Physics 238C **Spring 2011**

Condensed Matter Physics **due 2:00 pm Thursday May 26**

PROBLEM SET 8

Reading: Handout from Feynman's Statistical Mechanics

1. **BCS Theory** (T=0) Starting from the variational wavefunction

$$\psi = \prod_{\vec{k}} \left(u_{\vec{k}} |11\rangle_{\vec{k}} + v_{\vec{k}} |00\rangle_{\vec{k}} \right) \tag{1}$$

and the BCS Hamiltonian

$$H = \sum_{\vec{k}} \varepsilon_{\vec{k}} a^\dagger_{\vec{k}} a_{\vec{k}} + \sum_{\vec{k}'_1 \vec{k}'_2; \vec{k}_1 \vec{k}_2} V_{\vec{k}'_1 \vec{k}'_2; \vec{k}_1 \vec{k}_2} a^\dagger_{\vec{k}'_1} a^\dagger_{\vec{k}'_2} a_{\vec{k}_1} a_{\vec{k}_2} \tag{2}$$

show that

$$u^2_{\vec{k}} = \frac{1}{2}\left(1 - \frac{\varepsilon_{\vec{k}}}{E_{\vec{k}}}\right) \tag{3}$$

$$v^2_{\vec{k}} = \frac{1}{2}\left(1 + \frac{\varepsilon_{\vec{k}}}{E_{\vec{k}}}\right) \tag{4}$$

where $E_{\vec{k}} = \varepsilon^2_{\vec{k}} + \Delta^2_{\vec{k}}$ and $\Delta_{\vec{k}} = -\sum_{\vec{k}'} V_{\vec{k}\vec{k}'} u_{\vec{k}'} v_{\vec{k}'} > 0$. (You should work through the algebra.)

2. Find the self-consistent gap equation at $T = 0$:

$$\Delta_{\vec{k}} = -\sum_{\vec{k}'} V_{\vec{k}\vec{k}'} \frac{\Delta_{\vec{k}'}}{2\sqrt{\varepsilon^2_{\vec{k}'} + \Delta^2_{\vec{k}'}}} \tag{5}$$

Use the BCS approximation $V_{\vec{k}\vec{k}'} = -V < 0$ for $\left|\varepsilon_{\vec{k}}\right| < \hbar\omega_o$ to obtain

$$\Delta = \frac{\hbar\omega_o}{\sinh\left(\frac{1}{N(0)V}\right)} \tag{6}$$

where $V > 0$ and $N(0)$ is the density of states at the Fermi energy.