VERSION B

1. (20 points) 110 kJ of heat is added to a block of ice of mass $m = 720$ g initially at -10 ^{o}C. What is the final amount of ice in grams? What is the final amount of water in grams? What is the final temperature? The specific heat of ice is $c_{ice} = 2220$ J/kg-K, the specific heat of water is $c_{liq} = 4190$ J/kg-K, and the heat of fusion is $L_F = 333$ kJ/kg.

Answer: The strategy is similar to having to budget a finite amount of money and seeing if you can afford things. So split the problem into 2 stages. First calculate the amount of heat Q_h needed to raise the temperature from the initial temperature of -10 ^{o}C to 0 ^{o}C. If that does not totally expend the budget of 110 kJ, then we will calculate the heat Q_{melt} needed to melt the ice at 0 ^{o}C.

The heat Q_h needed to raise the temperature from the initial temperature of -10 ^{o}C to 0 ^{o}C:

$$\begin{aligned} Q_h &= c_{ice} m \Delta T \\ &= (2220 \text{ J/kg} - \text{K})\,(0.72 \text{ kg})\,(10\ ^o\text{C}) \\ &= 16 \text{ kJ} \end{aligned}$$

So there is enough heat to warm the ice to 0 ^{o}C. The amount of heat that remains is $\Delta Q = 110$ kJ $-$ 16 kJ = 94 kJ. This can be used to melt ice. Let us see how much heat Q_{melt} is needed to melt the ice:

$$\begin{aligned} Q_{melt} &= m L_F \\ &= (0.72 \text{ kg})(333 \text{ kJ/kg}) \\ &= 240 \text{ kJ} \end{aligned}$$

So there is not enough heat left to melt the whole block. Only part of it will melt. Let us calculate the fraction F that will melt:

$$\begin{aligned} F &= \frac{94 \text{ kJ}}{240 \text{ kJ}} \\ &= 0.392 \end{aligned}$$

So there will be a mixture of ice and water. The temperature of the mixture is 0 ^{o}C. The amount of water is (0.392)(720 g)=282 g. The amount of ice is the remainder; (original amount of ice) - (water) = (final amount of ice). (720 g) - (282 g) = 438 g of ice.

amount of ice = 438 g

amount of water = 282 g

$T_{final} = 0^o$ C

2. (20 points) 2 moles of an ideal diatomic gas undergoes an adiabatic expansion in 3 dimensions. The work done by the gas is -200 J. What is the temperature change ΔT in Kelvin? $\Delta T = T_f - T_i$ where T_f is the final temperature and T_i initial temperature. Ignore the vibrational degrees of freedom.

   Answer: From the first law of thermodynamics,

$$\begin{aligned} \Delta E_{int} &= Q + W \\ &= W \end{aligned}$$

   since $Q = 0$. A diatomic gas has 5 degrees of freedom (3 translational and 2 rotational) . So

$$\begin{aligned} \Delta E_{int} &= \frac{5}{2} nR\Delta T \\ \Delta T &= \frac{\Delta E_{int}}{\frac{5}{2}nR} \\ &= \frac{W}{\frac{5}{2}nR} \\ &= -\frac{200\text{J}}{\frac{5}{2}2R} \\ &= -\frac{200}{5R}\text{J} \\ &= -\frac{200\text{J}}{5 \cdot 8.314\text{J/mole} - \text{K}} \\ &= -4.81K \end{aligned}$$

$\Delta T = -4.81$ K

3. (20 points) **SHORT ANSWER**

(a) (4 points) What is the constant volume molar specific heat C_V of an ideal diatomic gas in 2 dimensions? (Ignore vibrational degrees of freedom.) Explain your reasoning.

Answer:

$$C_V = \left.\frac{dQ}{dT}\right|_V \quad (1)$$

From the first law of thermodynamics, for $V =$ const and $W = 0$,

$$\Delta E_{int} = Q \quad (2)$$

So we need to find ΔE_{int}. To do this, we use the equipartition theorem which says that the internal energy per molecule is $k_B T/2$ for each degree of freedom. Let's add up the degrees of freedom. There are 2 translational degrees of freedom since we are in 2 dimensions and 1 rotational degree of freedom which corresponds to rotating in the plane about an axis perpendicular to the plane. Thus there are 3 degrees of freedom and the internal energy per molecule is $3k_B T/2$. For 1 mole, the internal energy is

$$E_{int} = \frac{3}{2}RT \quad (3)$$

So the molar specific heat is

$$\begin{aligned} C_V &= \frac{dE_{int}}{dT} = \frac{3}{2}R \\ &= \frac{3}{2}(8.314\text{J/mol})) = 12.47\text{J/mol} \end{aligned}$$

where R is the gas constant.

$C_V = \frac{3}{2}R = 12.47$ J/mol

(b) (4 points) What is the first law of thermodynamics? Be sure to define all variables.

Answer: The change in the internal energy of a system equals the heat Q it absorbs plus the work W done *on* the system.

$$\Delta E_{int} = Q + W \quad (4)$$

$\Delta E_{int} = E_{int}$(final) $-E_{int}$(initial) = change in the internal energy of the system
Q = heat absorbed by the system
W = work done *on* the system. (Work done by the system would have the opposite sign.)
Differential form: $dE_{int} = dQ + dW$

(c) (4 points) A hollow ball of aluminum floats in water. If the ball is heated (but does not melt), will the ball

i. ride lower in the water
ii. ride higher in the water
iii. stay at the same height

ii. ride higher in the water

Explain your answer.

Answer: Before the ball is heated, the ball's weight balances the buoyancy force. The ball will expand when it is heated. The buoyancy force equals the weight of the water displaced. If the ball stays at the same level in the water, it will displace more water and the buoyancy force upward will increase. But the ball's weight (mg) stays the same. So the buoyancy force B pushes the ball upward until $B = mg$. So the ball rides higher in the water.

(d) (4 points) An ideal gas is initially at a temperature of 300 K. If the temperature of the gas is increased to 1200 K, the rms molecular speed v_{rms}

i. increases
ii. decreases
iii. remains the same

i. increases

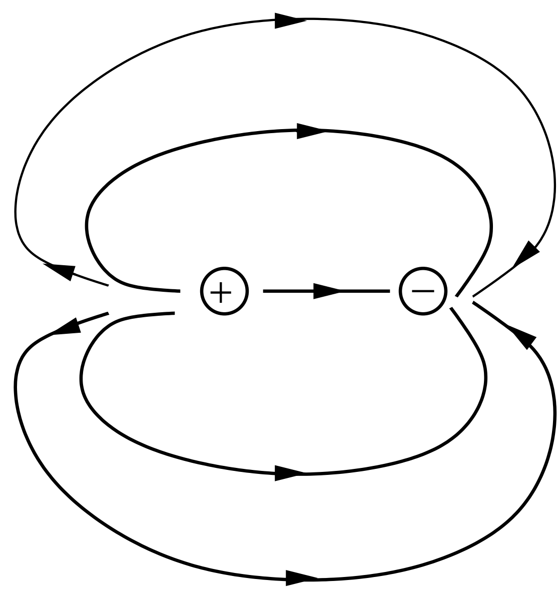
If the speed changes, by what factor does it change? Show your work.

Answer: The final temperature $T_f = 4T_i$ where T_i is the initial temperature. From the equipartition theorem:

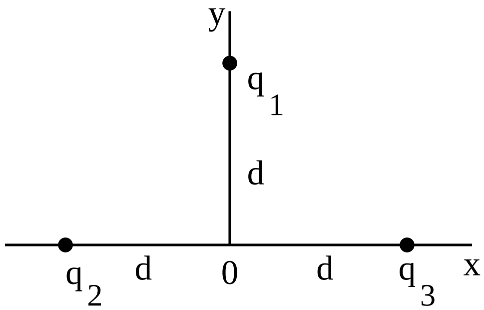
$$\begin{aligned}
\frac{3}{2}k_B T &= \frac{1}{2}m\overline{v}^2 \\
v_{rms}^2 &= \frac{3k_B T}{m} \\
v_{rms,i} &= \sqrt{\frac{3k_B T_i}{m}} \\
v_{rms,f} &= \sqrt{\frac{3k_B T_f}{m}} \\
&= \sqrt{\frac{3k_B \cdot 4T_i}{m}} \\
&= \sqrt{4}v_{rms,i} \\
&= 2v_{rms,i}
\end{aligned}$$

factor = 2

(e) (5 points) Sketch the electric field lines for 2 equal and opposite point charges.



4. (20 points) In the figure charges $q_1 = q_2 = +e$, and charge $q_3 = +3e$. What are the components of the force $\mathbf{F}_{1,tot}$ on charge q_1? State your answer in terms of the electron charge e, the distance d, and $k_e = 1/(4\pi\varepsilon_o)$.



Answer: From the principle of superposition,

$$\vec{F}_{1,tot} = \vec{F}_{1\leftarrow 2} + \vec{F}_{1\leftarrow 3} \tag{5}$$

From Coulomb's Law:

$$\vec{F}_{1\leftarrow 2} = k_e \frac{q_1 q_2}{r_{12}^2}\,\hat{r}_{12} \qquad \hat{r}_{12} = \frac{\vec{r}_1 - \vec{r}_2}{|r_1 - r_2|}$$

Method I: Add vectors The coordinates of q_1 are $(0, d)$. The coordinates of q_2 are $(-d, 0)$. The coordinates of q_3 are $(d, 0)$.

$$\begin{aligned}
\vec{F}_{1\leftarrow 2} &= k_e \frac{e^2}{2d^2}\left[\frac{0-(-d)}{\sqrt{2}d}\,\hat{\imath} + \frac{d-0}{\sqrt{2}d}\,\hat{\jmath}\right] \\
&= k_e \frac{e^2}{2d^2}\left[\frac{1}{\sqrt{2}}\,\hat{\imath} + \frac{1}{\sqrt{2}}\,\hat{\jmath}\right] \\
\vec{F}_{1\leftarrow 3} &= k_e \frac{3e^2}{2d^2}\left[\frac{0-d}{\sqrt{2}d}\,\hat{\imath} + \frac{d-0}{\sqrt{2}d}\,\hat{\jmath}\right] \\
&= k_e \frac{3e^2}{2d^2}\left[-\frac{1}{\sqrt{2}}\,\hat{\imath} + \frac{1}{\sqrt{2}}\,\hat{\jmath}\right] \\
\vec{F}_{1,tot} &= \vec{F}_{1\leftarrow 2} + \vec{F}_{1\leftarrow 3} \\
&= k_e \frac{e^2}{2d^2}\frac{1}{\sqrt{2}}\left[(1-3)\,\hat{\imath} + (1+3)\,\hat{\jmath}\right] \\
&= k_e \frac{e^2}{2\sqrt{2}d^2}\left[-2\,\hat{\imath} + 4\,\hat{\jmath}\right] \\
&= k_e \frac{e^2}{\sqrt{2}d^2}\left[-\,\hat{\imath} + 2\,\hat{\jmath}\right]
\end{aligned}$$

Method II: Trigonometry

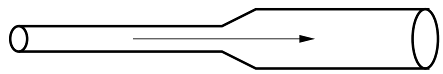
$$\sin\theta = \frac{d}{\sqrt{2}d}$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{2}} \\
\cos\theta &= \frac{d}{\sqrt{2}d} \\
&= \frac{1}{\sqrt{2}} \\
\vec{F}_{1\leftarrow 2} &= k_e \frac{e^2}{2d^2}\left[\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}\right] \\
&= k_e \frac{e^2}{2d^2}\left[\frac{1}{\sqrt{2}}\,\hat{\imath} + \frac{1}{\sqrt{2}}\,\hat{\jmath}\right] \\
&= k_e \frac{e^2}{2\sqrt{2}d^2}\left[\,\hat{\imath} + \,\hat{\jmath}\right] \\
\vec{F}_{1\leftarrow 3} &= k_e \frac{3e^2}{2d^2}\left[-\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}\right] \\
&= k_e \frac{3e^2}{2d^2}\left[-\frac{1}{\sqrt{2}}\,\hat{\imath} + \frac{1}{\sqrt{2}}\,\hat{\jmath}\right] \\
&= k_e \frac{3e^2}{2\sqrt{2}d^2}\left[-\,\hat{\imath} + \,\hat{\jmath}\right] \\
\vec{F}_{1,tot} &= \vec{F}_{1\leftarrow 2} + \vec{F}_{1\leftarrow 3} \\
&= k_e \frac{e^2}{2\sqrt{2}d^2}\left[(1-3)\,\hat{\imath} + (1+3)\,\hat{\jmath}\right] \\
&= k_e \frac{e^2}{2\sqrt{2}d^2}\left[-2\,\hat{\imath} + 4\,\hat{\jmath}\right] \\
&= k_e \frac{e^2}{\sqrt{2}d^2}\left[-\,\hat{\imath} + 2\,\hat{\jmath}\right]
\end{aligned}
$$

$F_x = -k_e \frac{e^2}{\sqrt{2}d^2}$

$F_y = k_e \frac{2e^2}{\sqrt{2}d^2} = k_e \frac{\sqrt{2}e^2}{d^2}$

5. (20 points) Water flows at a speed of 2.0 m/s through a horizontal and level pipe with a circular cross-sectional area of 10 cm^2. The water then flows into a wider pipe with twice the diameter. What is the pressure difference ΔP between the two sections of pipe? The density of water is 1000 kg/m^3.



Answer: Let the left side of the pipe be 1 and the right side be 2. From the principle of continuity

$$
\begin{aligned}
A_1 v_1 &= A_2 v_2 \\
v_2 &= \frac{A_1}{A_2} v_1
\end{aligned}
$$

where v_1 and v_2 are the velocities and A_1 and A_2 are the cross sectional areas of the sections of pipe. The cross sectional area is

$$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2 \tag{6}$$

where r is the radius and d is the diameter. Since $d_2 = 2d_1$,

$$A_2 = 4A_1 \tag{7}$$

So

$$
\begin{aligned}
v_2 &= \frac{A_1}{A_2} v_1 \\
&= \frac{A_1}{4A_1} v_1 \\
&= \frac{v_1}{4}
\end{aligned}
$$

Bernoulli's principle:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \tag{8}$$

The pipe is horizontal, so $y_1 = y_2 = 0$. So we have for the pressure difference

$$
\begin{aligned}
\Delta P &= P_1 - P_2 \\
&= \frac{1}{2}\rho \left(v_2^2 - v_1^2\right) \\
&= \frac{1}{2}\rho \left[(v_1/4)^2 - v_1^2\right] \\
&= \frac{1}{2}\rho v_1^2 \left(\frac{1}{16} - 1\right) \\
&= \frac{1}{2}\left(1000 \text{ kg/m}^3\right)(2 \text{ m/s})^2 \left(-\frac{15}{16}\right) \\
&= -1875 \text{ Pa}
\end{aligned}
$$

$\Delta P = 1875$ Pa

Which section of pipe has the fluid at higher pressure?

(a) Narrow section of pipe
(b) Wide section of pipe
(c) Pressures are equal in the two sections

(b) Wide section of pipe