LECTURE 14

Cosmology

(Reference: M. S. Turner and J. A. Tyson, "Cosmology at the Millennium," in *More Things in Heaven and Earth: A Celebration of Physics at the Millennium*, edited by Benjamin Bederson (1999); and S. Weinberg, *The First Three Minutes* (Updated Edition), published by Basic Books (1993).)

Cosmology is the study of the origin of the universe and its subsequent evolution. "Where did the universe come from?" is one of the oldest questions that man has asked. For example, the book of Genesis in the Bible begins with "In the beginning . . ." The amazing thing is that science can actually say intelligent things about the beginning of the universe based on observations.

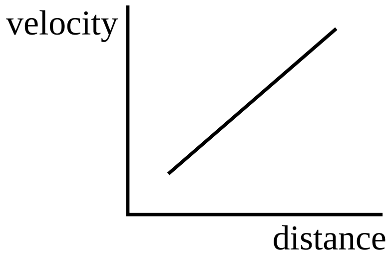
If you look up at the night sky, it looks pretty much the same as it did centuries ago. In fact, until this century, it was believed that the heavens were unchanging. We now know that the universe began between 13 and 17 billion years ago and has been expanding ever since. The initial event when the universe began is called the Big Bang. The Big Bang is marked by a tremendous release of energy; so much energy, in fact, that the 4 basic forces (gravity, electromagnetic, strong and weak) were unified into one. One tends to think of the universe starting as a point and expanding out like a balloon. But one can equally well think of the universe as starting infinitely big with a cosmic scale factor $R(t)$ that grows with time. One can say the universe is infinitely big because one always takes the integrals over space to go from $-\infty$ to $+\infty$. So the universe starts out infinite and gets bigger! Which picture is correct is not known and the question may not have any real meaning. As the universe expanded, it cooled. The 4 basic forces differentiated (GUTs symmetry was broken), matter was formed (baryons, leptons) from the energy, atoms (primarily hydrogen and helium) were formed, and eventually stars and galaxies. We'll discuss the details of this timeline later. I want first to tell you about some of the history of cosmology.

Evidence for the Big Bang

1. General Relativity and Hubble's Observations

How do we know the universe is expanding? What evidence do we have for the Big Bang? As I said, at the beginning of the 20th century, people assumed the universe was unchanging and was the way it had always been. In 1917 Einstein worked out the theory of general relativity which deals with gravity in terms of a curved space-time. The Einstein equations said that the universe was expanding and decelerating. (Gravity was causing the deceleration.) Einstein believed that his equations were coming to the wrong conclusion. He believed that the universe was unchanging. So he fixed his equations by adding a constant of integration called the "cosmological constant" which is often denoted by Λ.

In 1929 Edwin Hubble presented observational evidence that the universe was expanding. He found a correlation between the velocities of galaxies and their distance from us. Basically it's a linear relation that says the farther a galaxy is from us, the faster it's moving.



We can tell the speed of a galaxy from its red shift. The larger the red shift, the faster it's moving away from us. The red shift (often denoted by z) can be determined by passing the light from a galaxy through a spectrum analyzer. The spectra of spiral galaxies have bright emission lines coming from hydrogen gas while elliptical galaxies have dark thin absorption lines which correspond to the atomic transitions by atoms in the atmospheres of stars in the galaxy. By comparing a galaxy's spectrum with the known atomic spectra of various elements, we can determine what the red shift is. (All the lines are shifted by the same amount.) So once we have the red shift, we have the velocity. Now how do we find the corresponding distance? We must have some independent way of finding the distance. Hubble used the Cepheid variable stars in galaxies. These are bright stars whose luminosity varies periodically in time. Leavitt and Shapley had worked out a relation between the period and the absolute luminosity of a Cepheid variable. Using this and the apparent luminosity of the Cepheids, Hubble was able to determine the distance to various galaxies. We refer to the Cepheids as "standard candles." (Absolute luminosity refers to the total power emitted by an object; apparent luminosity refers to what we see.) Hubble found the linear relationship shown in the figure. The farther the galaxy, the bigger the red shift, and the larger its velocity away from us. This was the first evidence that the universe is expanding. Einstein kicked himself (figuratively speaking) for not predicting that the universe was expanding, said that introducing the cosmological constant was his biggest mistake, and figured that $\Lambda = 0$. Note that just because all the distant galaxies we see are moving away from us, does not mean that we are at the center of the galaxy. If we lived in some other galaxy (far, far away) we would see the same thing. Think of a balloon with polka dots painted on it. Suppose there is an ant sitting on one of the dots looking around at the other dots. As the balloon is blown up, the ant sees all the other dots moving away from him. This idea is embodied in the cosmological principle which is the hypothesis that the universe is isotropic and homogeneous.

One can view the red shift as a Doppler shift associated with the expansion of the universe. The red shifts are characterized by a parameter z which is defined by

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} \tag{1}$$

where λ is the wavelength of emitted or observed light. It is also valid to think of the red shift as due to increasing or conformally stretching the wavelengths of light which are embedded in the expanding fabric of spacetime. Let $R(t)$ be the cosmic scale factor which has dimensions of length. The conformal stretching of wavelengths can be expressed by the fact that the wavelength is larger now by a factor

$$\frac{R(\text{now})}{R(\text{then})} = \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = 1 + z \tag{2}$$

If we take relativistic time dilation into account, then the red shift due to the Doppler shift can be expressed as

$$1 + z \equiv \frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} = \sqrt{\frac{c+v}{c-v}} \tag{3}$$

where v is the velocity of the source. Notice that it is possible for z to have very large values. In the limit that $v/c \ll 1$, this reduces to the classical limit

$$\frac{\lambda_{\text{observed}}}{\lambda_{\text{emitted}}} \approx 1 + \frac{v}{c} \tag{4}$$

The slope of Hubble's velocity versus distance plot is called Hubble's constant and is denoted by H_o. Let d be the distance and let the velocity be zc where c is the velocity of light. Then for $z \ll 1$, we can write Hubble's relationship as

$$z \simeq \frac{H_o d}{c} \tag{5}$$

The value of H changes with time because the rate of expansion of the universe changes and because our measurements get better. The current value is $H_o = (67 \pm 10)$ km/sec-Mpc. 1Mpc = 1 megaparsec $=3.09\times10^{24}$ cm $\simeq$ 3 million light years. (1 parsec $\approx$ 3 light years. This is the typical distance betweeen stars.) The inverse of the Hubble constant–the Hubble time–sets a timescale for the age of the Universe: $H_o^{-1} = (15 \pm 2)$ billion years. (We use H to denote the value of Hubble's constant at any time and H_o to denote the value at the present time.)

Until recently it was generally believed that $\Lambda = 0$ or that Λ was very small. One assumed that the initial outward push was provided by the energy released in the Big Bang and that the gravitational attraction of the matter and energy of the universe was causing the rate of expansion to decrease. However, in 1998, 2 groups of astrophysicists used type 1a supernovae as standard candles. Type 1a supernovae result when a white dwarf accretes matter and reaches the Chandrasakar limit of 1.4 solar masses. At that point, it explodes. (A white dwarf is a hot, dim star that has burned all of its hydrogen and almost all of its helium. It has a carbon oxygen core surrounded by a thin layer of helium rich gas. The only thing holding it up against gravitational collapse is the electron degeneracy pressure (Pauli principle for electrons). The Chandrasakar limit is when gravity beats the electron degeneracy pressure.) Since all type 1a supernovae start from the same mass, we know their absolute luminosity. The characteristic shape of their light curves are powered by the radioactive decay of Ni^{56}. Using these type 1a supernovae as standard candles, 2 groups of astrophysicists determined that the best fit to their data gave a finite value for Λ ($\Omega_\Lambda \sim 2/3$, where $\Omega_\Lambda = \Lambda/3H_o^2$) corresponding to a universe whose expansion is accelerating. This value of Ω_Λ also agrees with the data from Boomerang which is a recent survey of anisotropies in the cosmic microwave background radiation. The source of this accelerating force comes from the energy density of the quantum vacuum or from something else. No one really knows. One problem with field theory calculations which try to estimate the cosmological constant which would arise from the vacuum energy is that they find $\Omega_{\rm vac} \sim 10^{55}$. Actually the vacuum energy diverges due to zero point energies ("the ultraviolet catastrophe"), so this estimate of $\Omega_{\rm vac}$ comes from putting in a short wavelength cutoff of $\sim 10^{-17}$ cm.

2. Cosmic Microwave Background Radiation

The cosmic microwave background radiation (CMBR) is the remnant of the energy released by the Big Bang. Think of baking something in your oven at home. After you turn off the oven, it cools down. If you come back a few hours later, it may still be a little bit warm and those few left over photons are the analog of the cosmic microwave background radiation. The CMBR was discovered in 1964 by 2 scientists at Bell Labs, Arno Penzias and Robert Wilson, who were working with an antenna that was to be used in communications via satellite. Their antenna was sensitive to microwave radiation. They found that there was a persistent background noise that they couldn't get rid of. They even tried cleaning the bird droppings from their antenna. Eventually they heard that Peebles and Dicke at Princeton had theorized that the remnant of the Big Bang should be observable and their colleagues (Roll and Wilkinson) were setting up an experiment to look for this remnant. Penzias and Wilson then realized that they had detected the cosmic microwave background, though their Nobel–prize winning paper is modestly entitled "A Measurement of Excess Antenna Temperature at 4,080 Mc/s." Radio engineers describe radio noise in terms of a so–called antenna temperature, which roughly corresponds to the black body radiation temperature that would produce such noise. In fact the cosmic microwave background radiation matches the blackbody spectrum of a blackbody at a temperature of $T = 2.73$ K. It's often referred to as the "3 degree blackbody radiation." The COBE (Cosmic Background Explorer) satellite measured the radiation at various frequencies and found that the deviations from a blackbody spectrum are less than 300 parts per million. The only viable explanation for such perfect blackbody radiation is the hot, dense conditions that are predicted to exist at early times in the hot Big Bang model. The CMBR consists of photons from when the universe was 300,000 years old. As expected, it is all around us and does not come from a certain part of the sky. The uniform isotropy is impressive. The anisotropy on angular scales of 10^o is about 30 μK or $\delta T/T \simeq 10^{-5}$. Presumably these anisotropies have been amplified by gravity over the years and have given rise to the large scale structure of the universe (large groupings of galaxies).

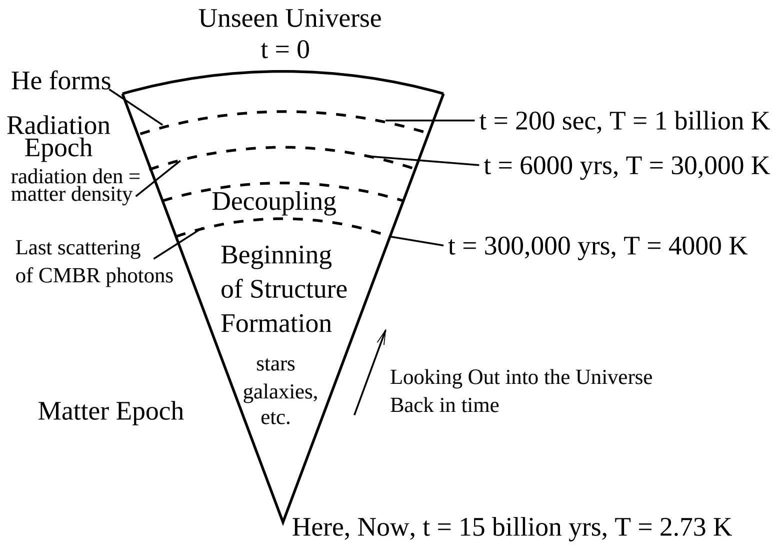
3. Big Bang Nucleosynthesis

The final observational pillar of standard cosmology is Big Bang nucleosynthesis. When the universe was seconds old and the temperature was around 1 MeV a sequence of nuclear reactions led to the production of the light elements D, ^{3}He, ^{4}He, and ^{7}Li.

(Hydrogen is just a proton, so it doesn't count. At high temperatures, the electrons are stripped off and there are just ionic nuclei.) Let me sketch how nucleosynthesis goes. Deuterium is made when a neutron and a proton come together; add a proton to deuterium to get ^{3}He; add another neutron to ^{3}He to get ^{4}He; etc. Coulomb barriers (electrostatic repulsion of additional protons) and the lack of stable nuclei with mass numbers of 5 and 8 prevent further nucleosynthesis. (Heavier elements were created in stars and stellar explosions billions of years after the Big Bang.) The abundance of the light elements observed in the cosmos is consistent with that predicted by the model of Big Bang nucleosynthesis. Almost all of the hydrogen and helium in the universe is a product of the Big Bang. Observations indicate that when stars form, they consist mostly of hydrogen with about 20–30% helium. This is consistent with estimates based on Big Bang nucleosynthesis. This then is further confirmation of the Big Bang.

Evolution of the Universe

The history of the universe so far can be divided into 2 epochs: the radiation dominated phase and the matter dominated phase.



- *Radiation Dominated Phase* The radiation dominated phase ($t \lesssim$ 10,000 years, $k_BT \gtrsim 3$ eV) was when the energy density contained in radiation and relativistic particles exceeded that in matter. The scale factor $R(t) \sim t^{1/2}$ and the temperature decreased as $k_BT \sim 1\ \mathrm{MeV}(t/\mathrm{sec})^{-1/2}$ (see appendix). At the earliest times, the energy in the universe consists of radiation and relativistic particle–antiparticle pairs. When $k_BT \gg mc^2$, pair creation from photons makes particle–antiparticle pairs as abundant as photons. The standard model of particle physics provides the input for our understanding of what happened at $t = 10^{-11}$ sec when $k_BT \sim 300$ GeV. At this time the sea of relativistic particles includes the 6 species of quarks and antiquarks, leptons and antileptons as well as the 12 gauge bosons (photons, $W^\pm$, Z^o, and 8 gluons). When the temperature drops below the mass of a particle species, those particles and their antiparticles annihilate and disappear. As the temperature fell below $k_B T \sim 200$ MeV, a phase transition occurred from a quark–gluon plasma to neutrons, protons, and pions, along with the leptons, antileptons, and photons. At a temperature of $k_B T \sim 100$ MeV, the muons and antimuons disappeared. When the temperature was around 1 MeV, a sequence of events and nuclear reactions began that ultimately resulted in the synthesis of D, ^{3}He, ^{4}He, and ^{7}Li. During this time, since $k_B T \sim m_e c^2 \sim 0.5$ MeV, the last of the particle–antiparticle pairs, the electrons and positrons, annihilated.

- *Matter Dominated Phase* The matter dominated phase ($t \gtrsim$ 10,000 years, $k_B T \lesssim$ 3 eV) began when the energy density in matter exceeded that in radiation. The cosmic scale factor began to grow as $R(t) \propto t^{2/3}$ (see appendix). Shortly after matter domination begins, at a redshift of $1 + z \simeq 1100$, the universe has cooled enough to allow electrons and ions (mostly free protons) to combine into neutral atoms. This occurs at $k_B T \sim 3000$ K which is roughly 300,000 years after the beginning of the universe. Because neutral atoms do not scatter photons nearly as much as charged particles (electric and magnetic fields do not couple to neutral particles), matter and radiation decouple. Thus the photons we see in blackbody radiation have been traveling ballistically since this time. We are still in the matter dominated phase. Over the last 13 billion years or so, the primeval inhomogeneities in the density of matter have been amplified by gravity to form the structures that we see today: galaxies, clusters of galaxies, superclusters, great walls, and voids. A galaxy has a few hundred billion stars and there are at least a hundred billion galaxies in the universe. Light from a star on one edge of the galaxy takes about 100,000 years to reach the opposite side. Our Milky Way galaxy is part of a local group of some 30 galaxies. The Local Group extends some 4 million light years across. Our Local Group is part of a supercluster of galaxies some 150 million light years across. Our supercluster is centered on the Virgo cluster which contains thousands of galaxies. Outside the supercluster is a nearly galaxy–free region called a cosmic void.

Future of the Universe

Will the universe expand forever or will it eventually stop and then contract? No one really knows. But we can describe the different scenarios. Let's define a critical density $\rho_c = 3H_o^2/8\pi G$ where G is the gravitational constant. ρ_c corresponds to about 5 protons per cubic meter.

For simplicity let's first suppose that the cosmological constant is zero. (This was a fine assumption until 1998.) If the density ρ of matter and energy is less than the critical density ρ_c, the universe will expand forever. We say that the universe has negative curvature. General relativity relates gravity to a geometric picture of spacetime. If the universe has $\rho > \rho_c$, the expansion will eventually stop expanding and will contract. This ends in the "big crunch." In this case we say that the universe has positive curvature. If $\rho = \rho_c$, we say that the universe is flat. In this case the universe will expand forever, though the expansion slows down (due to gravity) and eventually stops at $t = \infty$.

It is convenient to scale energy densities to the critical density:

$$\Omega_i = \frac{\rho_i}{\rho_c} \tag{6}$$

$$\Omega_o = \sum_i \Omega_i \tag{7}$$

where we are summing over different contributions to the total Ω_o. For example, baryonic matter, photons, neutrinos, the cosmological constant, etc. The different curvatures correspond to

$$\begin{array}{lll} \Omega_o & < 1 & \text{negatively curved universe} \\ \Omega_o & = 1 & \text{flat universe} \\ \Omega_o & > 1 & \text{positively curved universe} \end{array}$$

Sometimes a positively curved universe is called a closed universe and a negatively curved universe is called an open universe. But this nomenclature is only valid if the cosmological constant is zero.

In the case of a nonzero cosmological constant, things get more complicated. For a flat or negatively curved universe, the cosmological constant eventually dominates over the gravitational pull of matter because the matter density is decreasing with the expansion. Ultimately the universe enters an exponential expansion phase driven by the cosmological constant. This also occurs for a positively curved universe if the cosmological constant is large enough. If it isn't large enough, recollapse occurs.

Dark Matter

We don't know the density of energy and matter well enough to know whether Ω_o is greater than, less than, or equal to 1, though it's generally felt that Ω_o is close to 1. The theory of Big Bang nucleosynthesis and the measured abundance of primordial deuterium implies that the mass density contributed by baryons is $\Omega_B \simeq 5\%$. Together photons and neutrinos (assuming all 3 species are massless, or very light, $\ll 10^{-3}$ eV) contribute a very small energy density $\Omega_{\nu\gamma} \simeq 10^{-4}$. Most of the matter in the universe is of unknown form and dark. This mystery matter is called "dark matter." It's dark in the sense that it isn't in shining stars. Stars and closely related material contribute $\Omega_{\rm lum} \simeq 0.004$. While we can't directly see the dark matter, we can observe the effect of its gravitational pull on other objects. For example, we can watch the motion of a galaxy in a cluster of galaxies and deduce the gravitational potential that it is moving in by using the virial theorem (KE$\simeq |\text{PE}|/2$). From these observations, we deduce that the galaxies in the cluster have more mass than we can see.

In addition we can map out the rotational velocity v of a galaxy as a function of distance r from the center. If m is the mass of a star in the galaxy and M is the mass of the galaxy, then roughly speaking we expect

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \tag{8}$$

Or

$$v^2 = \frac{GM}{r} \tag{9}$$

If most of the mass is near the center of the galaxy where most of the luminous material is, then we expect the velocity $v \sim 1/\sqrt{r}$ which is what happens with the velocity of planets in our solar system. But what is observed is $v \sim$ const with respect to r, which implies that the mass M of the galaxy increases linearly with r. Since luminous matter doesn't increase linearly with r, it must be dark matter that is responsible for the rotational behavior of most galaxies.

Another observation involves gravitational lensing. Gravity bends light, so a cluster of galaxies can bend the light from much more distant galaxies. Close to the center of the cluster, lensing is strong enough to produce multiple images; farther out, lensing distorts the shape of distant galaxies. From its performance as a lens, we can deduce the mass density of the cluster. Again we find that the mass is much greater than than the luminous matter. Taking into account the large amount of hot intracluster gas deduced from x–ray measurements, we estimate $\Omega_M \sim 1/3$. Recall that $\Omega_\Lambda \sim 2/3$. Thus the total $\Omega_o \simeq 1$.

Pulling this all together: stars contribute 0.4% of the critical density, baryons contribute 5%, nonrelativistic (nonbaryonic) particles of unknown type contribute 30%, and vacuum energy contributes 64% for a total equaling the critical density. The unseen baryonic matter could be in the form of diffuse gas or dark stars (faint, low mass stars; white dwarfs, neutron stars, or black holes); we don't really know. We also don't know what the nonbaryonic dark matter is. Particle physics suggests 3 dark–matter candidates: a 10^{-5} eV axion; a 10 GeV–500 GeV neutralino; and a 30 eV neutrino. Efforts are currently underway to search for all this missing matter.

There are 2 basic types of dark matter: hot dark matter and cold dark matter. If most of the matter is hot, e.g. 30 eV neutrinos, then the structure of the universe formed from the top down: large things, like superclusters, formed first, and fragment into smaller objects such as galaxies. This is because fast moving neutrinos smooth out density perturbations on small scales by moving from regions of high density into regions of low density. On the other hand, cold dark matter particles cannot move far enough to damp perturbations on small scales, and structure then formed from the bottom up: galaxies, followed by clusters of galaxies, and so on. Observations clearly indicate that galaxies formed first (at red shifts of $z \sim 2 - 4$), before superclusters which are just forming today. That rules out hot dark matter and leaves cold dark matter.

Fundamental Questions

There are a number of fundamental questions that are left unanswered by the standard model of the Big Bang in which the universe expands adiabatically (i.e., without changing its entropy).

1. *Matter–Antimatter Asymmetry:* The laws of physics are very nearly symmetric with respect to matter and antimatter. Yet everywhere we look, we see matter, not antimatter. If the early universe had equal amounts of matter and antimatter,

these would have annihilated as the universe cooled, leaving only trace amounts of nucleons and antinucleons. Instead what we see is a small net baryon number. The ratio of baryons to photons is $n_B/n_\gamma = (n_b - n_{\bar{b}})/n_\gamma = \eta = 5 \times 10^{-10}$. A possible solution was suggested by Sakharov in 1967: baryon–number violating and matter–antimatter symmetry violating interactions occurring in a state of nonequilibrium allows for a small net baryon number to develop. The Grand Unified Theories (GUTs) of particle physics allow for violation of baryon number (proton decay), and matter–antimatter symmetry is known to be violated slightly in the neutral Kaon system and in B meson systems. (CP violation at the level of 10^{-3}).

2. *The heat of the Big Bang:* The entropy associated with the CMBR and the three neutrino seas is enormous. Within the observable universe the entropy is 10^{88} k_B (the number of nucleons is 10 orders of magnitude smaller). Where did all the heat come from?

3. *Origin of the Smoothness:* The universe is very smooth and isotropic. The anisotropy of the CMBR is tiny. Yet the different parts of the universe are causally disconnected, i.e., different parts of the universe have not had enough time to communicate with one another given the speed of light. So why are these different parts so alike?

4. *Origin of the Flatness:* Today it appears that $\Omega_o \simeq 1$. (The subscript o means the value of Ω now.) The universe was even more flat in the past, i.e., Ω was even closer to one: $|\Omega - 1| < 10^{-16}$ at 1 second. To arrive at the universe we see today, the universe must have begun very flat. If in the beginning the universe was slightly above the critical density, it would have collapsed from the gravity. If it was slightly below ρ_c, matter and energy would have flown apart too fast to allow condensation into stars and galaxies. Why is the universe so flat?

   The flatness and smoothness problems are not indicative of any inconsistency of the standard model of Big Bang cosmology, but they do require fine tuning of the initial conditions. As stated by Collins and Hawking (1973), the set of initial conditions that evolve to a universe qualitatively similar to ours is of measure zero.

5. *Origin of the Big Bang:* Where did the Big Bang come from? How did it arise?

Inflation

A possible solution to the last four questions is inflation. The idea is that the universe begins with a brief period of tremendous expansion (inflation) in which the scale factor increases by 10^{27} in 10^{-32} seconds. The precise details of this “inflationary phase” are not understood, but in most models the exponential expansion occurs when a scalar field ϕ that represents or pervades the universe is initially displaced from the old minimum of its potential energy curve and moves to a new minimum. (Some models postulate a first or second order phase transition, but other models, such as chaotic and stochastic inflation, do not require a phase transition. The phase transition models tend to have problems. For example, a second order phase transition requires very fine tuning of the potential near the origin as well as strong coupling of ϕ to other matter fields which would tend to destroy the fine tuning. See the book by Peter Coles and Francesco Lucchin, *Cosmology: The Origin and Evolution of Cosmic Structure* for more details.) Inflation blows up a small (subhorizon–sized) portion of the universe to a size much greater than that of the observable universe today. Because this tiny region was causally connected before inflation, it can be expected to be smooth and homogeneous–including the very small portion of it that is our observable part of the universe. Likewise, because our Hubble volume (i.e., our observable part of the universe) is but a small part of the region that inflated, it looks flat, regardless of the initial curvature of the region that inflated which implies that $\Omega_o = 1$. This is analogous to saying that a tiny piece of a curved line looks straight if the piece is small enough, no matter how curved the line.

It is while this scalar field responsible for inflation rolls slowly down its potential that the exponential expansion takes place. As the field reaches the minimum of the potential energy curve, it overshoots and oscillates about it. The quanta of these oscillations decay into lighter particles which thermalize and provide the tremendous heat content of the universe.

Quantum fluctuations arise in the scalar field that drives inflation; they begin as truly microscopic ($\lesssim 10^{-23}$ cm). However, they are stretched in size by the tremendous expansion during inflation to astrophysical scales. (This results in fluctuations in the local curvature of spacetime which are equivalent to fluctuations in the gravitational field.) This gives rise to the inhomogeneities seen in the distribution of energy and matter density in the universe. A limit on the amount of anisotropy was set by the COBE observations of the CMBR. Further refinements have been measured by Boomerang which flew a balloon with a very sensitive detector around Antarctica.

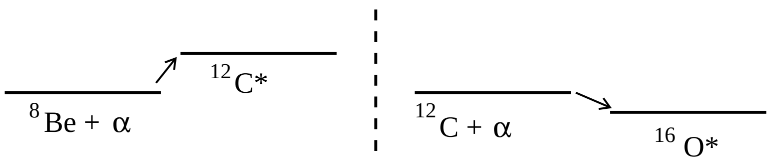
To give you some idea of length scales, fluctuations on length scales of ~ 1 Mpc give rise to galaxies, on scales of ~ 10 Mpc give rise to clusters of galaxies, and on scales of ~ 100 Mpc give rise to great walls of clusters of galaxies.

Stellar Nucleosynthesis

As we mentioned earlier, the light elements such as hydrogen and helium were produced by the Big Bang. This is called Big Bang nucleosynthesis. The heavier elements are produced in stars and stellar explosions. I just want to briefly mention how this goes. Stars are powered by fusion which is a process whereby nuclei of lighter elements are fused together to make heavier nuclei and energy is released in the process. For example our sun produces most of its power by fusing 4 hydrogen atoms together to make helium ($4\,{}^1H \rightarrow {}^4He$). Heavier elements such as carbon and oxygen can be made as follows:

$$\begin{aligned}
{}^4He + {}^4He &\rightarrow {}^8Be \quad ({}^8Be \text{ is unstable. } \tau \sim 10^{-15} \text{ sec}) \\
{}^8Be + \alpha &\rightarrow {}^{12}C \\
{}^{12}C + \alpha &\rightarrow {}^{16}O
\end{aligned}$$

An α particle is a ^{4}He nucleus consisting of 2 protons and 2 neutrons. The chain keeps going. Carbon and oxygen are crucial to living organisms. It is an interesting coincidence that the nuclear energy levels are such that we get both carbon and oxygen, rather than mostly one or the other. If it was too easy to make oxygen, all the carbon would be turned into oxygen. If it was too hard to make oxygen, there would be too much carbon and not enough oxygen to sustain life.



Appendix
Critical Density

In this appendix we derive the expression for the critical density using Newtonian mechanics. This is the density where the kinetic energy of outward expansion is balanced by the inward pull of gravity. Let's assume that the gravitational constant $\Lambda = 0$ and that the universe is flat (no curvature). Conside the motion of a galaxy of mass m by assuming that it sits on the surface of a sphere of radius $R(t)$. (One can also think of $R(t)$ as the cosmic scale factor.) The sphere has uniform density $\rho(t)$ and fixed mass M given by:

$$M = \frac{4\pi R^3}{3}\rho \tag{10}$$

The gravitational attraction of the mass inside the sphere produces the potential energy:

$$PE = -\frac{mMG}{R} = -\frac{4\pi R^3 G\rho m}{3R} = -\frac{4\pi R^2 G\rho m}{3} \tag{11}$$

where m is the mass of the galaxy and the gravitational constant $G = 6.67 \times 10^{-8}$ cm^3/gm–sec^2. The velocity of this galaxy is given by the Hubble law as

$$v = HR \tag{12}$$

where H is Hubble's constant (though it changes with time). Thus its kinetic energy is given by

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}mH^2R^2 \tag{13}$$

The total energy is then

$$E = KE + PE = mR^2\left[\frac{1}{2}H^2 - \frac{4}{3}\pi\rho G\right] \tag{14}$$

The energy remains constant as the universe expands. When $E = 0$, the kinetic energy for expansion exactly balances the potential energy for contraction. This is the condition for the critical density ρ_c.

$$\frac{1}{2}H^2 = \frac{4}{3}\pi\rho_c G \tag{15}$$

Thus the critical density is given by

$$\rho_c = \frac{3H^2}{8\pi G} \tag{16}$$

Note that from eq. (15), we have

$$H^2 = \frac{8\pi G\rho}{3} \tag{17}$$

for a flat universe with $\Lambda = 0$. If the universe is not flat and if $\Lambda \neq 0$, then eq. (15) becomes

$$H^2 = \frac{8\pi G\rho}{3} \pm \frac{1}{R_{\rm curv}^2} + \frac{\Lambda}{3} \tag{18}$$

where $R_{\rm curv}$ is the spatial curvature radius which grows as the cosmic scale factor.

Expansion Time Scales

In this appendix we will show how the various parameters change with time. In particular we want to show how the density $\rho(t)$, Hubble's constant $H(t)$, and the cosmic scale factor $R(t)$ change as the universe evolves. We know that very close to $t = 0$, $\Omega = 1$, which means that the kinetic and potential energies were equal. So from eq. (15) we have

$$\frac{1}{2}H^2 = \frac{4}{3}\pi\rho_c G \tag{19}$$

The characteristic expansion time is just the reciprocal of the Hubble constant:

$$t_{\rm exp}(t) \equiv \frac{1}{H(t)} = \sqrt{\frac{3}{8\pi\rho(t)G}} \tag{20}$$

Notice that $\rho(t) \sim t_{\rm exp}^{-2}$ and that $H(t) \sim \sqrt{\rho(t)}$.

Now, how does $\rho(t)$ vary with the scale factor $R(t)$? It's different for the matter dominated and radiation dominated eras. In the matter dominated era, the density $\rho(t)$ is proportional to 1/volume:

$$\rho(t) \propto \frac{1}{R(t)^3} \tag{21}$$

On the other hand, in the radiation dominated era,

$$\rho(t) \sim \frac{h\nu}{R(t)^3} \tag{22}$$

where $h\nu$ is the energy of radiation of frequency ν. But recall that the wavelength of the radiation goes with the cosmic scale factor, i.e., $\nu \sim 1/\lambda \sim 1/R(t)$. So

$$\rho(t) \propto \frac{1}{R(t)^4} \tag{23}$$

The other way to see that the energy density goes as $1/R(t)^4$ is to recall that the total energy density for black body radiation goes as T^4 (see eq. (19) in lecture 4). Since the temperature T goes as $1/R(t)$, $\rho(t) \propto 1/R(t)^4$. To summarize

$$\rho(t) \propto \frac{1}{R(t)^n} \tag{24}$$

where

$$\begin{aligned} n &= 3 \quad \text{matter dominated era} \\ n &= 4 \quad \text{radiation dominated era} \end{aligned}$$

Since $H(t) \sim \sqrt{\rho(t)}$,

$$H(t) \propto \left[\frac{1}{R(t)}\right]^{n/2} \tag{25}$$

Hubble's law tells us that the velocity of a typical galaxy is

$$v(t) = H(t)R(t) \propto [R(t)]^{1-n/2} \tag{26}$$

Since $v = dR/dt$, we have a differential equation

$$\frac{dR}{dt} = [R(t)]^{1-n/2} \tag{27}$$

We can integrate this

$$\int_{t_1}^{t_2} dt = \int_{R_1}^{R_2} dR\,[R(t)]^{n/2-1} \tag{28}$$

We find

$$\begin{aligned} t_2 - t_1 &= \frac{2}{n}\left[R_2^{n/2} - R_1^{n/2}\right] \\ &= \frac{2}{n}\left[\frac{1}{H(t_2)} - \frac{1}{H(t_1)}\right] \\ &= \frac{2}{n}\sqrt{\frac{3}{8\pi G}}\left[\frac{1}{\sqrt{\rho(t_2)}} - \frac{1}{\sqrt{\rho(t_1)}}\right] \end{aligned}$$

So, whatever the value of n, the time elapsed is proportional to the change in $\rho^{-1/2}$. So the time t required for the density to drop from a very high value to a small value of ρ is

$$t = \frac{2}{n}\sqrt{\frac{3}{8\pi G\rho}} \tag{29}$$

Now recall that the density $\rho \propto 1/R^n$. So we get

$$t \propto \frac{1}{\sqrt{\rho}} \propto R^{n/2} \tag{30}$$

or

$$R(t) \propto t^{2/n} \tag{31}$$

So

$$R(t) \propto \begin{cases} t^{1/2} & \text{radiation dominated era} \\ t^{2/3} & \text{matter dominated era} \end{cases}$$

Since the temperature $T \sim 1/R(t)$ (dimensional analysis: $k_B T \sim h\nu \sim hc/\lambda \sim hc/R(t)$), we have

$$T \sim \frac{1}{R(t)} \sim \begin{cases} t^{-1/2} & \text{radiation dominated era} \\ t^{-2/3} & \text{matter dominated era} \end{cases}$$