Physics 224 **Fall 1998**

Phenomenology **due 11:00 am Thursday Nov. 12**

PROBLEM SET 6

November 6 Colloquium: "Cosmic Gamma-Rays and the Emerging High Energy Universe"

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3 pm, 101 Rowland Hall (formerly PS I)

1. **Meissner Effect** In deriving flux quantization in a superconductor, we found that the electric current is given by

$$\vec{j} = q\psi^*\vec{v}\psi = \frac{qn_p}{m}\left(\hbar\nabla\theta - \frac{q}{c}\vec{A}\right) \tag{1}$$

   (a) Use this and the appropriate Maxwell equation to show that

$$\nabla^2\vec{B} = \lambda^{-2}\vec{B} \tag{2}$$

   What is λ in terms of the density of Cooper pairs n_p, e, the mass of the electron m, and c? λ is called the London penetration depth. (Hint: Use some vector identities to simplify the equations. See inside cover of Jackson's *Classical Electrodyamics*, for example.)

   (b) Suppose that $\vec{B}$ points along the z axis and only varies in the x direction. Suppose the superconductor fills the half space $x > 0$ and there is vacuum for $x < 0$. Show that B_x dies out exponentially as it penetrates the superconductor in the x direction. (Don't worry about the prefactor of the exponential.) In other words the magnetic field dies out exponentially as you go into the superconductor. This is the Meissner effect.

2. **AC Josephson Effect** When a static DC voltage V is applied across a Josephson junction, an AC current results. To see how this comes about, notice that an electron pair experiences a potential energy difference qV on passing across the junction, where

$q = -2e$. We can say that a pair on one side is at potential $-eV$ and a pair on the other side is at $+eV$. Thus the equations of motion become

$$i\hbar \frac{\partial \psi_1}{\partial t} = \hbar T \psi_2 - eV\psi_1 \qquad i\hbar \frac{\partial \psi_2}{\partial t} = \hbar T \psi_1 + eV\psi_2 \tag{3}$$

where ψ_1 is the superconducting order parameter on side 1:

$$\psi_1 = \sqrt{n_1} e^{i\theta_1} \tag{4}$$

n_1 is the density of superconducting pairs on side 1. Similarly

$$\psi_2 = \sqrt{n_2} e^{i\theta_2} \tag{5}$$

Assume that the superconductors are identical. Find the current density J as a function of time and of the phase difference $\delta(0)$. $\delta(0) = \theta_2 - \theta_1$ is the phase difference at $V = 0$. What is the angular frequency ω at which the current oscillates when a voltage V is applied?

3. Eisberg and Resnick Problem 13.18.

4. Eisberg and Resnick Problem 13.24.