LECTURE 9

Aside: Maxwell Velocity Distribution

Before we talk about the laser cooling of atoms, I just want to mention that if you have a gas of atoms in equilibrium, and you treat them as classical particles, then they will have Maxwell–Boltzmann statistics ($\sim e^{-\beta E}$). In particular, their energies are kinetic energies ($E = \frac{1}{2}mv^2$), so that their velocity distribution is given by

$$P(v) = Ce^{-\beta mv^2/2} \tag{1}$$

where $\beta = 1/k_B T$ and C is a constant that can be determined from the normalization condition. Notice that the velocity distribution is a Gaussian centered at $v = 0$ (the gas isn't going anywhere in particular) and that a range of velocities are represented.

Laser Cooling

(References: Nobel lectures of Chu, Cohen–Tannoudji, and Phillips, Rev. Mod. Phys. **70**, 685–742 (1998).)
Lasers are reminiscent of the ray guns that one sees in science fiction movies. As we mentioned in the last lecture, a laser with less power than a typical light bulb can burn a hole through a metal plate. However, in recent years, lasers have been used, not to heat, but to cool atomic gases to the lowest temperatures that man has been able to achieve: a few nanokelvin above absolute zero. I want to describe some the tricks that have been devised to accomplish this cooling. These developments are the basis of the 1997 Nobel prize in physics which was awarded to Steven Chu, Claude N. Cohen–Tannoudji, and William Phillips.

We mentioned in lecture 4 that light can apply radiation pressure to matter. This gives comets their tails. Microscopically when an atom absorbs a photon of energy $h\nu$, it will receive a momentum impulse $h\nu/c$ along the direction of the incoming photon $\vec{p}_{in}$ ($p_{in} = h\nu/c$). In order to absorb a photon of frequency ν, the atom must have an allowed transition between 2 energy levels where $\Delta E = h\nu$. If the atom emits a photon with momentum $\vec{p}_{out}$, the atom will recoil in the opposite direction. Thus the atom experiences a net momentum change $\Delta\vec{p}_{atom} = \vec{p}_{in} - \vec{p}_{out}$ due to this incoherent scattering process. Since the scattered photon has no preferred direction, the net effect is due to the absorbed photons, resulting in a scattering force $\vec{F}_{scatt} = N\vec{p}_{in}$, where N is the number of photons scattered per second. Typical scattering rates for atoms excited by a laser tuned to a strong resonance line are on the order of 10^7 to 10^8/sec. As an example, the velocity of a sodium atom changes by 3 cm/sec per absorbed photon. The scattering force can be 10^5 times the gravitational acceleration on earth, feeble compared to electromagnetic forces on charged particles, but stronger than any other long–range force that affects neutral particles.

Doppler Cooling

Recall how the doppler shift works. If you move along a beam of light toward its source with velocity v, the light is blue shifted. This is because the distance a wave travels in one period T of oscillation is cT while the distance you move in one period is vT. You can think of vT as the amount by which a wavelength is squeezed. The doppler shifted wavelength is

$$\lambda' = cT - vT = (c-v)T = \frac{c-v}{\nu} \tag{2}$$

Notice that this is shorter (bluer) than the original wavelength. The doppler shifted frequency is higher and is given by

$$\nu' = \frac{c}{\lambda'} = \nu\left(\frac{c}{c-v}\right) \tag{3}$$

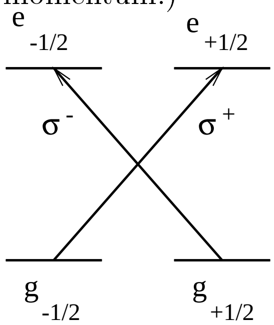
If you move away from the source of light, then the light is red shifted. Just change $v \to -v$ in the above equations to see that the wavelength gets longer and the frequency drops. The red shift of the spectral lines from stars and galaxies is how we know that the universe is expanding.

Now let's apply this to laser cooling. Take 2 identical laser beams counterpropagating in opposite directions. Let's put some atoms in the beams' path and tune the lasers below the atomic resonance. If the atoms are standing still, they won't absorb any photons because the frequency is less than what is needed for electrons to make a transition to a higher energy level. But a moving atom doppler shifts the beam opposing its motion closer to resonance and shifts the beam co–propagating with the motion away from resonance. So chances are that it will absorb more photons from the beam opposing its motion than from the beam in the direction of its motion. Thus the atom will experience a net force opposing its motion. By slowing the atoms down, we are cooling them. In the limit where atoms are moving slowly enough so that the difference in the absorption due to the Doppler effect is linearly proportional to the velocity, this force will result in viscous damping $\vec{F} = -\alpha\vec{v}$. We can upgrade to 6 laser beams pointed at the origin, with each pair of counterpropagating beams along one of the coordinate axes x, y, and z. There is also a heating effect due to the random kicks an atom receives by randomly scattering photons from counterpropagating beams that surround the atoms. Scattering consists of absorbing a photon and then emitting another photon in some random direction and recoiling as a result. Balancing the effects of heating and cooling, one gets an estimate of how cold the atoms can get. For alkali atoms the temperature is of order 100 μK. Not only does the laser light cool the atoms, it also tends to confine them. An atom receiving random kicks in a viscous medium is analogous to the Brownian motion of a dust particle immersed in water. In the laser case, the viscous medium is a sea of photons which have been dubbed "optical molasses." The first experiments showing that optical molasses works was done by Steve Chu and his collaborators. They achieved a temperature of 185 μK.

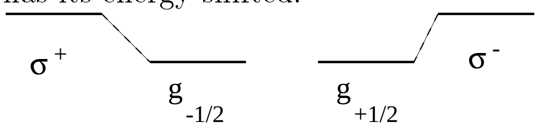
Sisyphus Cooling

100 μK may sound pretty cold, but laser cooling can get even colder thanks to the Sisyphus effect. To understand this effect, let's first consider the following case of optical pumping. It uses resonant excitation of atoms by circularly polarized light for transferring to the atoms part of the angular momentum carried by the light beam. It is based on the fact that different Zeeman sublevels in the atomic ground state have in general

different absorption rates for incoming polarized light. (Notation: σ^+-polarized light has positive helicity and the photons carry $+1$ angular momentum. If you face the oncoming wave, the polarization vector rotates counterclockwise and the wave is left–circularly polarized. Similarly $\sigma^-$$-$polarized light has negative helicity, the photons carry -1 angular momentum, and it is right–circularly polarized. π polarized light is linearly polarized light which is a linear combination of σ^+ and σ^- and carries no net angular momentum.)



For example, for a $J_g = 1/2 \leftrightarrow J_e = 1/2$ transition, only atoms in the sublevel $M_g = -1/2$ can absorb σ^+-polarized light. They are excited into the sublevel $M_e = +1/2$ of $J_e = 1/2$ from which they can fall back into the sublevel $M_g = +1/2$ by spontaneous emission of a linearly polarized photon. They then remain trapped in this state because no further σ^+-transition can take place. It is possible in this way to obtain high degrees of spin orientation in atomic ground states. Because of this coupling between the $M_g = -1/2$ sublevel and the excited state via σ^+ photons, the energy of the $M_g = -1/2$ sublevel is shifted. Similarly $M_g = +1/2$ atoms can absorb $\sigma^-$$-$polarized light and be excited into the $M_e = -1/2$ sublevel from which they can fall back into the sublevel $M_g = -1/2$ by spontaneous emission of a linearly polarized photon. Thus the $M_g = +1/2$ sublevel has its energy shifted.

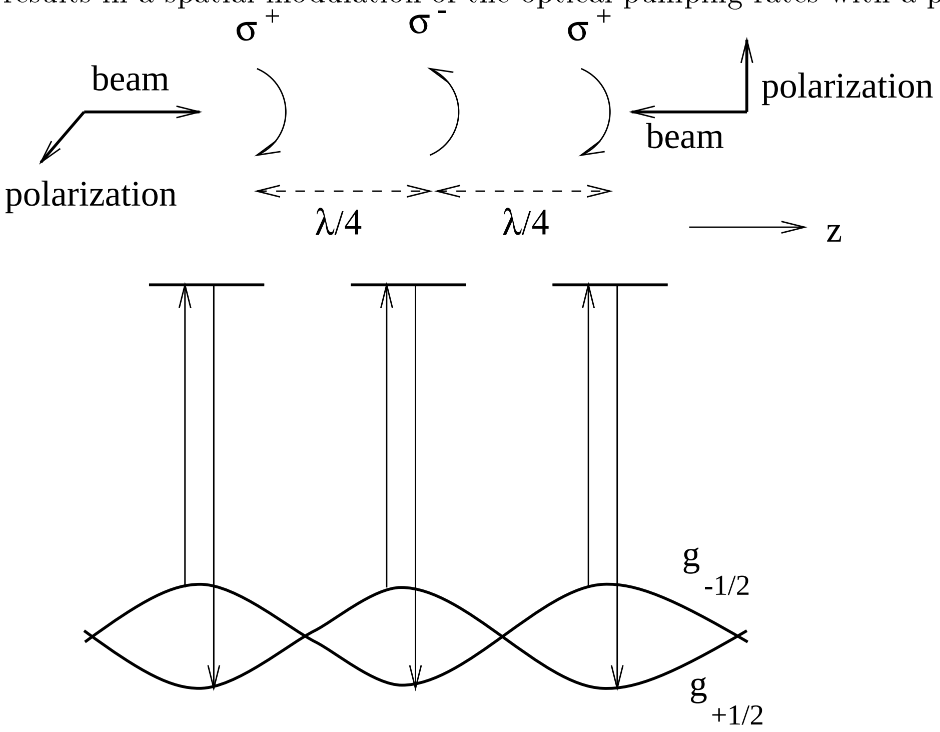


Now consider a laser configuration consisting of 2 counterpropagating plane waves along the z axis, with orthogonal linear polarizations and with the same frequency and the same intensity. Because the phase shift between the 2 waves increases linearly with z, the polarization of the total field changes from σ^+ to σ^- and vice versa every $\lambda/4$. In between it is elliptical or linear.

Consider now the simple case where the atomic ground state has an angular momentum $J_g = 1/2$. The two Zeeman sublevels $M_g = \pm 1/2$ undergo different energy level shifts (called "light shifts") depending on the laser polarization, so that the Zeeman degeneracy in zero magnetic field is removed. This gives the energy diagram showing spatial modulations of the Zeeman splitting between the two sublevels with a period $\lambda/2$.

When the atom absorbs a photon followed by spontaneous emission of a photon, optical pumping transfers between the two sublevels occurs with the direction depending on the polarization: $M_g = -1/2 \rightarrow M_g = +1/2$ for σ^+ polarization, $M_g = +1/2 \rightarrow M_g =$

$-1/2$ for σ^- polarization. Here also, the spatial modulation of the laser polarization results in a spatial modulation of the optical pumping rates with a period of $\lambda/2$.



Spatial modulation of the light polarization results in correlation between the spatial modulation of light shifts and optical pumping rates. By properly tuning the frequency of the laser light, optical pumping always transfers atoms from the higher Zeeman sublevel to the lower one. One always loses energy. Suppose now that the atom is moving to the right, starting from the bottom of an energy valley, for example in the state $M_g = +1/2$ at a place where the polarization is σ^+. As the atom climbs the potential energy hill, its kinetic energy is converted into potential energy. At the top of the hill it has the maximum probability to be optically pumped into the lower sublevel, i.e., the bottom of the energy valley. And the process starts all over again. This is like the story of Sisyphus in Greek mythology, who was condemned by the gods to roll the stone up the hill, only to have it roll back down before he reached the top. He would then have to start all over again. Dissipation occurs because the spontaneously emitted photon has an energy higher than the absorbed photon. (This is an example of an anti–Stokes Raman process.) So the atom is always losing energy and hence is being cooled. Using Sisyphus cooling, temperature of a few μK can be achieved. Cooling is further aided by the fact that one can arrange it so that when an atom has velocity $v \simeq 0$, it no longer absorbs light, and so won't be heated by photon absorption. This is called subrecoil laser cooling.

(In case you're wondering why an atom at the top of the hill (in the $M_g = +1/2$, say) doesn't make a direct transition into the valley (to the $M_g = -1/2$, say) and avoid the trouble of making an intermediate stop in the excited state, it's because the density of final states is too small. Recall Fermi's Golden Rule from lecture 6:

$$R_{f\leftarrow i} = \frac{2\pi}{\hbar} \mid < f|H|i > |^2 \; N(E) \tag{4}$$

where $N(E)$ is the density of final states with energy E. Recall from lecture 1 that the density of photon states $N(\omega) \sim \omega^2$. The energy difference between $M_g = +1/2$ and $M_g = -1/2$ is very small, so the density of final photon states is very tiny as well. This makes the transition rate very small.)

Raman Effect

In our discussion of the Sisyphus effect, we saw an example where the atom went from one state to another via an excited state. The atom absorbed a photon of frequency ν, made a transition from state 1 to an excited state, then spontaneously emitted another photon of frequency ν' and made a transition down to state 2. The frequency difference between the absorbed and emitted photons is $\nu' - \nu = (E_1 - E_2)/\hbar$. The scenario we have just described is called the Raman effect. Raman scattering refers to the process where the incident and emitted photons have a frequency difference that corresponds to a characteristic frequency of the atom, molecule, or solid. In some cases the frequency difference corresponds to a phonon (vibrational quantum) or, for molecules, to a characteristic rotational frequency.

Bose–Einstein Condensation and Superfluidity

(Reference: Robert B. Leighton, *Principles of Modern Physics*, McGraw–Hill (1959).)

Interesting things happen at very low temperatures and Bose–Einstein condensation is one of them. Recall that there is no statistical limit to the number bosons that can occupy a single state. In a Bose condensed state, an appreciable fraction of the particles is in the lowest energy level at temperatures below T_C. These particles are in the same state and can be described by the same wavefunction. In other words a macroscopic number of particles are in one coherent state. (We saw this in the case of the photons in a laser beam.) If we write $\psi = |\psi|e^{i\phi}$, then this state is described by a given phase ϕ.

The oldest known physical manifestation of Bose condensation is superfluid ^4He. A ^4He atom has total angular momentum zero and is therefore a boson. At $T_C = 2.18$ K liquid helium becomes superfluid. The transition temperature is called the $\lambda-$point because the shape of the specific heat curve at T_C is shaped like λ. One cools liquid helium by pumping on it to get rid of the hot atoms (evaporative cooling). It boils a little. Then at the transition it boils vigorously and suddenly stops. Eisberg and Resnick has a picture of this on page 403. The reason for this behavior is that the thermal conductivity increases by a factor of about 10^6 at the transition, so that the superfluid is no longer able to sustain a temperature gradient. To make a bubble, heat has to locally vaporize the fluid and make it much hotter than the surrounding fluid. This is no longer possible in the superfluid state.

Perhaps the hallmark of a superfluid is that it has no viscosity. As a result the superfluid can flow through tiny capillary tubes that normal liquid can't get through. Superfluid ^4He is often described by a two–fluid model, i.e., it is thought of as consisting of

2 fluids, one of which is normal and the other is superfluid. It's the superfluid component which is able to flow through the capillary tube. So if you use this method to measure the coefficient of viscosity, you find that it suddenly drops to zero at the $\lambda-$point.

One can see the effect of both components by putting a torsional oscillator consisting of a stack thin, light, closely spaced mica disks immersed in the liquid. If the liquid has a high viscosity, the liquid between the disks is dragged along and contributes significantly to the moment of inertia of the disks. If the viscosity is small, the moment of inertia is more nearly equal to that of the disks alone. Using this method, no discontinuity is found in the coefficient of viscosity at the $\lambda-$point.

Another weird thing that superfluid helium does is escape from a beaker by crawling up the sides, flowing down the outside, and dripping off the bottom. The helium atoms are attracted by the van der Waals forces of the walls of the container, and they are able to flow up the walls because of the lack of viscosity. The rate of flow can be 30 cm per second or more. The superfluid helium can surmount quite a high wall, on the order of several meters in height.

(Brief aside to explain the van der Waals force: As an electron moves in a molecule, there exists at any instant of time a separation of positive and negative charge in the molecule. The latter has, therefore, an electric dipole moment p_1 which varies in time. If another molecule exists nearby, it will have a dipole moment induced by the first molecule. These two dipoles are attracted to each other. This is the van der Waals force.)

We can show mathematically that there is a macrosopic population of the lowest energy state in the following way. Consider a gas of noninteracting bosons. Let the energy levels be measured from the lowest energy level, i.e., let the zero point energy be the zero of energy. Then the chemical potential μ must be negative, otherwise the Bose–Einstein distribution would be negative for some of the levels. Recall from lecture 3 that the Bose–Einstein distribution gives the average number of particles in state s:

$$< n_s >= \frac{1}{e^{\beta(E_s-\mu)} - 1} \tag{5}$$

μ is adjusted so that the total number of particles is N:

$$N = \sum_s < n_s > \tag{6}$$

Let me give a sneak preview: If we assume a continuous distribution of states, μ starts out negative and gets bigger as the temperature decreases. μ equals its upper limit of zero at some temperature T_C, below which we can no longer satisfy (6) because the right hand side can't deliver enough particles. This leads us to treat the lowest energy level separately and we find that we can satisfy (6) by keeping the extra particles we need in the lowest state.

Now let's do the math. In order to turn the sum over s in (6) into an integral, let's assume a continuous density of states. In lecture 1 we found that if k–space is isotropic, i.e., the same in every direction, then the number of states in a spherical shell lying between radii k and $k + dk$ is

$$\rho_k dk = \frac{V}{(2\pi)^3}(4\pi k^2 dk) = \frac{V}{2\pi^2} k^2 dk \tag{7}$$

Now if the energy of the bosons is purely kinetic energy and continuous, then

$$E = \frac{\hbar^2 k^2}{2m} \tag{8}$$

and

$$dE = \frac{\hbar^2 k dk}{m} \tag{9}$$

or

$$k dk = \frac{m dE}{\hbar^2} \tag{10}$$

Also (8) implies that

$$k = \frac{\sqrt{2mE}}{\hbar} \tag{11}$$

Plugging (10) and (11) into (7) yields

$$\rho_E dE = V \left(\frac{m^3}{2}\right)^{1/2} \frac{E^{1/2}}{\pi^2 \hbar^3} dE \tag{12}$$

So we can rewrite (6) as

$$\begin{aligned} N &= \int \frac{1}{e^{\beta(E-\mu)} - 1} \rho_E dE \\ &= \frac{V}{\pi^2 \hbar^3} \left(\frac{m^3}{2}\right)^{1/2} \int_0^\infty \frac{E^{1/2}}{e^{\beta(E-\mu)} - 1} dE \end{aligned}$$

Now recall that for a geometric series

$$\sum_{p=0}^{\infty} e^{-p\beta(E-\mu)} = \frac{1}{1 - e^{-\beta(E-\mu)}} \tag{13}$$

So

$$\frac{1}{e^{\beta(E-\mu)} - 1} = \frac{1}{1 - e^{-\beta(E-\mu)}} \frac{1}{e^{\beta(E-\mu)}} = \frac{1}{e^{\beta(E-\mu)}} \sum_{p=0}^{\infty} e^{-p\beta(E-\mu)} \tag{14}$$

Plugging this into (13) leads to

$$N = \frac{V}{\pi^2 \hbar^3} \left(\frac{m^3}{2}\right)^{1/2} \int_0^\infty E^{1/2} e^{-\beta(E-\mu)} \sum_{p=0}^{\infty} e^{-p\beta(E-\mu)} dE \tag{15}$$

Let $x = \beta E$. Then

$$N = \frac{V}{\pi^2 \hbar^3} \left(\frac{m^3}{2} \right)^{1/2} (k_B T)^{3/2} \sum_{p=0}^{\infty} e^{\beta(p+1)\mu} \int_0^{\infty} x^{1/2} e^{-(p+1)x} dx \tag{16}$$

Let $s = p + 1$ and $y = sx$. Then

$$N = \frac{V}{\pi^2 \hbar^3} \left(\frac{m^3}{2} \right)^{1/2} (k_B T)^{3/2} \sum_{s=1}^{\infty} e^{\beta s \mu} \frac{1}{s^{3/2}} \int_0^{\infty} y^{1/2} e^{-y} dy \tag{17}$$

The definition of a gamma function is

$$\Gamma(n) = \int_0^{\infty} y^{n-1} e^{-y} dy \quad n > 0 \tag{18}$$

So

$$\int_0^{\infty} y^{1/2} e^{-y} dy = \Gamma(\frac{3}{2}) \tag{19}$$

Thus

$$N = \frac{V}{\pi^2 \hbar^3} \left(\frac{m^3}{2} \right)^{1/2} \Gamma(\frac{3}{2}) (k_B T)^{3/2} \sum_{s=1}^{\infty} \left(\frac{e^{\beta s \mu}}{s^{3/2}} \right) \tag{20}$$

Now on the right hand side, as the temperature T decreases, μ must increase to keep the product constant and equal to N. T can be made as small as we wish, but μ, which we said must be negative, cannot be greater than zero. But the product must be a constant. So (20) is only valid above a certain critical temperature T_C. Below this temperature, our treatment breaks down. Where did we go wrong? The flaw lies in the fact that we assumed that the states are continuously distributed. However, since we are interested in very low temperatures which involves the occupation of the lowest lying energy levels, we may expect that the actual discrete nature of the level distribution might play an essential role in the lowest temperature range. So let us treat the lowest level $E_1 = 0$ separately. We will assume that it is not degenerate with any other levels and we will assume that the remaining levels are continuously distributed from $E = 0$ to $E = \infty$ as described by (12). So in our summation (6) we will treat the lowest level separately:

$$\begin{aligned} N &= \frac{1}{e^{-\beta\mu} - 1} + \frac{V}{\pi^2 \hbar^3} \left(\frac{m^3}{2} \right)^{1/2} \int_0^{\infty} \frac{E^{1/2}}{e^{\beta(E-\mu)} - 1} dE \\ &= \frac{1}{e^{-\beta\mu} - 1} + \left(\frac{T}{T_C} \right)^{3/2} f(\mu) N \qquad (21) \\ & \qquad (22) \end{aligned}$$

where the second quantity on the right is just the right hand side of (20), written in terms of the critical temperature T_C. $f(\mu = 0) = 1$ because T_C is defined such that the right hand side of (20) equals N with $T = T_C$ and $\mu = 0$.

We see that it is now possible to satisfy this new equation with negative values of μ for all temperatures, since the first term becomes infinite as $\mu \to 0$. The inclusion of the lowest energy level as a separate term in our treatment has thus removed the previous difficulty of not being able to account for all of the particles at temperatures below T_C. If we now inquire into what this equation means physically, we see that, at temperatures below T_C, the chemical potential μ will take on such values that those particles which are not included in the continuous distribution will be found in the lowest level. That is, a kind of condensation occurs; it is such that an appreciable fraction of the particles is in the lowest energy level at temperatures below T_C.

If we write (22) as

$$N = n_1 + \left(\frac{T}{T_C}\right)^{3/2} f(\mu)N \tag{23}$$

and realize that $f(\mu \approx 0) \approx 1$ at low temperatures, then we find the population n_1 of the lowest level to be, approximately,

$$n_1 = N\left[1 - \left(\frac{T}{T_C}\right)^{3/2}\right] \tag{24}$$

At $T = T_C$, $n_1 = 0$ while at $T = 0$, $n_1 = N$. Using $n_1 = (e^{-\beta\mu} - 1)^{-1}$, we find that at low temperatures

$$\mu = -k_B T \ln\left(\frac{1}{n_1} + 1\right) \tag{25}$$

Notice that μ is negative. As $T \to 0$, $\mu \to 0$:

$$\mu(T \to 0) \to -0^+ \ln(\frac{1}{N} + 1) \to 0 \quad \text{as } N \to \infty \text{ and } T \to 0 \tag{26}$$

Considering superfluid helium as a 2 component fluid with normal and superfluid components is consistent with having some of the particles in the lowest energy level and the rest in higher energy levels. There is no microscopic theory of superfluid helium, though computer simulations by Ceperley have been quite successful in reproducing its properties. One of the complications is that the helium atoms are so closely packed that they are strongly interacting; they're in a liquid state. It would be closer to the ideal case to have a system of bosons which are weakly interacting.

(Reference: H.–J. Miesner and W. Ketterle, "Bose–Einstein Condensation in Dilute Atomic Gases," Solid State Communications **107**, 629 (998) and references therein.) This has recently been achieved in the case of alkali atoms such as rubidium, sodium, and lithium. Using a combination of optical and magnetic traps together with laser cooling and evaporative cooling, several research groups have achieved Bose condensation in dilute weakly interacting vapors of alkali atoms. In these systems the thermal deBroglie wavelength exceeds the mean distance between atoms. Nanokelvin temperatures and densities of 10^{15} cm^{-3} have been achieved. (Compare this to a mole of liquid which has a typical density of 10^{23} cm^{-3}.) At nanokelvin temperatures the thermal deBroglie wavelength exceeds 1 μm which is about 10 times the average spacing between atoms. In these experiments they have actually been able to directly observe the macroscopic population of the zero momentum eigenstate. In addition the coherence resulting from being in macroscopic wavefunctions has been demonstrated by observing the interference between two independent condensates. Two spatially separated condensates were released from the magnetic trap and allowed to overlap during ballistic expansion of the gases. Interference patterns were observed that are analogous to the pattern produced in a double–slit experiment in optics.