Key Points on Chapter 15: Fluid Mechanics

- Pressure is force per unit ________________________.
- Pressure just depends on the ________________________ in the fluid.
- The ________________________ force on an object is an upward force produced by the liquid.
- The buoyant force is equal to the ________________________ of the fluid displaced by the object.
- Fluid flows ________________________ in a narrower pipe.
- Moving fluids exert ________________________ pressure than stationary fluids.

Lecture on Chapter 15: Fluid Mechanics

Discuss Syllabus.

States of Matter

Matter is normally classified as being in one of 3 states: solid, liquid, or gas. A solid maintains its volume and shape. A liquid has a definite volume but assumes the shape of its container. A gas assumes the volume and shape of its container. We've all heard of these catagories, but not everything can be easily put into one of these catagories. For example, what about you? Your bones are solid, and your blood is a liquid (suspension). But what about your flesh? What about sand? Each grain is a solid but pour a pile of sand into a bucket and it assumes the shape of its container like a liquid. Setting these netteling questions aside, let us consider fluids. A fluid is a collection of molecules that are randomly arranged and held together by weak cohesive forces between molecules and by forces exerted by the walls of a container. Both liquids and gases are fluids.

Pressure

Suppose a force is applied to the surface of an object with components parallel and perpendicular to the surface. Assume that the object does not slide. Then the force parallel to the surface may cause the object to distort. Do this to a book. The force parallel to the surface is called a shearing force or a shear force. A fluid cannot sustain a shear force. If you put your hand on the surface of a pool of water and move your hand parallel to the surface, your hand will slide along the surface. You will not be able to distort the water in the same way as the book. This provides an operational definition of a fluid; a fluid cannot sustain an infinitely slow shear force. If the shear force is fast enough, the water can display some elasticity. For example, consider skipping a stone across a pond. But let's just consider the slow case. Since a fluid cannot sustain a (slow) shear, let's ignore the shear force.

Key Point: Pressure is force per unit ________________________.

This leaves the force that is perpendicular to the surface. Suppose the fluid is in a container. The fluid exerts a force on the walls of the container because the molecules of the fluid collide and bounce off the walls. By the impulse–momentum theorem and Newton's third law, each collision exerts a force on the wall. There are a huge number of collisions every second resulting in a constant macroscopic force. The force is spread over the area of the wall. **Pressure** is defined as the ratio of the force to the area:

$$P = \frac{F}{A} \tag{1}$$

Pressure has units of N/m^2. Another name of this is the pascal (Pa):

$$1 \ \text{Pa} = 1 \ \text{N/m}^2 \tag{2}$$

Force and pressure are 2 different things. Force is a vector; it has a direction. Pressure is a scalar (a number); it doesn't have a direction. A small amount of force applied to a very tiny area can produce a large pressure. Consider a hypodermic needle which easily punctures your skin. Contrast this with lying on a bed of nails. If you lie on 1 nail, you get impaled. But if you lie on a steel mattress, there is no problem. Even if you lie on a bed of nails, your weight is distributed over the nails and the force per unit area or pressure is small. Snow shoes, skis and snowboards work this way. So does a surfboard. You can't stand on water but you can stand on a surfboard and slide down the face of a wave because the board distributes the force of your weight over the area of the board. The atmosphere produces pressure. Atmospheric pressure is given by

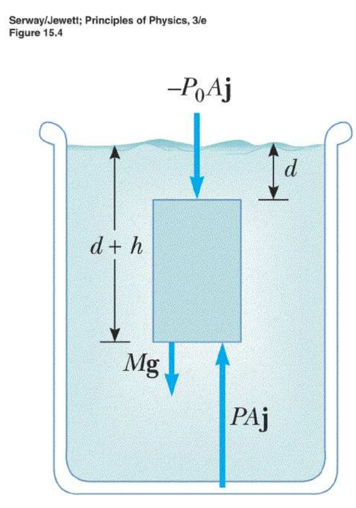
$$P_{\text{atm}} = 1.00 \text{ atm } \approx 1.013 \times 10^5 \text{ Pa} \tag{3}$$

Atmospheric pressure is what allows suction cups to work. You press the cup onto a flat surface which pushes the air out from under the cup. When you let go of the cup, it tries to spring back. There is not much air under the cup so it doesn't apply much pressure to the underside of the cup. The atmospheric pressure outside the cup is much stronger, so it pushes the cup against the flat surface.

Variation of Pressure with Depth

Key Point: Pressure just depends on the ________________________ in the fluid.

Pressure in a fluid varies with the depth of the fluid. The deeper down you go in the ocean, the higher the water pressure because there is more water on top of you and water weighs a lot (8 pounds per gallon). This is why deep sea divers must wear diving suits. The higher up you go in the atmosphere, the lower the atmospheric pressure. This is why airplanes must pressurize their cabins. Also the air becomes less dense at high altitudes, so your lungs would not get enough oxygen if the cabin were not pressurized.



We can derive a mathematical relation showing how the pressure varies with depth. Consider a liquid of density ρ at rest. Consider a portion of the liquid contained within an imaginary cylinder of cross–sectional area A. The height of the cylinder is h. So if the top of the cylinder is submerged a distance d below the surface of the liquid, the bottom of the cylinder is a distance $d + h$ below the surface of the liquid. Since the sample of the liquid is at rest, the net force on the sample must be zero by Newton's second law. So let's add up all the forces on the sample and set the sum equal to 0. The pressure on the bottom face of the cylinder is P. This is the pressure from the liquid below the cylinder. The associated force PA is pushing up. The liquid on top of the cylinder is pushing down. The pressure is P_0 and the associated force is $-P_0A$. The minus sign means the force is pointing down. There is also gravity which applies a force of $-mg$ where m is the mass of the liquid in the cylinder. So we can write

$$PA - P_0A - mg = 0 \tag{4}$$

The density of the liquid is ρ. Density is the mass per unit volume. So mass of the liquid sample is $m = \rho V = \rho Ah$ where V is the volume of the cylinder. This means that $mg = \rho gAh$. Thus,

$$PA = P_0A + \rho gAh \tag{5}$$

Cancelling the area A on each side of the equation gives:

$$P = P_0 + \rho gh \tag{6}$$

This equation indicates that the pressure in a liquid depends only on the depth h within the liquid. The pressure is therefore the same at all points having the same depth, independent of the shape of the container. Eq. (6) also indicates that any increase in pressure at the surface must be transmitted to every point in the liquid. This was first recognized by the French scientist Blaise Pascal and is called Pascal's law: *A change in the pressure applied to an enclosed liquid is transmitted undiminished to every point of the fluid and to the walls of the container.*

This explains how a hydraulic press works. A force $\vec{F}_1$ is applied to a small piston of cross sectional area A_1. The pressure is transmitted through a liquid to a larger piston of area A_2, and force $\vec{F}_2$ is exerted by the liquid on the piston. Because the pressure is the same at both pistons, we must have

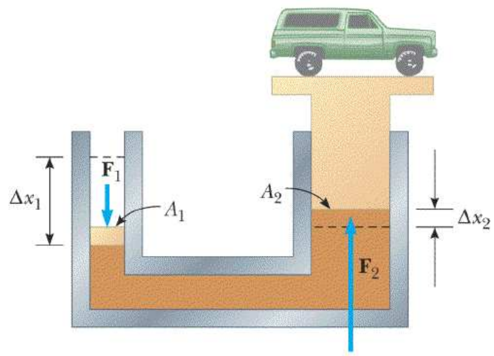
$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \tag{7}$$

or

$$F_2 = F_1\frac{A_2}{A_1} \tag{8}$$

Thus $F_2 > F_1$ if $A_2 > A_1$. The hydraulic lift amplifies the force F_1. Hydraulic brakes, car lifts, hydraulic jacks, and forklifts all use this principle.

Serway/Jewett; Principles of Physics, 3/e
Figure 15.5a



(a)

Pressure Measurements

Barometric pressure is often mentioned as part of weather reports. This is the current pressure of the atmosphere which varies a little from the standard pressure of 1 atm. A barometer is used to measure the atmospheric pressure. It was invented by Evangelista Torricelli (1608–1647). A long tube closed at one end is filled with mercury and then inverted into a dish of mercury. The pressure at the closed end is basically zero since it's a vacuum there. The pressure at point A at the mouth of the tube must be the same as point B outside the tube on the surface of the mercury since point A and point B are at the same height above the ground. If this were not the case, mercury would move until the net force was zero. So the atmospheric pressure must be given by

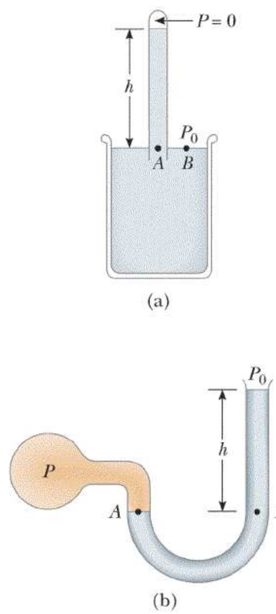
$$P_0 = \rho_{\mathrm{Hg}} g h \tag{9}$$

or

$$h = \frac{P_0}{\rho_{\mathrm{Hg}} g} \tag{10}$$

If $P_0 = 1$ atm $= 1.013 \times 10^5$ Pa, $h = 0.760$ m $= 760$ mm.

Serway/Jewett; Principles of Physics, 3/e
Figure 15.7



The open–tube manometer is a device for measuring the pressure of a gas contained in a vessel. One end of the U–shaped tube containing a liquid is open to the atmosphere, and the other end is connected to a system of unknown pressure P. The pressures at points A and B must be the same otherwise there would be a net force and the liquid would accelerate. The pressure at A is the unknown pressure P of the gas. Equating the pressures at A and B, we can write

$$P = P_0 + \rho g h \tag{11}$$

where P_0 is the atmospheric pressure. P is called the *absolute pressure* and $P - P_0$ is called the *gauge pressure*. The pressure you measure in a car tire is gauge pressure.

Example: Problem 15.12. Imagine Superman attempting to drink water through a very long straw. With his great strength he achieves maximum possible suction. The walls of the tubular straw do not collapse. (a) Find the maximum height through which he can lift the water. (b) Still thirsty, the Man of Steel repeats his attempt on the Moon, which has no atmosphere. Find the difference between the water levels inside and outside the straw.

Answer: Superman can produce a perfect vacuum in the straw. Take point B at the water surface in the basin and point A at the water surface in the straw. (see Figure 15.7 for location of points A and B)

$$\begin{aligned} P_B + \rho g y_B &= P_A + \rho g y_A \\ 1.013 \times 10^5\ \mathrm{N/m^2} + 0 &= 0 + \left(1000\ \mathrm{kg/m^3}\right)\left(9.80\ \mathrm{m/s^2}\right) y_A \\ y_A &= 10.3\ \mathrm{m} \end{aligned}$$

Buoyant Forces and Archimedes' Principle

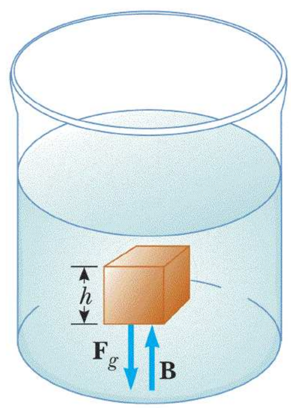
Key Point: The ________________________ force on an object is an upward force produced by the liquid.

A buoyant force is an upward force exerted on an object by the surrounding fluid. Buoyant forces are what keep ships and boats afloat. They also are the reason it's easier to lift someone in the water than on dry ground (JFK example).

Key Point: The buoyant force is equal to the ________________________ of the fluid displaced by the object.

The magnitude of the buoyant force is given by Archimedes's principle: *Any object completely or partially submerged in a fluid experiences an upward buoyant force whose magnitude is equal to the weight of the fluid displaced by the object.*

Serway/Jewett; Principles of Physics, 3/e
Figure 15.8



We can write down a formula to express this. Consider a cubic volume of fluid at rest inside a container full of fluid. Gravity exerts a force $-mg$ on the cube of fluid. Since it's at rest, there must be an upward force cancelling gravity. This upward force is the buoyant force B. Thus we can write

$$B = mg \tag{12}$$

If the density of the fluid is ρ_f, then $m = \rho_f V$, and we can write

$$B = \rho_f g V \tag{13}$$

where V is the volume of the cube. Remember how we said that the pressure in a fluid varies with depth? The buoyant force B is just the difference in the force between the top and bottom of the cube:

$$B = F_{bot} - F_{top} = P_{bot} A - P_{top} A \tag{14}$$

where the pressure P_{bot} on the bottom of the cube is greater than the pressure P_{top} on the top.

Now suppose we replace the cubic volume of fluid with a cube of steel (same volume). The buoyant force on the steel is the same as the buoyant force was on the cube of fluid with the same dimensions. This is true for a submerged object of any shape, size, or density. Suppose we have a submerged object with volume V_O and density ρ_O. The force due to gravity is $Mg = \rho_O V_O g$. The buoyant force is $B = \rho_f V_O g$. So the net force on the object is

$$F_{\text{net}} = B - Mg = (\rho_f - \rho_O)\, V_O g \tag{15}$$

This equation implies that if the density of the object is less than the density of the liquid, the net force will be positive, so the object rises and will float. If the density of the object is greater than the fluid, the net force will be negative (downward) and the object will sink.

Now consider an object that floats on the surface of the fluid. That means that it is only partially submerged. So the volume V of displaced fluid is only a fraction of the total volume V_O of the object. Because the object is in equilibrium, the buoyant force must balance gravity:

$$\begin{aligned} B &= Mg \\ \rho_f V g &= Mg \\ \rho_f V g &= \rho_O V_O g \\ \frac{V}{V_O} &= \frac{\rho_O}{\rho_f} \end{aligned} \tag{16}$$

Thus, the fraction of the volume of the object that is submerged under the surface of the fluid is equal to the ratio of the object density to the fluid density.

Fish are able to change the depths at which they swim by changing the amount of gas in its swim bladder which is a gas–filled cavity inside the fish. Increasing the size of the bladder increases the amount of water displaced and hence the buoyant force. So the fish rises. Decreasing the size of the bladder allows the fish to sink deeper.

A ship floats because the buoyant force balances the weight of the ship. If the ship takes on extra cargo, it rides lower in the water because the extra volume of displaced water means the buoyant force is increased to compensate for the increased weight.

Fluid Dynamics

So far we've considered fluids at rest. Now let's turn our attention to fluid dynamics, i.e., fluids in motion. There are 2 ways in which fluids can flow. The first way is steady, smooth flow that is called *laminar* flow. Each bit of fluid follows a smooth path so that different paths never cross each other. We can imagine a "velocity field" in which each point in the fluid is associated with a vector that corresponds to the velocity of the fluid at that point. For laminar flow, the velocity of the fluid at each point remains constant in time. In other words, the velocity field is does not change with time. The other type of flow is turbulent flow. Turbulent flow is irregular and has whirlpools. Think of white water rapids.

Viscosity is often used to characterize the ease of flow. Honey is very viscous. Highly viscous fluids resist flow and do not flow easily, e.g., ketchup. There is a sort of internal friction as parts of the fluid try to flow or move past other parts.

Fluids can be very complicated. To make things simpler, we make the following 4 assumptions about the fluids we will consider:

1. **Nonviscous fluid** which has no resistance to flow.

2. **Incompressible fluid** in which the density of the fluid is assumed to remain constant regardless of the pressure in the fluid.

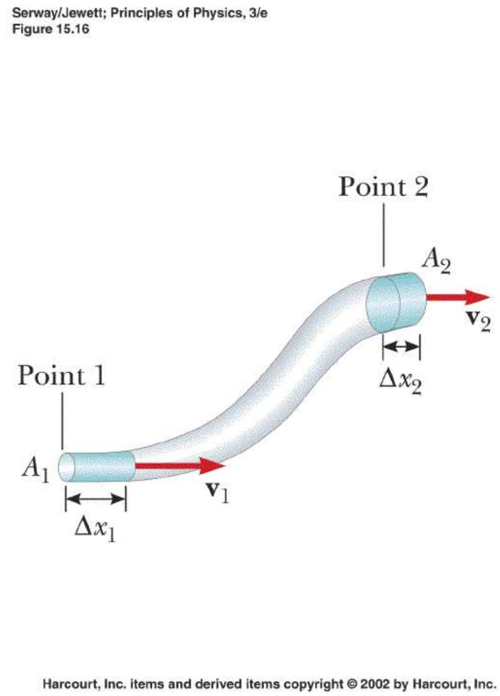
3. **Steady flow** where the velocity field remains constant in time.

4. **Irrotational flow** which means the fluid about any point has no angular angular momentum. So a small paddle wheel placed anywhere in the fluid does not rotate.

The first 2 assumptions are properties of our ideal fluid. The last 2 describe the way the fluid flows.

Streamlines and the continuity equation for fluids

The path taken by a particle during steady flow is called a *streamline.* The velocity of the particle is tangent to the streamline. Streamlines cannot cross because if they did, a particle would have a choice of paths and would sometimes take one path and sometimes the other path. Then the flow wouldn't be steady. A set of streamlines forms a *tube of flow.* It's like a pipe with invisible walls.



Consider fluid flowing in a pipe which is wider at the exit than at the entrance. Steady flow means that the amount or volume of fluid entering a pipe in a time interval δt must equal the volume leaving it in this time interval δt. Let the cross–sectional area at the entrance be A_1 and let the cross–sectional area at the exit be A_2. Suppose that in a time interval Δt, a volume V of fluid enters the pipe. Since the cross–sectional area at the entrance is A_1, the length of the fluid segment must be Δx_1 such that $V = A_1 \Delta x_1$. Since the fluid is incompressible, this same volume of fluid must exit the pipe. The volume of the exiting fluid is $V = A_2 \Delta x_2$ where Δx_2 is the length of the segment of departing fluid. Thus we can write

$$V = A_1 \Delta x_1 = A_2 \Delta x_2 \tag{17}$$

If we divide this equation by the time interval Δt, we have

$$\frac{A_1 \Delta x_1}{\Delta t} = \frac{A_2 \Delta x_2}{\Delta t} \tag{18}$$

In the limit that $\Delta t \to 0$, $\Delta x / \Delta t \to v$ and we can write

$$A_1 v_1 = A_2 v_2 \tag{19}$$

This equation is called the **continuity equation for fluids**. It says that the product of the cross–sectional area of the fluid and the fluid speed is a constant at all points along the pipe. So if A decreases, then v must increase.

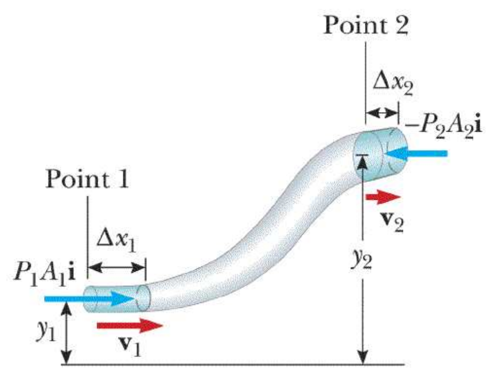
Key Point: Fluid flows ________________________ in a narrower pipe.

This is why water squirts out faster from a nozzle or when you put your thumb across part of the opening of a hose. You decrease the area by constricting the opening and so the water speeds up. The product Av has the dimensions of volume per time and is called the *volume flow rate*.

Bernoulli's Principle

You may have noticed that when you take a shower, the shower curtain moves inwards toward you. This is because when water and air flow past the shower curtain, they produce less pressure on the curtain than when the air is still as it is on the outside of the curtain. Since the pressure is greater outside than inside the shower, the curtain moves inward.

Serway/Jewett; Principles of Physics, 3/e
Figure 15.17



This is an example of Bernoulli's principle which explicitly shows the dependence of pressure on speed and elevation. Consider a section of fluid between point 1 and 2 in a pipe through which fluid is flowing. At the beginning of the time interval Δt, the section is between point 1 and point 2. At the end of the time interval, the section has moved to be between (point 1 + Δx_1) and (point 2 + Δx_2). Suppose the pipe changes elevation and that its diameter changes so that it has cross–sectional area A_1 at the left end and cross–sectional area A_2 at the right end of the section we are considering. Energy is conserved so we can write

$$\Delta K + \Delta U = W \tag{20}$$

Since the cross–sectional area is different between points 1 and 2, the velocities and hence the kinetic energies are different. So

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 \tag{21}$$

Here m is the mass of fluid in a little chunk of volume $V = A_2\Delta x_2 = A_1\Delta x_1$. The chunk at point 1 and point 2 have the same volume and the same mass because the fluid is incompressible.

The change in elevation means that the gravitational potential energy changes:

$$\Delta U = mgy_2 - mgy_1 \tag{22}$$

Finally we evaluate the work done on the section of fluid. The fluid to the left of our section is pushing on our section and so is doing work $F_1\Delta x_1$ on it. The fluid to the right is pushing against the flow and so the displacement is opposite to the direction of the force which gives negative work $-F_2\Delta x_2$. The net work done on the system is

$$\begin{aligned} W &= F_1\Delta x_1 - F_2\Delta x_2 \\ &= P_1A_1\Delta x_1 - P_2A_2\Delta x_2 \\ &= P_1V - P_2V \end{aligned} \tag{23}$$

Plugging into Eq. 20, we get

$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgy_2 - mgy_1 = P_1V - P_2V \tag{24}$$

Dividing both sides by V gives

$$\frac{1}{2}\left(\frac{m}{V}\right)v_2^2 - \frac{1}{2}\left(\frac{m}{V}\right)v_1^2 + \left(\frac{m}{V}\right)gy_2 - \left(\frac{m}{V}\right)gy_1 = P_1 - P_2 \tag{25}$$

Using $\rho = m/V$, we obtain

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gy_2 \tag{26}$$

This is **Bernoulli's equation**. It is often expressed as

$$P + \frac{1}{2}\rho v^2 + \rho gy = \text{ constant} \tag{27}$$

Bernoulli's equation says that the sum of the pressure P, the kinetic energy per unit volume $\frac{1}{2}\rho v^2$, and the gravitational potential energy per unit volume ρgy has the same value at all points along a streamline. This means that if one terms increases, the other terms must decrease to compensate and keep the sum of terms constant.

When the fluid is at rest, $v_1 = v_2 = 0$ and Bernoulli's equation becomes

$$P_1 - P_2 = \rho g\left(y_2 - y_1\right) = \rho gh \tag{28}$$

which agrees with Eq. (6). Notice that if there is no change in height, then

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \tag{29}$$

So a velocity difference means there is a pressure difference.

Key Point: Moving fluids exert ________________________ pressure than stationary fluids.

If $v_2 > v_1$, $P_1 > P_2$. This explains the shower curtain. The air and water are moving faster inside the shower than outside, so the presure is less inside the shower curtain and the curtain moves inward.

Some other applications of fluid dynamics

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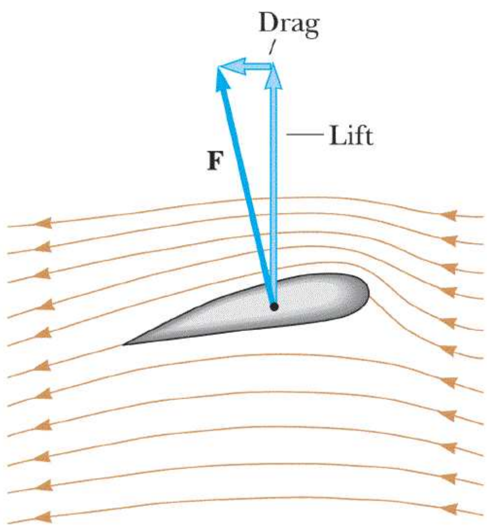
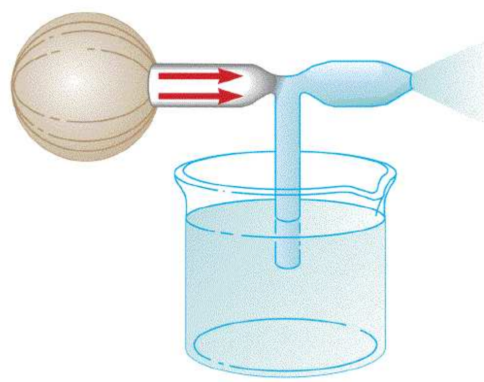
Figure 15.19



Another application of fluid dynamics is aerodynamic lift. This why airplanes fly. When an airplane wing is propelled through the air, there is a force upward on the wing due to Bernoulli's principle because air passes more quickly over the top of the wing than the bottom of the wing. This produces a lower pressure on the upper surface, and hence lift. To see why air passes more quickly over the top of the wing, consider a piece of air in front of the wing. When the wing goes through it, part of the air passes on top of the wing and part passes below the wing. Both pieces meet up at the rear of the wing so they took the same amount of time to pass by the wing. But the piece on the top had to go further since the upper part of the wing is arched and the bottom part is flat. So the air on the top moved more quickly and exerted less pressure.

There is also an upward force due to the air deflected downward by the wing. According to Newton's third law, the wing applies a force on the air to deflect it downward and so the air applies an equal and opposite force to push the wing upward. These two forces taken together tend to lift the wing against gravity and are therefore known as LIFT.

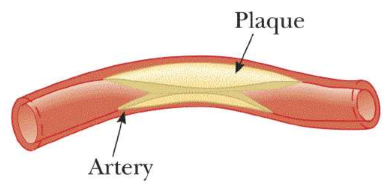
Serway/Jewett; Principles of Physics, 3/e
Figure 15.21



Another application is atomizers and paint sprayers. A stream of air passing over an open tube reduces the pressure above the tube. This reduction in pressure causes the liquid to rise into the air stream. The liquid is then dispersed into a fine spray of droplets.

Bernoulli's principle also explains vascular flutter. In advanced arteriosclerosis, plaque accumulates on the wall of a blood vessel and constricts the opening where the blood flows. As a result, the blood flows faster which in turn reduces the pressure on the inside of the blood vessel. The external pressure can cause the walls of the blood vessel to collapse, blocking the flow of blood. Since the blood momentarily stops flowing, the pressure inside the blood vessel rises, the walls are restored and the vessel reopens restoring blood flow. Then the cycle repeats itself. Such variations in blood flow can be heard with a stethoscope.

Serway/Jewett; Principles of Physics, 3/e
Figure 15.22



One final example is the "bar room bet." Take a cocktail napkin or fold a piece of paper. Put it on the table like a little tent. What is the best way to blow on it to get it to flatten onto the table? Bet someone that you can flatten it better than they can by just blowing on it.

Example: Problem 15.38 A legendary Dutch boy saved Holland by plugging a hole in a dike with his finger, 1.20 cm in diameter. If the hole was 2.00 m below the surface of the North Sea (density 1030 kg/m^3), (a) what was the force on his finger? (b) If he pulled his finger out of the hole, during what time interval would the released water fill 1 acre of land to a depth of 1 ft? Assume that the hole remained constant in size. (A typical U.S. family of four uses 1 acre-foot of water, 1234 m^3, in 1 year.)

Solution: (a) The force $F = PA$ where P is the pressure on his finger and $A = \pi r^2$ is the area of the hole. The radius $r = d/2$ where the diameter $d = 1.2$ cm $= 0.012$ m. So the area of the hole is

$$A = \pi r^2 = \pi (d/2)^2 = \frac{\pi}{4} d^2 = \frac{\pi (0.012 \text{ m})^2}{4} = 1.13 \times 10^{-4} \text{ m}^2 \tag{30}$$

We need the net pressure on his finger, so we calculate the pressure from the sea and the pressure from the air around his hand. The pressure on his hand from the air is $P_0 = 1$ atm $= 1.013 \times 10^5$ Pa $= 1.013 \times 10^5$ N/m^2. The sea pressure $P_{sea} = P_0 + \rho g h$. So the net pressure on his finger is $P_{net} = \rho g h$. Therefore the net force on his finger is

$$\begin{aligned} F &= P_{net} A \\ &= \rho g h A \\ &= \left(1030 \text{ kg/m}^3\right) \left(9.8 \text{ m/s}^2\right) (2 \text{ m}) \left(1.13 \times 10^{-4} \text{ m}^2\right) \\ &= 2.28 \text{ N} \end{aligned}$$

(b) To find the time to fill a volume $V = 1234$ m^3, we need the volume rate of flow dV/dt where t is time. We note that

$$\frac{dV}{dt} = Av \tag{31}$$

where the area A is the area of the hole out of which the water flows and v is the velocity of the water coming out of the hole. You can check this relation by seeing if the units are the same on each side of the equation. We can integrate this equation to get the volume in terms of the time:

$$\begin{aligned} \Delta V &= \int Av dt \\ &= Av\Delta t \end{aligned}$$

Alternatively, since Av is independent of time, we can rewrite Eq. (31) to get

$$\frac{\Delta V}{\Delta t} = Av \tag{32}$$

where $\Delta V = 1234$ m^3 and Δt is the time needed to fill that volume. Solving for Δt gives

$$\Delta t = \frac{\Delta V}{Av} \tag{33}$$

We have the volume V and the area A. To find the velocity v, we use Bernoulli's equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \tag{34}$$

Let the left side represent the sea side of the hole and the right side represent the air side of the hole. The velocity of the sea $v_1 = 0$ on the sea side of the hole. We can set $y_2 = 0$ (assume the hole is on the ground or is where we measure height from). We also have $P_1 = P_2 = P_0 = 1$ atm. So we have

$$\begin{aligned} P_1 + \rho g y_1 &= P_2 + \frac{1}{2}\rho v_2^2 \\ P_0 + \rho g h &= P_0 + \frac{1}{2}\rho v_2^2 \\ \rho g h &= \frac{1}{2}\rho v_2^2 \\ g h &= \frac{1}{2} v_2^2 \\ 2gh &= v_2^2 \\ v_2 &= \sqrt{2gh} \end{aligned}$$

Plugging into Eq. (33), we get

$$
\begin{aligned}
\Delta t &= \frac{\Delta V}{Av} \\
&= \frac{\Delta V}{A\sqrt{2gh}} \\
&= \frac{1234\ \mathrm{m}^3}{(1.13 \times 10^{-4}\ \mathrm{m}^2)\sqrt{2\left(9.8\ \mathrm{m/s}^2\right)(2\ \mathrm{m})}} \\
&= 1.743 \times 10^6\ \mathrm{s} \\
&= 1.743 \times 10^6\ \mathrm{s} \times \frac{1\ \mathrm{min}}{60\ \mathrm{sec}} \times frac1\ \mathrm{hr}60\ \mathrm{min} \times \frac{1\ \mathrm{day}}{24\ \mathrm{hrs}} \\
&= 20.17\ \mathrm{days}
\end{aligned}
$$