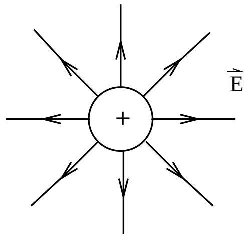
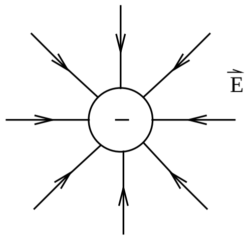
Lectures on Chapter 24: Electromagnetic Waves

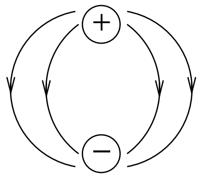
Absence of Magnetic Monopoles



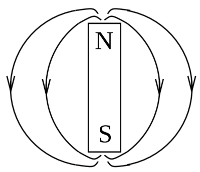
Recall that for electrostatics, a positive point charge has electric field lines that all point out. It acts as a "source" of $\vec{E}$ field lines. A negative point charge has electric field lines that all point toward it. It acts as a "sink" of $\vec{E}$ field lines.



We can regard point charges as electric "monopoles." If we put "+" and "−" charge near each other, we get an electric dipole where the electric field lines begin on the "+" and end on the "−."



Are there magnetic monopoles or magnetic charges? The answer is no, though scientists have searched for them. The simplest magnetic structure is a dipole. The field lines of a bar magnet are those of a dipole. They start at N and end at S.



We could try to isolate magnetic charges by breaking the magnet up into tiny pieces, but we would just get tiny dipoles.

To express this mathematically, recall that Gauss' law

$$\varepsilon_o \Phi_E = \varepsilon_o \oint \vec{E} \cdot d\vec{a} = q_{enc} \tag{1}$$

states that the flux of electric field through a closed surface is proportional to the charge enclosed. Since there are no magnetic charges, there can be no net magnetic flux through any closed surface:

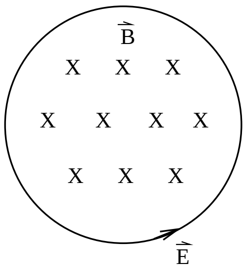
$$\Phi_B = \oint \vec{B} \cdot d\vec{a} = 0 \tag{2}$$

This is one of Maxwell's equations. Any closed surface will have equal amounts of outgoing and incoming magnetic flux.

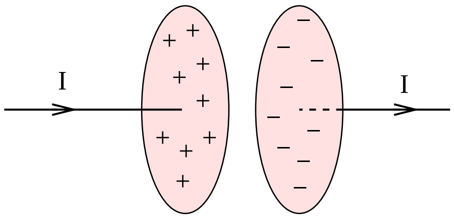
Changing $\vec{E}$ produces $\vec{B}$

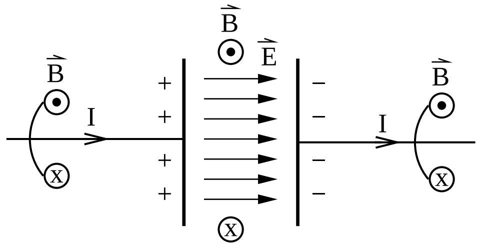
Faraday's law tells us that a changing magnetic field can induce an electric field.

$$\mathcal{E} = \oint_C \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{a} \tag{3}$$



So if $\vec{B}$ is increasing and pointing into the page, $\vec{E}_{\text{ind}}$ is counterclockwise. It turns out that the converse is also true. Namely, a changing $\vec{E}$ field can produce a $\vec{B}$ field. Think of a parallel plate capacitor that is charging up.





As the charge on the plates increases, the $\vec{E}$ field between the plates increases. This changing (increasing) $\vec{E}$ field produces a $\vec{B}$ field. Just as $\vec{B}$ field lines circle around a current carrying wire, so the induced $\vec{B}$ field circles around the changing $\vec{E}$ field.

Mathematically, we describe what's going on by adding a term to Ampere's law.

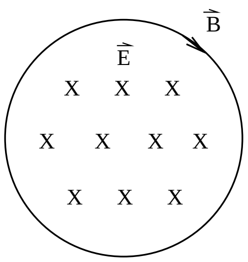
$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

or

$$\oint_C \vec{B}\cdot d\vec{s} = \mu_0 I + \mu_0\varepsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{a}$$

Here the surface S is an imaginary surface pierced by the $\vec{E}$ field lines. The closed curve C is the boundary of S. For the parallel plate capacitor, the surface S lies between the plates and is parallel to the plates.

Notice that the added term has a "+" sign in front: $+\mu_0\varepsilon_0 d\Phi_E/dt$. (This is in contrast to Faraday's law where we had a minus sign and Lenz's law.) The "+" sign means that the $\vec{B}$ field goes in the same direction around the capacitor as it does around the current carrying wire.



So if $\vec{E}$ is increasing into the page, $\vec{B}_{\text{ind}}$ is clockwise. (Notice that $\vec{B}_{\text{ind}}$ is opposite to $\vec{E}_{\text{ind}}$ in the first figure which illustrated Faraday's law.) We can get the direction of $\vec{E}$ from the right hand rule: if your thumb is in the direction of increasing $\vec{E}$, your fingers curl in the direction of $\vec{B}$.

Displacement Current:

We can write Ampere's law

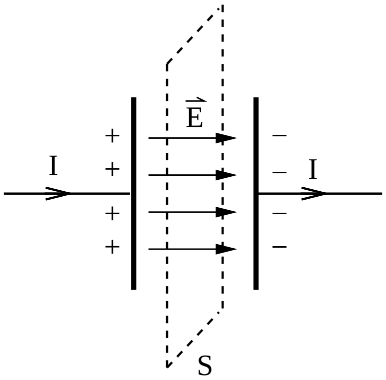
$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I + \mu_0\varepsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{a}$$

in the form

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I + \mu_0 I_d$$

where the **displacement current** I_d is given by

$$I_d = \varepsilon_0 \frac{d}{dt}\int_S \vec{E}\cdot d\vec{a} \tag{4}$$



Let's calculate I_d for a parallel plate capacitor where the plates each have area A. Then the flux through a surface S between the plates is

$$\int_S \vec{E} \cdot d\vec{a} = EA \tag{5}$$

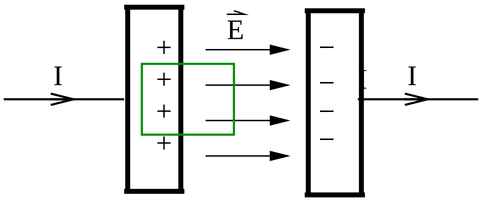
So

$$I_d = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt} \tag{6}$$

It turns out that this I_d is equal to the conduction current I that is flowing in the wires connected to the capacitor. To see this, note that

$$I = \frac{dq}{dt}$$

where q is the charge flowing through the wire and onto the capacitor plate.



We can relate q and the $\vec{E}$ field between the plates with Gauss' law:

$$\begin{aligned} \varepsilon_0 \oint \vec{E} \cdot d\vec{a} &= q_{enc} \\ \varepsilon_0 E a &= \sigma a \end{aligned}$$

where $\sigma = q/A$ and a is the area of the face of the Gaussian box.

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A} \qquad \Longrightarrow \qquad q = \varepsilon_0 EA$$

(or have the Gaussian surface cover the inner surface of the capacitor plate to get $\varepsilon_0 \oint \vec{E} \cdot d\vec{a} = \varepsilon_0 EA = q$, ignoring fringing fields). So the current in the wires is

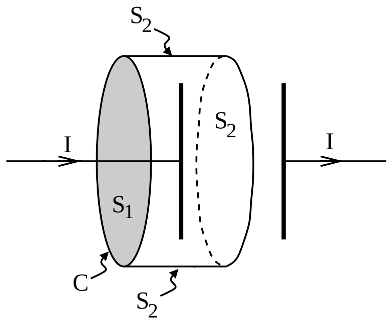
$$\begin{aligned} I &= \frac{dq}{dt} = \varepsilon_0 \frac{d(EA)}{dt} = \varepsilon_0 A \frac{dE}{dt} \\ I &= I_d \end{aligned}$$

There is another way to see the importance of the displacement current. Suppose we use

$$I = \int_S \vec{J} \cdot d\vec{a} \tag{7}$$

where $\vec{J}$ = current density, to write Ampere's law in the form

$$\begin{aligned} \oint_C \vec{B} \cdot d\vec{s} &= \mu_0 \underbrace{\int_S \vec{J} \cdot d\vec{a}}_{I} + \mu_0 \varepsilon_0 \frac{d}{dt} \int_S \vec{E} \cdot d\vec{a} \\ &= \mu_0 \int_s \vec{J} \cdot d\vec{a} + \mu_0 \varepsilon_0 \int_S \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a} \end{aligned}$$



Ampere's law tells us that the line integral of the magnetic field around any closed loop C is equal to μ_0 times the current passing through a surface S bounded by the loop plus $\mu_0\varepsilon_0$ times the rate of change of the electric field integrated over the surface S. Now we have a choice of surfaces. If we choose S_1, which lies in the plane of the loop, then $\partial\vec{E}/\partial t = 0$ at S_1 but $\vec{J} \neq 0$. (If the current I is steady in the wire, then $\vec{E}$ in the wire is time independent.) So

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 \int \vec{J} \cdot d\vec{a} = \mu_0 I \tag{8}$$

But if we choose the bag-shaped surface S_2 that encloses one plate of the capacitor, then $\vec{J}(S_2) = 0$ because no electrons go through S_2. However, $\partial E/\partial t|_{S_2} \neq 0$, so

$$\oint_C \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \int_s \frac{\partial \vec{E}}{\partial t} \cdot d\vec{a} = \mu_0 I_d \tag{9}$$

Since we've already shown that $I = I_d$, we get the same result for $\oint_C \vec{B} \cdot d\vec{s}$, which is good. Ampere's law holds no matter which surface we choose.

Maxwell's Equations

It was James Clerk Maxwell who added the term to Ampere's law. He realized that a changing electric field generates a magnetic field. We can now write down the four basic equations of electricity and magnetism. These are called Maxwell's equations:

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{q_{\text{enc}}}{\varepsilon_0} \qquad \text{Gauss' law}$$

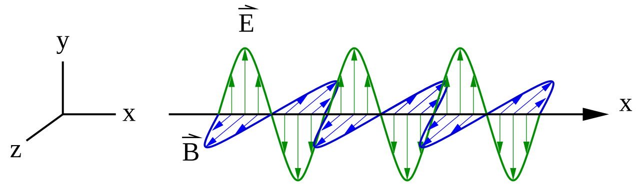
$$
\begin{aligned}
\oint_S \vec{B} \cdot d\vec{a} &= 0 && \text{No magnetic monopoles} \\
\oint_C \vec{E} \cdot d\vec{s} &= -\frac{d}{dt}\int_S \vec{B} \cdot d\vec{a} && \text{Faraday's law} \\
\oint_C \vec{B} \cdot d\vec{s} &= \mu_0 I + \mu_0\varepsilon_0 \frac{d}{dt}\int_S \vec{E} \cdot d\vec{a} && \text{Ampere's law}
\end{aligned}
$$

Gauss' law and Faraday's law tell us that electric charges and changing magnetic fields generate electric fields. Ampere's law tells us that electric currents and changing electric fields generate magnetic fields. The coefficient $\mu_0\varepsilon_0$ in Ampere's law is related to the speed of light:

$$
c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}
$$

In fact, from Maxwell's equations one can derive a wave equation that describes electromagnetic waves and light. Maxwell discovered that light consists of electromagnetic waves. Radio waves, microwaves, and x-rays are all forms of electromagnetic waves described by Maxwell's equations. An electromagnetic wave consists of oscillating $\vec{E}$ and $\vec{B}$ fields. $\vec{E} \perp \vec{B} \perp \vec{k}$ where $\vec{k}$ points in the direction that the wave is travelling. The changing $\vec{E}$ field generates a changing $\vec{B}$ field and vice–versa. One example of a (linearly polarized) electromagnetic wave traveling in the $\hat{x}$ direction is

$$
\begin{aligned}
\vec{E} &= E_{\max}\cos(kx - \omega t)\hat{y} \\
\vec{B} &= B_{\max}\cos(kx - \omega t)\hat{z}
\end{aligned}
$$



Notice that $\vec{E}$ and $\vec{B}$ oscillate in space and time and that they are perpendicular to each other and to the direction that the wave is travelling in. Maxell's equations are incredibly powerful. They are the basis for the operation of all electromagnetic and optical devices such as electric motors, telescopes, cyclotrons, eyeglasses, television transmitters and receivers, telephones, electromagnets, radar, and microwave ovens. These equations help to explain natural phenomena such as rainbows and lightening. That so much can be summed up in just four "simple" equations is the epitome of elegance.