LECTURE 13

Elementary Particle Physics

(The chart on the classroom wall can be found at http://www-pdg.lbl.gov/cpep/cpep_sm_large.html.)

Four Fundamental Forces

As far as we know, there are just four fundamental forces in nature: strong, weak, electromagnetic, and gravitational. When we think of these forces in terms of quantum mechanics, we say that each of these forces is mediated by the exchange of a particle. These mediating particles are bosons (integer spin). You are familiar with the electromagnetic force. The mediator is the photon and it has spin 1. The photon is massless and has no electric charge. The gravitational force is mediated by the graviton which has spin 2, is massless, and has no electric charge. The fact that the graviton and the photon are massless means that the electromagnetic and gravitational forces are long range. The strong force holds protons and neutrons together in the nucleus. Until about 1972, the strong force between nucleons was thought to be mediated by pions. This is still a good picture at low energies. As we shall see, the proton and the neutron are each made of 3 quarks; the strong force holds these quarks together. The strong force between quarks is mediated by the gluon which is massless and has spin 1 (like the photon). Even though the gluon is massless, the strong force is short range because the gluons attract each other and would rather clump up and not spread over all space. The weak force is responsible for particle decays such as the decay of the neutron:

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{1}$$

The weak force is mediated by spin 1 intermediate vector bosons: W^+, W^-, and Z^0. The superscripts denote their electric charges. The W^+ and W^- each have a mass of 80,280 MeV/c^2 while the Z^0 has a mass of 91,188 MeV/c^2. The relative strengths of the 4 forces are

strong	10
electromagnetic	10^{-2}
weak	10^{-13}
gravitational	10^{-42}

Leptons

The leptons are fermions that are insensitive to the strong force and have spin 1/2. The most familiar lepton is the electron. There are 6 known leptons:

Lepton	Symbol	Mass (MeV/c^2))	Charge
electron	e	0.511	-1
electron neutrino	ν_e	≈ 0	0
muon	μ	105	-1

muon neutrino	ν_μ	≈ 0	0
tau	τ	1771	-1
tau neutrino	ν_τ	≈ 0	0

Particles have corresponding antiparticles which have the opposite spin and charge. Each of the leptons we listed has an antilepton. For example there are antineutrinos. Antielectrons are called positrons. Antiparticles are denoted by a bar over their symbol. For example $\bar{\nu}_e$ is an electron antineutrino.

Helicity: If neutrinos are massless, then they move at the speed of light. Suppose we choose the direction of motion as the z axis, and look at S_z, the component of spin along the z axis. If $S_z = +1/2$ so that the z component of spin points along the direction of motion, then the helicity is $+1$ and the neutrino is right–handed. If $S_z = -1/2$ so that the z component of spin points opposite to the direction of motion, then the helicity is -1 and the neutrino is left–handed. All neutrinos are left–handed and all antineutrinos are right–handed (if neutrinos are massless).

Hadrons and Quarks

The lifetime of states which decay via the strong force is on the order of 10^{-23} seconds. Particles are defined as those entities which live much longer than this, say 10^{-20} seconds or longer. All particles which are sensitive to the strong force are called *hadrons*. Hadrons are divided into 2 classes: *baryons* and *mesons*. The baryons are fermions and have half integer spin. The mesons are bosons and have integer spin. Hadrons are made of quarks. A quark is a fermion with spin 1/2. A baryon consists of 3 quarks, while a meson consists of a quark and an antiquark. (All the hadrons and quarks have antimatter counterparts.) There are 6 types or "flavors" of quarks: up, down, charm, strange, top and bottom. The quarks have fractional charges.

Quark	Symbol	Mass (MeV/c*c)	Charge
up	u	5	2/3
down	d	10	-1/3
charm	c	1600	2/3
strange	s	180	-1/3
top	t	180,000	2/3
bottom	b	4500	-1/3

For example, a proton consists of *uud*. If we add the charges of the constituent quarks, we get Q = +1. If we add the spins of the quarks, S=1/2 is allowed. A neutron consists of *udd*. The sum of the charges is Q = 0 and the spin is 1/2. A π^+ meson consists of $u\bar{d}$; it has charge +1 and spin 0. A π^- meson consists of $d\bar{u}$; it has charge -1 and spin 0. A π^- meson is the antiparticle of a π^+ meson. A π^0 meson consists of $(u\bar{u} - d\bar{d})/\sqrt{2}$; it has charge and spin 0, and is its own antiparticle.

Notice that there are 6 quarks and 6 leptons which is a nice symmetry.

Quantum Numbers

In the early days of particle physics people found a whole plethora of particles (things living much longer than 10^{-23} seconds). They tried to make sense of these particles by arranging them in sensible ways. They also tried to understand why they saw the reactions they did and why they didn't see other reactions. So they came up with selection rules that said that certain quantities were conserved in a reaction or decay. Here is a list of the conserved quantities:

- *Energy and Momentum.*
- *Total Angular Momentum* = sum of spin and orbital angular momentum.
- *Electric Charge* Q.
- *Baryon Number* B: Baryons have baryon number $B = +1$ and their antiparticles have $B = -1$.
- *Lepton Number* L: Leptons have lepton number $L = +1$ and their antiparticles have $L = -1$. So $L = +1$ for the electron and for ν_e, and $L = -1$ for the positron and for the electron antineutrino. Until recently it was believed that electron lepton number L_e was conserved, as was muon lepton number L_μ and tau lepton number L_τ. However, recent evidence for neutrino oscillations indicates that one type of neutrino can become another type of neutrino. It turns out that this can only happen if neutrinos acquire mass. We will learn more about this later.

The 4 quantum numbers Q, B, and L are "good" in the sense of being exactly conserved in every known reaction. (By the way, there is no conservation of mesons. For example a π^o can decay into 2 photons.) So if we have a reaction that has the following form

$$A + B \rightarrow C + D \tag{2}$$

then the total Q, B, and L before the reaction will equal the total Q, B, and L after the reaction. We will now list quantities that are conserved in strong interactions but not conserved in electromagnetic and/or weak interactions.

- *Strangeness* S *and Hypercharge* Y: The strangeness quantum number was invented to account for the unusually long decay lifetime of certain baryons (e.g., Λ^o, the Σ's, and the Ξ's,) and certain mesons (kaons). It turns out that these particles all have at least one strange quark. If a particle has one strange quark, it has $S = -1$. If it has 2 strange quarks, then its $S = -2$, etc. If a particle has a strange antiquark, then it has $S = +1$, etc. Hypercharge Y is defined as the sum of the baryon number and the strangeness:

$$Y = B + S \tag{3}$$

This is just a special case of the fact that strong and electromagnetic interactions conserve quark flavor, but the weak interaction does not. For example, the strong interaction is involved in the reaction

$$\pi^-(d\bar{u}) + p^+(uud) \rightarrow K^+(u\bar{s}) + \Sigma^-(dds) \tag{4}$$

Notice that the total strangeness (S=0) is the same on both sides. In decays, however, the nonconservation of strangeness is very conspicuous, because for many particles this is the only way they can decay. The Λ, for instance, is the lightest strange baryon; if it is to decay into lighter particles, strangeness cannot be conserved. Thus the decay occurs via the weak interaction. Its decay paths are $\Lambda(uds) \rightarrow p^+(uud) + \pi^-(d\bar{u})$ 64% of the time, and $\Lambda(uds) \rightarrow n(udd) + \pi^o((u\bar{u} - d\bar{d})/\sqrt{2})$ 36% of the time. Similarly charm*ness*, bottom*ness*, and top*ness* are conserved in strong and electromagnetic interaction, but not in weak interactions. Hypercharge was defined before charm, bottom, and top were discovered. It should be upgraded, but it's easier just to think in terms of the constituent quarks.

- *Isotopic spin I and I_3*: In nuclear physics isotopic spin I and its third component I_3 were introduced so that the neutron and the proton, with nearly equal masses, could be treated as charge states of the same particle, the nucleon. Thus the neutron and the proton were part of an isospin doublet with $I = 1/2$. The proton has $I_3 = 1/2$ and the neutron has $I_3 = -1/2$. In particle physics also the observed masses of the hadrons form clusters at particular values; there are many families (n, p; Ξ^-, Ξ^o; Σ^-, Σ^o, Σ^+) within which the masses are nearly the same. Apparently, the strong interactions are nearly independent of charge. It is convenient to regard these families as isospin multiplets. We use the algebra of ordinary spin. So a multiplet with isotopic spin I has a multiplicity $2I + 1$. The third component I_3 can be used to determine the charge of the particle using the formula

$$Q = I_3 + \frac{Y}{2} \tag{5}$$

  This is true as long as the particle does not have charm, bottom, or top quarks.

  The up quark has $I_3 = 1/2$, the down quark has $I_3 = -1/2$. They comprise an $I = 1/2$ doublet. The other quark flavors have $I = 0$. Electromagnetic interactions do not conserve I, e.g., the proton and the neutron both have $I = 1/2$ but they behave differently electromagnetically (one has charge and the other doesn't).

- *C, P, and T*: Charge conjugation C changes a particle to its antiparticle. In particular, it reverses the sign of the electric charge and magnetic moment of the particle. Any particle that is its own antiparticle has a $C-$parity associated with its wavefunction. The $C-$parity of a state is odd if the wavefunction changes sign under charge conjugation, and is even otherwise. The $C-$parity of a photon is -1 because the electric field produced by a charge (say an electron) changes sign if the particle is replaced by its antiparticle (a positron). The $C-$parity of the spinless π^o is even because the π^o decays into 2 photons, the $C-$parity of 2 photons is $(-1)^2 = +1$, and $C-$parity is conserved in electromagnetic interactions.

As we have seen, the parity operation P reverses the direction of any spatial vector $\vec{r}$, that is, $P\vec{r} = -\vec{r}$. Invariance of an interaction under P implies that the interaction is symmetric under mirror reflection. The intrinsic parity of a particle is odd or even depending on whether the wavefunction of the particle is odd or even under P. The parity of a fermion is opposite to that of its antiparticle while the partiy of a boson is the same as its antiparticle. Quarks have positive intrinsic parity, so antiquarks have negative parity. The parity of a composite system in its ground state is the product of the parities of its constituents. For excited states, there is an extra factor of $(-1)^\ell$ where ℓ is the orbital angular momentum. Thus, in general, the mesons carry a parity of $(-1)^{\ell+1}$. So in the ground state, a pion has $P = -1$, 2 pions have $P = +1$ and 3 pions have $P = -1$. The intrinsic parity of a photon is -1.

T stands for time reversal.

Experimentally, both strong and electromagnetic interactions are found to be invariant under the operations C, P, and T carried out separately. The weak interaction is not invariant under P. (T. D. Lee and C. N. Yang received the 1957 Nobel Prize for predicting this.) Most reactions are found to be invariant to the combined operations of CP, except for the very special but important case of kaon decay and, more recently, B^o meson decay. (Val Fitch and James Cronin won the 1980 Nobel Prize for discovering CP violation, observed in K^o decay.) The CPT theorem states that all interactions are invariant under the combined operation CPT. Its proof is based on fundamental assumptions used in field theory. As a consequence of the CPT theorem, CP violation implies T violation.

CP Violation

Let us discuss CP violation in more detail. We will discuss the kaon system but similar considerations apply to the B^o system. CP violation was discovered by studying the neutral kaon system. Kaons are typically produced by the strong interactions, in eigenstates of strangeness. K^o has strangeness +1 and its antiparticle $\overline{K}^o$ has $S = -1$. The weak interaction allows these particles to interconvert:

$$K^o \rightleftharpoons \overline{K}^o \tag{6}$$

As a result the particles we normally observe in the laboratory are not K^o and $\overline{K}^o$, but rather some linear combination of the two. In particular, we can form eigenstates of CP as follows. The K^o's are pseudoscalars which means they are odd under parity:

$$P|K^o\rangle = -|K^o\rangle, \qquad P|\overline{K}^o\rangle = -|\overline{K}^o\rangle \tag{7}$$

Under charge conjugation,

$$C\,|K^o\rangle = |\,\overline{K}^o\rangle, \qquad C|\overline{K}^o\rangle = |K^o\rangle \tag{8}$$

So if we apply CP, we obtain

$$CP|K^o\rangle = -|\overline{K}^o\rangle, \qquad CP|\overline{K}^o\rangle = -|K^o\rangle \tag{9}$$

So we can construct normalized eigenstates of CP:

$$\begin{aligned} |K_1\rangle &= \frac{1}{\sqrt{2}}\left(|K^o\rangle - |\overline{K}^o\rangle\right) \\ |K_2\rangle &= \frac{1}{\sqrt{2}}\left(|K^o\rangle + |\overline{K}^o\rangle\right) \end{aligned} \tag{10}$$

Thus

$$CP|K_1\rangle = |K_1\rangle \qquad \text{and} \qquad CP|K_2\rangle = -|K_2\rangle \tag{11}$$

Neutral kaons decay via the weak interaction. Assuming CP is conserved in the weak interactions, K_1 can only decay into a state with CP=+1, whereas K_2 must decay into a state with CP=–1. Typically neutral kaons decay into two or three pions. But CP conservation requires that K_1 decay into two pions (never 3); K_2 decays into 3 pions (never 2):

$$K_1 \longrightarrow 2\pi, \qquad K_2 \longrightarrow 3\pi \tag{12}$$

This is because both 2 and 3 pion states have $C = +1$ but 2π has $P = +1$ and 3π has $P = -1$. The 2π decay is much faster because greater energy is released (corresponding to greater phase space for the final states). In fact K_1 has a lifetime of 0.89×10^{-10} sec while K_2 has a lifetime of 5×10^{-8} sec. So if we start with a beam of K^o's

$$|K^o\rangle = \frac{1}{\sqrt{2}}\left(|K_1\rangle + |K_2\rangle\right) \tag{13}$$

the K_1 component will quickly decay away, and down the line we shall have a beam of pure K_2's. Near the source where the kaons are made, we expect to see a lot of 2π events, but farther along we expect only 3π decays. If at this point we observe a 2π decay, we shall know that CP has been violated. Such an experiment was reported by James Cronin and Val Fitch in 1964. At the end of a beam 57 feet long, they observed 45 two–pion events in a total of 22,700 decays. That's a tiny fraction (1 part in 500), but it provided unmistakable evidence of CP violation. Evidently the long–lived neutral kaon K_L is not a perfect eigenstate of CP after all, but contains a small admixture of K_1:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}\left(|K_2\rangle + \epsilon|K_1\rangle\right) \tag{14}$$

The coefficient ϵ is a measure of nature's departure from perfect CP invariance; experimentally its magnitude is about 2.3×10^{-3}. Further measurements have confirmed the violation of CP. In particular although 34% of all K_L's decay by the 3π mode, some 39% go to

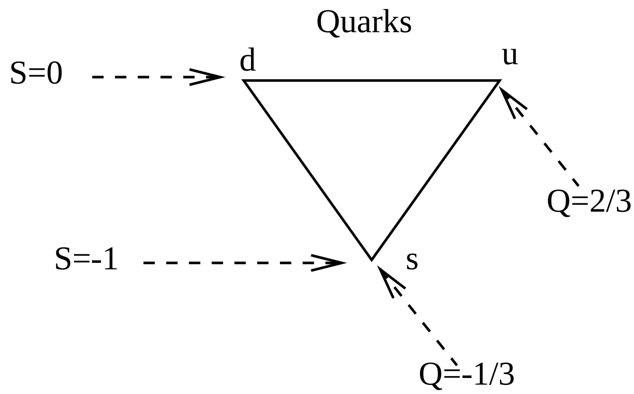
$$\begin{array}{ll} \text{(a)} & \pi^+ + e^- + \overline{\nu}_e \\ \text{(b)} & \pi^- + e^+ + \nu_e \end{array} \tag{15}$$

Both reactions have $CP = -1$. Notice that CP takes (a) into (b), so if CP were conserved, and K_L were a pure eigenstate, (a) and (b) would be equally probable. But experiments show that K_L decays more often into a positron than into an electron, by a fractional amount 3.3×10^{-3}. Here for the first time is a process that makes an absolute distinction between matter and antimatter, and provides an unambiguous, convention–free definition of positive charge: it is the charge carried by the lepton preferentially produced in the decay of the long–lived neutral K meson. CP violation may be responsible for the matter–antimatter asymmetry observed in the universe.

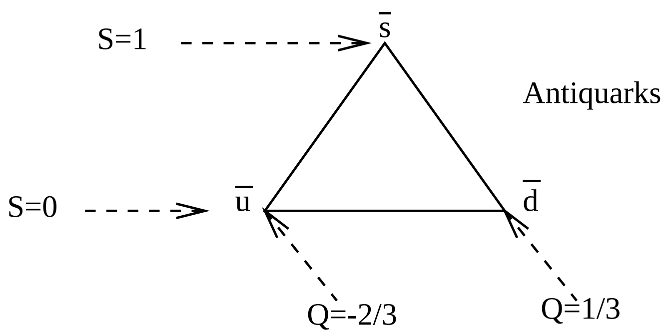
As we mentioned above, there is no evidence of CPT violation which is based on very general assumptions like Lorentz invariance, quantum mechanics and the idea that interactions are carried by fields. If CPT is conserved and CP is violated, then time reversal invariance (T) must be violated.

Hadron Spectroscopy

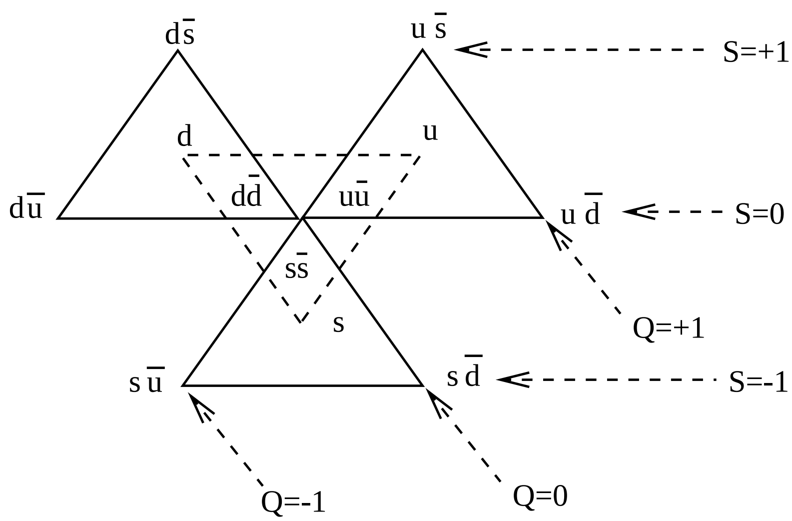
Back in the days when people just knew about 3 quark flavors (u, d, and s), Gell–Mann pointed out that there was a systematic way to put the 3 quarks together to get the known isospin multiplets. This has been dubbed "The Eightfold Way." Recall that when we discussed adding angular momenta, we said that adding 2 spin–1/2 objects gives a singlet ($S = 0$) and a triplet ($S = 1$). So when we combine a quark and an antiquark to make a meson, we can have a singlet (pseudoscalar meson) or a triplet (vector meson). (A pseudoscalar changes sign under a parity operation.) In group theory notation we can write $2 \times 2 = 1 + 3$ where you can regard the numbers as denoting the number of components. (Actually, it's the dimension of the matrices in the representation of the group. Spin–1/2 objects are represented by the 2×2 Pauli matrices which is a representation of the group called "SU(2)." $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$ and the identity matrix are the elements of the group SU(2). In general, spin is described by SU(2); this is true for any value of the spin, not just spin–1/2.) If one regards the 3 quark flavors as components of a multiplet (they are described by the group SU(3)), it turns out that group theory tells us that a quark and an antiquark can combine into an octet and a singlet: $3 \times \bar{3} = 8 + 1$. That means that there is an octet and a singlet of pseudoscalar mesons, and an octet and a singlet of vector mesons. We can represent this pictorally by putting the 3 quark flavors at the corners of a triangle:



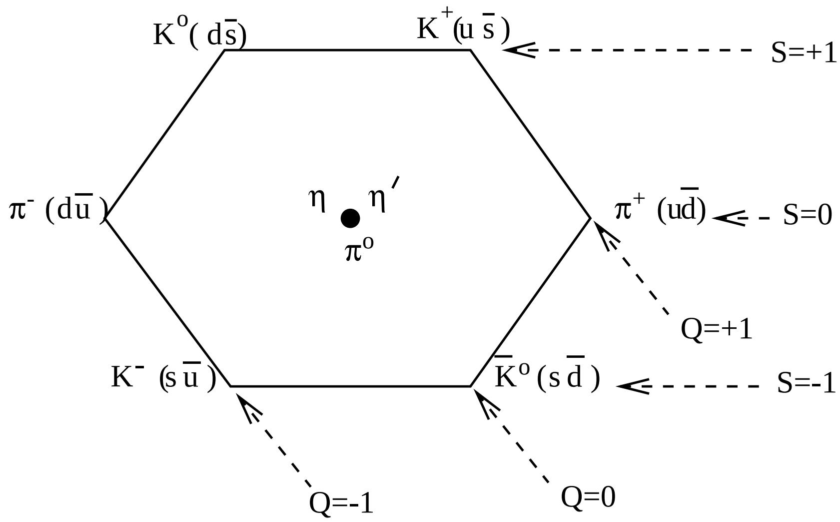
where S is the strangeness quantum number. Similarly the antiquarks are at the corners of an upside down triangle.



Now we can superpose the triangles to form an octet plus a singlet:



These give the spinless mesons:



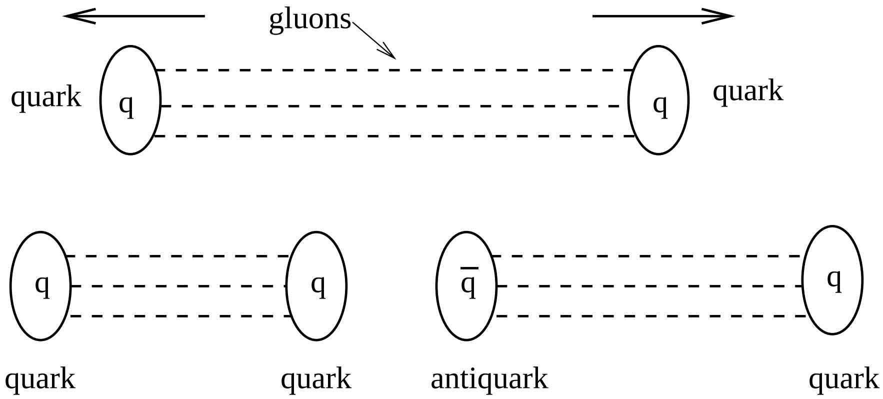
By superposing 3 quark triangles, one can get the baryon decuplet and baryon octet. (see pages 401–403 in *The New Physics*). In group theory language we would write $3 \times 3 \times 3 = 10 + 8 + 8 + 1$. The baryon decuplet has 10 baryons, each with spin 3/2. The baryon octet has 8 baryons, each with spin 1/2. This makes sense since we can get total spin 1/2 or 3/2 if we put together 3 quarks which each have spin 1/2. It turns out that there is only one octet of baryons, not two octets, because there is only one way to write a completely antisymmetric wavefunction for the octet.

$$\psi = \psi(color)\psi(flavor)\psi(spin)\psi(space) \tag{16}$$

Color

In the decuplet there are baryons such as Δ^{++} which is made of (uuu), Δ^- which consists of (ddd), and Ω^- which consists of (sss). This appears to violate the Pauli exclusion principle because we appear to be putting 3 identical quarks in the same state. To get out of this problem, the quarks were given an additional quantum number called "color". Each quark flavor comes in 3 colors (red, blue, and green, say). Antiquarks have anticolors. To make a baryon, we simply take one quark of each color, then the three u's in Δ^{++} are no longer identical (one is red, one is green, and one is blue). Since the exclusion principle only applies to identical particles, the problem evaporates. The allowed quark combinations must follow the rule that *all naturally occurring particles are colorless.* By "colorless" I mean that either the total amount of each color is zero or all three colors are present in equal amounts. (The latter case mimics the fact that white light is made of many colors.) A meson is colorless because it has a quark of one color (say, blue) and an antiquark with the anticolor (antiblue). This rule means that we won't find a particle made of 2 quarks or 4 quarks. It also tells us that we won't find individual quarks in nature. The only colorless combinations you can make are $q\bar{q}$ (mesons), qqq (baryons), and $\bar{q}\bar{q}\bar{q}$ (antibaryons). You could have 6 quarks, but we would interpret that as a bound state of 2 baryons.

Actually you can't separate quarks because the strong interaction between 2 quarks increases as the distance grows. So it takes too much energy to separate quarks. With all that energy, quark-antiquark pairs are created.

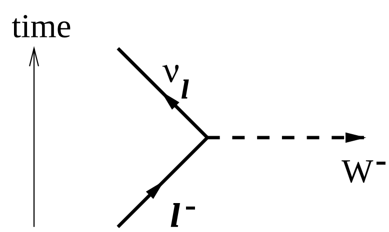


The gluons each carry a color and an anticolor. Since $3 \times \bar{3} = 8 + 1$, there is an octet of gluons: $(r\bar{b} + b\bar{r})/\sqrt{2}$, $(r\bar{g} + g\bar{r})\sqrt{2}$, $(r\bar{r} - b\bar{b})/\sqrt{2}$, etc. The color singlet ("1") is given by $(r\bar{r} + b\bar{b} + g\bar{g})/\sqrt{3}$. The color singlet is not a gluon but is represented by the colorless mesons which mediate the strong interaction at low energies. For example pions mediate the strong interaction between nucleons.

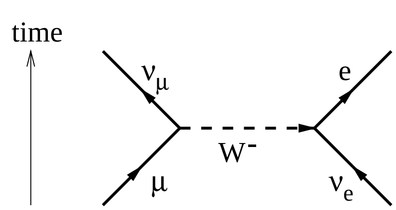
Weak Interactions

Both quarks and leptons are affected by weak interactions. There are two kinds of weak interactions: charged (mediated by $W^{\pm}$) and neutral (mediated by the Z^0). Let's look at leptons first.

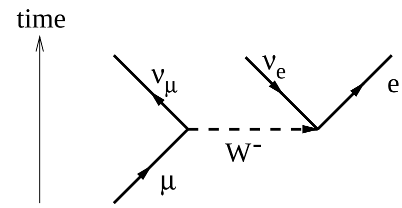
Leptons: The fundamental charged vertex looks like this:



A negative lepton (e^-, μ^-, or τ^-) is converted into the corresponding neutrino with the emission of a W^- (or absorption of a W^+): $\ell^- \to \nu_\ell + W^-$. This implies that $\ell^+ \to \bar{\nu}_\ell + W^+$ is also allowed. We can combine the primitive vertices together to make more complicated reactions. For example, $\mu^- + \nu_e \to e^- + \nu_\mu$ is represented by the diagram:

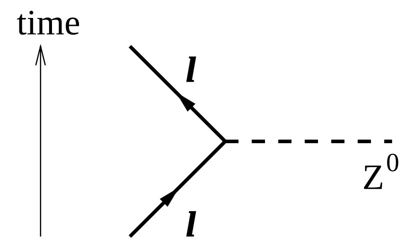


Such a neutrino–muon scattering event would be hard to set up in the laboratory, but with a slight twist essentially the same diagram describes the decay of the muon, $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$, which happens all the time:

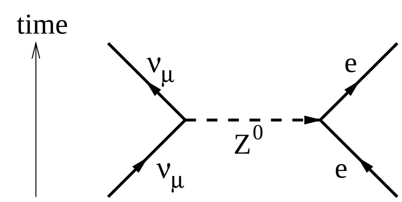


Note that a particle traveling backwards in time is an antiparticle. So in the diagram the neutrino ν_e traveling backwards in time is the antineutrino $\bar{\nu}_e$.

The fundamental neutral vertex is:



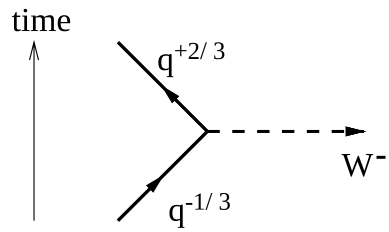
In this case ℓ can be any lepton, including neutrinos. The Z^0 mediates neutral weak processes such as neutrino–electron scattering ($\nu_\mu + e^- \to \nu_\mu + e^-$):



Neutrino scattering is extremely difficult to observe experimentally. Presumably scattering between 2 electrons can occur with the exchange of a Z^0, but this is masked by the much stronger electromagnetic interaction involving the exchange of a photon. Experiments at DESY (in Hamburg) studied the reaction $e^- + e^+ \to \mu^- + \mu^+$ at very high energy and found unmistakable evidence of a contribution from the Z^0. So neutral weak processes do occur.

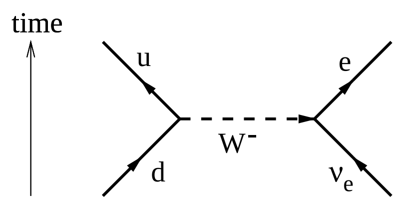
Note that the leptonic weak vertices connect members of the same generation: e^- converts to a ν_e with emission of a W^-, or $\mu^- \to \mu^-$ with the emission of a Z^0, but e^- never goes to μ^- nor does μ^- go to ν_e. In this way electron number, muon number, and tau number conservation are enforced at the leptonic weak vertices.

Quarks: For quarks the fundamental charged vertex is:

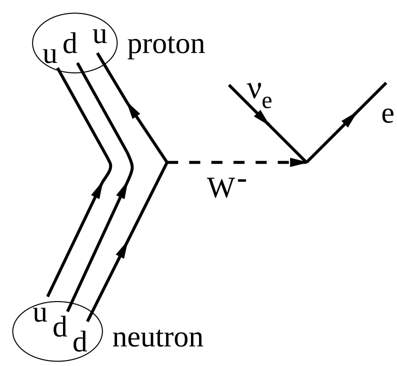


A quark with charge $-1/3$ (i.e., d, s, or b) converts into the corresponding quark with charge $+2/3$ (u, c, or t, respectively), with the emission of a W^-. The outgoing quark carries the same color as the ingoing one. But the flavor changes because flavor is not conserved in weak interactions.

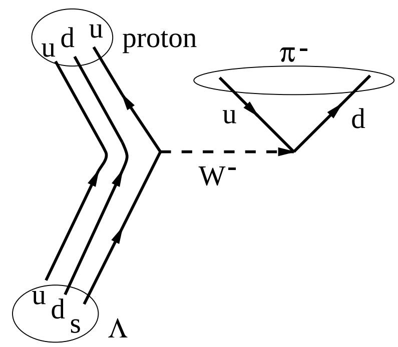
The far end of the W line can couple to leptons (a "semileptonic" process), or to other quarks (a purely hadronic process). The most important semileptonic process is $d + \nu_e \to u + e$:



Because of quark confinement, this process would never occur in nature as it stands. However, if we combine the quark with a u and a d quark, we obtain the beta decay of the neutron ($n \to p^+ + e^- + \bar{\nu}_e$):



The weak interaction is not confined to a quark generation. This allows strangeness changing weak interactions, such as the decay of the lambda ($\Lambda \to p^+ + \pi^-$), which involves the conversion of a strange quark into an up–quark.



The quark generations are "skewed" for the purposes of the weak interactions. Instead of

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

the weak force couples pairs

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix} \quad \begin{pmatrix} t \\ b' \end{pmatrix}$$

where d', s', and b' are linear combinations of the physical quarks d, s, and b:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

This 3×3 matrix is called the Kobayashi–Maskawa matrix. It is close to being the unit matrix; the diagonal elements are close to unity and the off–diagonal elements are small. This matrix allows mixing between quark generations. V_{ud} measures the coupling of u to d, V_{us} the coupling of u to s, and so on.

Neutrino Oscillations

Until recently it was believed that neutrinos are massless. However there is now evidence that neutrinos may indeed carry a small amount of mass. Among these experiments are those being carried out by Hank Sobel (UCI) and his collaborators at Super Kamiokande in Japan. How do you measure the mass of a neutrino? You can't stop it and weigh it. It turns out that if neutrinos have mass, they can transform into another neutrino flavor after a while. So as the neutrino goes along in space, it changes its flavor, e.g., from ν_μ to ν_τ. These flavor changes are called neutrino oscillations and it is these oscillations that experiments are trying to observe. To see why having mass corresponds to neutrino oscillations, let us assume that neutrinos have finite mass ($m_\nu \neq 0$). Further let us suppose that the neutrino eigenstates of the weak interaction Hamiltonian H_W are not the same as the neutrino mass eigenstates. Then

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}$$

are the eigenstates of H_W and

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

are the mass eigenstates. In other words their energy is given by

$$E = \sqrt{p^2c^2 + m^2c^4} \approx pc + \frac{m^2c^3}{2p} \tag{17}$$

As a result, Schroedinger's equation

$$-i\hbar\frac{d\psi}{dt} = H\psi \tag{18}$$

implies that these mass eigenstates will have a time dependence given by $\exp(-iEt/\hbar)$. These two representations are related by a unitary transformation:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

As a simple example, consider mixing two neutrinos.

$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}$$

(We choose ν_μ and ν_τ because that's what Super Kamiokande is looking at.) We can take $H' = UHU^\dagger$ and plug it into Schroedinger's equation $-i\hbar\frac{d\psi}{dt} = H'\psi$ to see how ν_μ and ν_τ will evolve in time. We find

$$|\nu_\mu(t)\rangle = |\nu_2(t=0)\rangle \cos\theta e^{-iE_2t/\hbar} + |\nu_3(t=0)\rangle \sin\theta e^{-iE_3t/\hbar} \tag{19}$$

$$|\nu_\tau(t)\rangle = -|\nu_2(t=0)\rangle \sin\theta e^{-iE_2t/\hbar} + |\nu_3(t=0)\rangle \cos\theta e^{-iE_3t/\hbar} \tag{20}$$

The probability $P(\nu_\mu)$ of detecting ν_μ at time t is given by

$$P(\nu_\mu) = |\langle\nu_\mu(t=0)|\nu_\mu(t)\rangle|^2 \tag{21}$$

where at $|\nu_\mu(t=0)\rangle$ is pure ν_μ. Plugging in eq. (19), we find that

$$P(\nu_\mu) = 1 - \frac{1}{2}\sin^2 2\theta \left[1 - \cos\frac{(E_2 - E_3)\,t}{\hbar}\right] \tag{22}$$

Using $E_i \approx pc + \frac{m_i^2c^3}{2p}$, we find that

$$E_2 - E_3 = \frac{(m_2^2 - m_3^2)\,c^3}{2p} \tag{23}$$

Here we are assuming that the neutrino momentum $p_2 \approx p_3 \approx p \gg m_\nu c$. Then

$$P(\nu_\mu) = 1 - \frac{1}{2}\sin^2 2\theta \left[1 - \cos\left(\frac{(m_2^2 - m_3^2)\, c^3}{2p\hbar}\right) t\right] \tag{24}$$

So the probability of detecting ν_μ oscillates in time if neutrinos have mass. Super K has found that $5 \times 10^{-4} < \Delta m^2 < 6 \times 10^{-3}$ eV^2 at the 90% confidence level.

There are other implications for massive neutrinos. Recall that all massless neutrinos are lefthanded and all massless antineutrinos are righthanded. If neutrinos have mass, it also means that there are some righthanded neutrinos and some lefthanded antineutrinos, though none have been found so far.