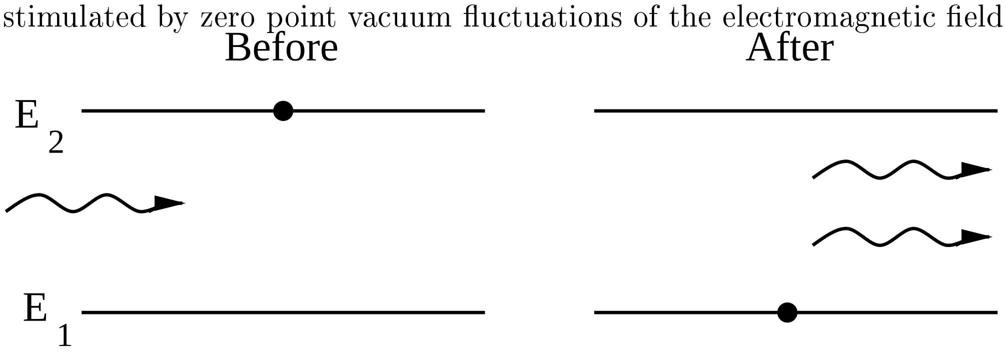
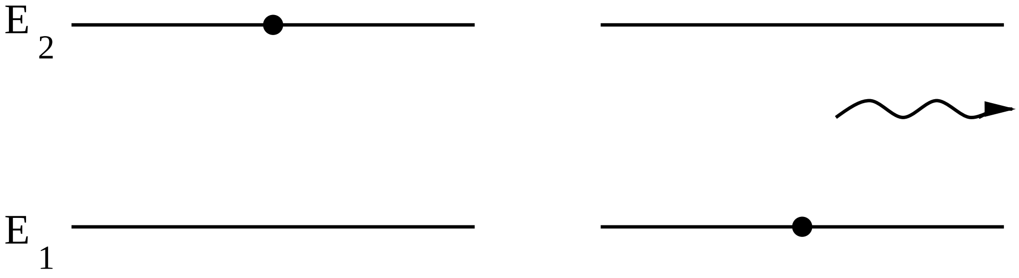
LECTURE 8

How a Laser Works

Laser is an acroynm that stands for Light Amplification by Stimulated Emission of Radiation. A maser is the corresponding system for microwaves. To understand how a laser works, consider an atom with 2 energy levels and one electron. Let's call this a two level system. Let E_1 be the energy of the ground state and E_2 be the energy of the excited state. Suppose the electron is in the lower energy state. If the atom absorbs a photon of frequency $\nu = (E_2 - E_1)/h$, the electron makes a transition to the excited state. From there it can decay back to the lower state by emitting a photon of frequency $\nu = (E_2 - E_1)/h$. There are 2 types of emission processes. In stimulated emission, an incident photon of frequency ν stimulates the atom to make a transition from the higher state to the lower energy state and emit another photon of frequency ν that is in phase with the first photon. The second photon is in phase because the first photon causes the electron charge of the atom to oscillate in phase with it and this oscillation causes the second photon to be in phase with the first. So one photon comes in and two photons leave. In spontaneous emission the excited atom spontaneously decays to the lower energy state and emits a photon of frequency ν. One can regard this as emission stimulated by zero point vacuum fluctuations of the electromagnetic field.



Stimulated Emission



Spontaneous Emission

Now suppose we start with an inverted population of atoms which are each in their excited state. This can be accomplished by optically pumping the atoms into their excited state with incident radiation. Now an incident photon of frequency $\nu = (E_2 - E_1)/h$, which may have been produced by spontaneous emission, stimulates one of the excited atoms to emit a photon in phase with it. These two photons of frequency ν then produce stimulated emission from other excited atoms. If we put the system in a cavity with reflecting walls at the ends, the photons bounce back and forth, gaining more and more cohorts via stimulated emission. If one of the walls is slightly transparent, rather than being totally reflecting, part of this coherent monochromatic beam escapes as a laser beam. Monochromatic means the photons are all of one frequency ν and coherent means that the photons are all in phase. The fact that the photons are in phase means that the amplitudes of the electromagnetic waves add constructively, and this enhances the intensity and energy density concentration of the beam. In fact a laser with less power than a typical light bulb can burn a hole in a metal plate. In contrast, white light is incoherent and nonmonochromatic because there is no correlation in the times that the atoms make transitions.

Let's make this more quantitative. Consider a set of two level systems (atoms) in the presence of electromagnetic radiation. Let N_ν be the number of photons of frequency ν. Let there be n_1 atoms in energy state E_1 and n_2 atoms in energy state E_2, where $E_2 > E_1$ and $\nu = (E_2 - E_1)/h$. The rate of absorption of photons is proportional to the number of photons of frequency ν and to the concentration of atoms in state 1 which can absorb photons:

$$\left(\frac{dN_\nu}{dt}\right)_{\text{absorption}} = -Bn_1N_\nu \tag{1}$$

where B is a constant of proportionality that depends on the properties of states 1 and 2. The minus sign indicates that the photon density decreases due to absorption by the atoms. The rate of stimulated emission is proportional to the number of photons of frequency ν and to the concentration of atoms in state 2 which can be induced to emit photons of frequency ν:

$$\left(\frac{dN_\nu}{dt}\right)_{\text{stimulated emission}} = +Bn_2N_\nu \tag{2}$$

The plus sign indicates that emission increases the number of photons. Spontaneous emission can occur even when there are no photons and the corresponding rate is

$$\left(\frac{dN_\nu}{dt}\right)_{\text{spontaneous emission}} = An_2 \tag{3}$$

So the total rate of change of the number of photons is the sum of all these processes.

$$\left(\frac{dN_\nu}{dt}\right)_{\text{total}} = B(n_2 - n_1)N_\nu + An_2 \tag{4}$$

This is an example of a rate equation. It tells us the rate of change of a population due to emission and absorption. A and B are called the Einstein coefficients.

Let us consider the steady state where the system is in equilibrium. This is not the case for a laser, but let's look at it anyway. In steady state the number of photons does not change with time, so

$$\left(\frac{dN_\nu}{dt}\right)_{\text{total}} = 0 \tag{5}$$

or

$$B(n_2 - n_1)N_\nu + An_2 = 0 \tag{6}$$

Solving for N_ν, we obtain

$$N_\nu = \frac{A}{B\left(\frac{n_1}{n_2} - 1\right)} \tag{7}$$

In equilibrium the ratio of the populations of the excited and lower states is given by the Boltzmann probability distribution

$$\frac{n_2}{n_1} = e^{-\beta(E_2 - E_1)} \tag{8}$$

or

$$\frac{n_1}{n_2} = e^{\beta(E_2 - E_1)} \tag{9}$$

Plugging this into (7) yields

$$N_\nu = \frac{A}{B\left(e^{\beta(E_2 - E_1)} - 1\right)} \tag{10}$$

On the other hand, we already know what the distribution is for photons in equilibrium. It's just the Planck distribution that we derived when we discussed black body radiation

$$N_\nu = \frac{1}{e^{\beta h\nu} - 1} = \frac{1}{e^{\beta(E_2 - E_1)} - 1} \tag{11}$$

Comparing (10) and (11) yields

$$A = B \tag{12}$$

Thus the coefficient in the spontaneous emission rate is exactly the same as in the induced emission rate; the total emission rate is

$$\left(\frac{dN_\nu}{dt}\right)_{\text{total emission}} = Bn_2(N_\nu + 1) \tag{13}$$

Note that the form of the emission rate was deduced without knowing the actual value of B; only *detailed balancing* arguments were used. Detailed balancing is simply the idea that in equilibrium the amount of energy absorbed and emitted must be equal.

Now let's consider the case of the laser. This is not an equilibrium situation. The ratio of the emission and absorption of photons is given by

$$\left|\frac{\left(\frac{dN_\nu}{dt}\right)_{\text{emission}}}{\left(\frac{dN_\nu}{dt}\right)_{\text{absorption}}}\right| = \frac{(A + BN_\nu)n_2}{BN_\nu n_1} \tag{14}$$

In equilibrium

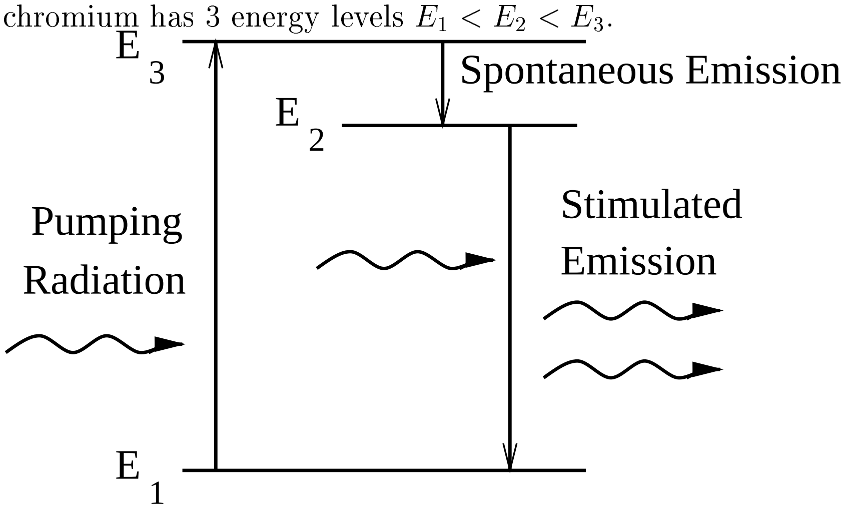
$$\left| \frac{\left(\frac{dN_\nu}{dt}\right)_{\text{emission}}}{\left(\frac{dN_\nu}{dt}\right)_{\text{absorption}}} \right| = 1 \tag{15}$$

but in a laser we have population inversion ($n_2 > n_1$) and

$$\left| \frac{\left(\frac{dN_\nu}{dt}\right)_{\text{emission}}}{\left(\frac{dN_\nu}{dt}\right)_{\text{absorption}}} \right| \sim \frac{n_2}{n_1} \tag{16}$$

So when the population is inverted, the rate of emission is greater than the rate of absorption. This means that the applied radiation of frequency $\nu = (E_2 - E_1)/h$ will be amplified in intensity because of stimulated emission with much more radiation emerging than entering. Of course, such a process will reduce the population of the upper state until equilibrium is established. In order to sustain the process, population inversion must be maintained. One common way to do this is via optical pumping.

As an example let's consider a solid state laser that operates with a ruby crystal in which some of the Al atoms in the Al_2O_3 molecules are replaced by Cr atoms. These "impurity" chromium atoms account for the laser action. The energy level scheme of chromium has 3 energy levels $E_1 < E_2 < E_3$.



The energy difference $E_3 - E_1$ corresponds to a wavelength of about 5500 Å. Optical pumping occurs in the following way. Pumping radiation with a wavelength 5500 Å is sent in and absorbed by Cr atoms which make a transition from their ground state to the level with energy E_3. This is a short lived state (lifetime is about 10^{-8} seconds) which decays spontaneously to the intermediate excited state which is long lived (about 3×10^{-3} seconds). The net result of the optical pumping is to invert the population such that $n_2 > n_1$. So when a Cr atom makes a transition from state 2 to state 1, it emits a photon of wavelength 6943 Å which will stimulate transitions in other Cr atoms. Because $n_2 > n_1$, emission dominates over absorption and an intensified coherent monochromatic beam results. The ruby laser consists of a cylindrical rod with parallel, optically flat reflecting ends, one of which is only partly reflecting. The laser beam exits through this partially reflecting end. The emitted photons that do not travel along the axis escape through the sides before they are able to cause much stimlated emission. But those photons that move exactly in the direction of the axis are reflected several times, and they are capable of stimulating emission repeatedly. Thus the number of photons is built up rapidly, those escaping from the partially reflecting end giving a unidirectional beam of great intensity and sharply defined wavelength.

There are many kinds of lasers–gas lasers, liquid lasers, semiconductor lasers, and solid state lasers made from crystals. These cover a wide range of the electromagnetic spectrum. Lasers have a wide range of uses including medical physics, biophysics, communications, and data storage (CD players). For example, semiconductor lasers that are the size of a grain of sand are used to send signals down optic fibers. I would like to spend a little time describing some of the more amazing applications of lasers that have been developed in recent years, namely laser trapping and laser cooling.

Laser Trapping and Tweezers

Investigators have been able to use lasers to trap and manipulate atoms and micron–size particles with astonishing control. For example, laser tweezers use a laser beam to hold and move organelles (like the mitochondria) inside of living cells without puncturing the intervening membranes. Steve Chu, who won the 1997 Nobel Prize in physics, has a cute video showing a single DNA molecule attached to a polystyrene bead. By using laser tweezers, they can whip the DNA molecule around at will. They can also fix the ends of the DNA molecule and pluck it like a violin string and study its modes of vibration.

Laser tweezers consist of a focussed laser beam. The particle is trapped at the focal point where the field is most intense. Laser tweezers work because of the "dipole force" on the trapped particle. The oscillating electric field $\vec{E}$ of the light induces a dipole moment $\vec{p}$ on the particle. If the induced dipole moment is in phase with $\vec{E}$, the interaction energy $-\vec{p}\cdot\vec{E}$ is lower in high field regions. If the induced dipole moment is out of phase with the driving field, the particle's energy is increased in the electric field and the particle will feel a force ejecting it out of the field. If we model the atom or particle as a damped harmonic oscillator, the sign change of the dipole force is easy to understand.

Recall that the electric susceptibility χ_e relates the polarization $\vec{P}$ to the electric field $\vec{E}$: $\vec{P} = \chi_e\vec{E}$. On a microscopic scale, χ_e is related to the polarizability α of an atom or molecule: $\vec{p} = \alpha\vec{E}$. Let's consider an atom (or molecule) that's in an oscillating electric field $\vec{E} = \vec{E}_o e^{-i\omega t}$. Since we're dealing with the macroscopic field $\vec{E}$, we can treat $\vec{E}_o$ as a constant over the size of the atom. We're also going to assume that $\vec{E}$ is small so that it causes only small displacements of the electrons in the atom from their equilibrium positions. This means that we can use the theory of small oscillations to treat the electrons as harmonic oscillators. Finally we'll assume that there is some damping force acting on the electrons that is proportional to their velocity. Let x be the displacement parallel to $\vec{E}$ of an electron whose oscillator frequency is ω_o. Then the equation of motion is

$$m\frac{d^2x}{dt^2} + m\gamma\frac{dx}{dt} + m\omega_o^2 x = qE_o e^{-i\omega t} \tag{17}$$

This is the equation for a driven dampled harmonic oscillator. In the steady state (after all transient behavior has stopped) $x = x_o e^{-i\omega t}$. Plugging this in (17) yields

$$e^{-i\omega t}\left[-m\omega^2 - im\gamma\omega + m\omega_o^2\right]x_o = qE_o e^{-i\omega t} \tag{18}$$

Solving for x_o, we obtain

$$x_o = \frac{qE_o}{m\left[(\omega_o^2 - \omega^2) - i\gamma\omega\right]} \tag{19}$$

The dipole moment associated with the displacement of an electron is $p = qx$ where q is the electric charge. So the induced dipole moment is the real part of

$$\vec{p} = \left[\frac{q^2}{m\left[(\omega_o^2 - \omega^2) - i\gamma\omega\right]}\right]\vec{E} \tag{20}$$

Comparing this with $\vec{p} = \alpha\vec{E}$, we see that the polarizability is

$$\alpha = \frac{q^2}{m\left[(\omega_o^2 - \omega^2) - i\gamma\omega\right]} \tag{21}$$

The real part is given by

$$\alpha_R = \frac{q^2\left(\omega_o^2 - \omega^2\right)}{m\left[\left(\omega_o^2 - \omega^2\right)^2 + (\gamma\omega)^2\right]} \tag{22}$$

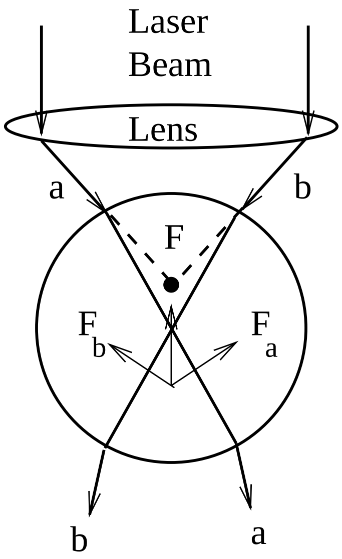
Notice that below resonance ($\omega < \omega_o$), α_R is positive whereas above resonance α_R is negative.

Going back to $U = -\vec{p}\cdot\vec{E}$, we can use $\vec{p} = \alpha\vec{E}$ to write

$$< U >= - < \mathrm{Re}\,\vec{p}(t)\cdot\mathrm{Re}\,\vec{E}(t) >= -\frac{1}{2}\alpha_R E_o^2 \tag{23}$$

where the average is over one period of the oscillating electric field. The factor of 1/2 comes from this average ($< \cos^2\omega t >= 1/2$). "Re" means take the real part of the complex number. So the particle will be trapped at the focal point of the laser beam (where E is largest) as long as the laser is tuned below resonance. The particle will be pushed away if the laser is tuned above resonance. One can use this repelling effect to make a trampoline of light with atoms bouncing on it.

Another way to understand how laser tweezers works is as a lensing effect. A lens alters the distribution of momentum of light, and by Newton's third law, the lens must experience a reaction force equal and opposite to the rate of momentum change of the light. A small particle can act as the lens.



Tweezers Trap