Physics 238C **Spring 2011**

Condensed Matter Physics **due 2:00 pm Thursday May 5**

PROBLEM SET 5

Reading: AM Chapter 34 or Marder (2nd Ed) Chapter 27 or Kittel (8th Ed) Chapter 10

1. Discuss how the following experiments can discern the existence and magnitude of the superconducting gap Δ.

   (a) Specific heat measurements.

   (b) Single particle tunneling across a thin insulator separating two identical superconductors.

2. **The London Equation** The London Equation is given by

$$\vec{j} = -\frac{ne^2}{mc}\vec{A} \tag{1}$$

   (a) Using the appropriate Maxwell equation, show that

$$\nabla^2\vec{B} = \vec{B}/\lambda^2 \tag{2}$$

   What is λ in terms of the density of Cooper pairs n, e, the mass of the electron m, and c? λ is called the London penetration depth. (Hint: Use some vector identities to simplify the equations. See inside cover of Jackson's *Classical Electrodyamics*, for example.)

   (b) Suppose that $\vec{B}$ points along the z axis and only varies in the x direction. Suppose the superconductor fills the half space $x > 0$ and there is vacuum for $x < 0$. Show that B_x dies out exponentially as it penetrates the superconductor in the x direction. (Don't worry about the prefactor of the exponential.) In other words the magnetic field dies out exponentially as you go into the superconductor. This is the Meissner effect.

(c) Show that $B(x)$ inside a superconducting plate perpendicular to the x-axis and of thickness δ is given by

$$B(x) = B_a \frac{\cosh(x/\lambda)}{\cosh(\delta/\lambda)} \tag{3}$$

where B_a is the field outside the plate and parallel to it; here $x = 0$ is at the center of the plate. ($\vec{B_a} = \vec{B_a}\hat{z}$)

(d) Show that the diamagnetic current density flowing in equilibrium is

$$j = -\frac{c}{4\pi\lambda} B_a \frac{\sinh(x/\lambda)}{\cosh(\delta/2\lambda)} \tag{4}$$

What is the direction of j?