Physics 238A | **Fall 2006**

Condensed Matter Physics | **due 9:30 am Tuesday Oct. 31**

PROBLEM SET 6

Reading: Ashcroft and Mermin (AM) Chapters 7 and 8

1. Show by direct calculation that the expectation value of momentum for an electron in a Bloch state is the mass times the group velocity $\hbar^{-1}\partial E/\partial \vec{k}$. (Hint: Write the Bloch wave in the form

$$\psi_k(\vec{r}) = \sum_{\vec{G}} C_{\vec{G}}^{(k)} e^{i(\vec{k}-\vec{G})\cdot\vec{r}}$$

where $\vec{G}$ runs over the reciprocal lattice vectors, take the expectation value of the energy and use the fact that it is minimized by the correct form of $C_{\vec{G}}^{(k)}$.)

2. A very good approximation to the form of the potential exerted on a neutron by a nucleus is

$$V(\vec{r}) = \frac{2\pi\hbar^2}{m} a\delta(\vec{r})$$

where a is the so–called scattering length and is typically of the order of the nuclear radius.

   (a) Using the Born approximation, find the total scattering cross–section in terms of a. Hence estimate the mean free path of a neutron in a typical disordered material. (Neglect the possibility of absorption by the nucleus.)

   (b) Consider now a perfect crystalline material of cubic symmetry where the nuclei are identical. Find the order of magnitude of the lowest energy gap ΔE for a neutron propagating in such a crystal in the (100) direction, and compare it with the width of the allowed band.

   (c) If a monochromatic beam of neutrons is normally incident from outside the crystal at exactly a Bragg wavevector, it will have an energy in the middle of the gap and hence cannot form a truly propagating state in the crystal. However, it turns out that it can form an exponentially attenuated wave which can propagate for a distance of the order of $\hbar v/\Delta E$, where v is its (initial) velocity. (This statement is strictly true only if the crystal is cut parallel to a (100) plane. Otherwise the situation is somewhat more complicated.) Estimate the distance for the neutron, and compare it with the result of part (a).