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PROBLEM SET 2

**Reading:** Ashcroft and Mermin (AM) Chapters 2 and 3

1. Fermi Gases in Astrophysics

- (a) Use the relativistic relation between energy and momentum to find the relationship between the Fermi energy and the density in an ultrarelativistic electron gas.
- (b) Given that the sun is approximately electrically neutral, is composed mainly of light elements and weighs approximately  $2 \times 10^{30}$  kg, estimate the total number of electrons it contains.
- (c) In a white dwarf star this number of electrons may be ionized and contained in a sphere of radius  $\sim 2 \times 10^6$  meters. Find the (order of magnitude of the) Fermi energy in eV. (Note: You will have to decide whether it is more sensible to apply the ultra-relativistic or nonrelativistic formula.)
- (d) Estimate *roughly* the temperature below which the electrons are degenerate.

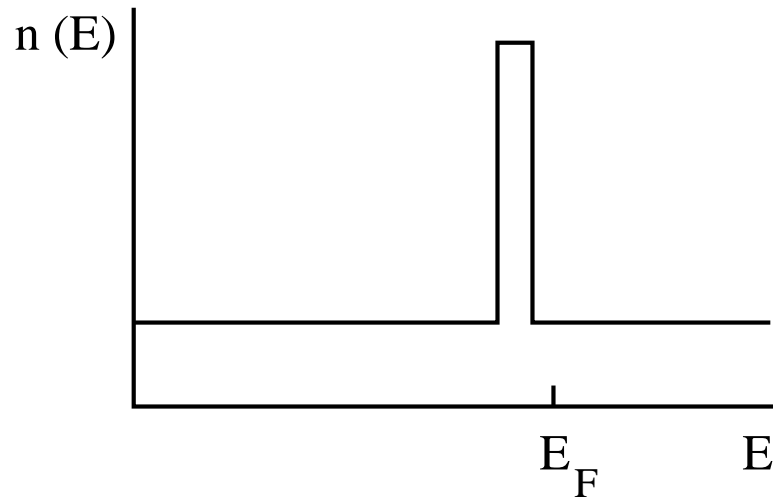
2. Deformation of the Fermi distribution by currents

Using general ideas about the way in which the distribution function  $f$  is distorted by current and heat flow, estimate the order of magnitude of the deviation of  $f$  from its equilibrium form (the Fermi–Dirac distribution function) in copper at room temperature

- (a) when an electric current of 1000 A/cm<sup>2</sup> flows, and
- (b) when it is subject to a thermal gradient of 1000 K/cm.

In the first case, estimate also the mean drift velocity of the electrons as a fraction of the Fermi velocity. (Look up any data you need on copper.) Note that the deviation from  $f_{eq}$  is appreciable only for  $E \lesssim kT$ .

3. A fictitious transition metal has the electronic density of states  $n(E)$  shown below.



As the temperature  $T$  increases, does the chemical potential  $\mu$  increase or decrease? Justify your answer.