat{z} = \frac{\mu_o I_w}{6\pi R}\hat{z} \\
-\frac{I_w}{2} + \frac{I_p}{4} &= \frac{I_w}{6} \\
-\frac{2I_w}{3} &= -\frac{I_p}{4} \\
I_w &= \frac{3}{2}\frac{I_p}{4} \\
I_w &= \frac{3I_p}{8}
\end{aligned}$$

direction of $I_w = -\hat{x}$ or into the page

magnitude of $I_w = 3I_p/8$

7. (15 points) The circuit shown has two ideal batteries with emfs $\varepsilon_1 = 600$ V and $\varepsilon_2 = 100$ V. There are 3 resistors: $R_1 = 50\ \Omega$, $R_2 = 50\ \Omega$, and $R_3 = 200\ \Omega$. How much power P_1 is dissipated in the resistor R_1?



Answer: From the loop rule

$$\begin{aligned}
\varepsilon_1 - I_1 R_1 - I_1 R_2 - \varepsilon_2 &= 0 \\
(\varepsilon_1 - \varepsilon_2) - I_1 (R_1 + R_2) &= 0 \\
(\varepsilon_1 - \varepsilon_2) &= I_1 (R_1 + R_2) \\
I_1 &= \frac{(\varepsilon_1 - \varepsilon_2)}{(R_1 + R_2)}
\end{aligned}$$

The power dissipated in R_1 is

$$\begin{aligned}
P_1 &= I_1^2 R_1 \\
&= \left(\frac{\varepsilon_1 - \varepsilon_2}{R_1 + R_2}\right)^2 R_1 \\
&= \frac{(600\text{ V} - 100\text{ V})^2 \cdot 50\ \Omega}{(50\ \Omega + 50\ \Omega)^2} \\
&= \frac{(500\text{ V})^2 \cdot 50\ \Omega}{(100\ \Omega)^2} \\
&= 1250\text{ W}
\end{aligned}$$

$P_1 = 1250$ Watts