Physics 224 **Fall 2001**

Discoveries and Inventions of Modern Physics **due 11:00 am Tuesday Nov. 6**

PROBLEM SET 6

Nov. 1 Colloquium: "Hunting for Dark-Matter Axions"

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3:30 pm, 101 Rowland Hall

1. (20 pts) In the Weiss (mean–field) theory of ferromagnetism the Gibbs free energy ($G = E(M) - HM - TS$, where H is the externally applied magnetic field, M is the magnetization, E is the internal energy, and S is the entropy) has the form

$$G = G_o(T) + a(T)M^2 + b(T)M^4 + O(M^6) - MH \tag{1}$$

where $G_o(T)$ is independent of M, and where the coefficient $b(T)$ is a slowly varying function of T but $a(T)$ is of the form $a_o(T - T_C)$, T_C being the critical temperature in Weiss theory. Assume that T is close to T_C and that M is small. Using the fact that in thermal equilibrium M will take the value which minimizes G, find (a) the equilibrium value of M for $H = 0$, for $T > T_C$ and $T < T_C$; (b) the form of M at T_C as a function of H; (c) the zero–field differential susceptibility $\chi = (\partial M/\partial H)_{T,H=0}$ for $T > T_C$ and $T < T_C$; and (d) the discontinuity in the specific heat at constant H ($C_H = -T(\partial^2 G/\partial T^2)_H$) at the point $T = T_C$ and $H = 0$. In other words find the difference between the limits of $C_H(T, H = 0)$ as $T \to T_C$ from above and from below.

2. **Meissner Effect** In deriving flux quantization in a superconductor, we found that the electric current is given by

$$\vec{j} = q\psi^*\vec{v}\psi = \frac{qn_p}{m}\left(\hbar\nabla\theta - \frac{q}{c}\vec{A}\right) \tag{2}$$

   (a) Use this and the appropriate Maxwell equation to show that

$$\nabla^2\vec{B} = \lambda^{-2}\vec{B} \tag{3}$$

What is λ in terms of the density of Cooper pairs n_p, e, the mass of the electron m, and c? λ is called the London penetration depth. (Hint: Use some vector identities to simplify the equations. See inside cover of Jackson's *Classical Electrodyamics*, for example.)

(b) Suppose that $\vec{B}$ points along the z axis and only varies in the x direction. Suppose the superconductor fills the half space $x > 0$ and there is vacuum for $x < 0$. Show that B_x dies out exponentially as it penetrates the superconductor in the x direction. (Don't worry about the prefactor of the exponential.) In other words the magnetic field dies out exponentially as you go into the superconductor. This is the Meissner effect.

3. **AC Josephson Effect** When a static DC voltage V is applied across a Josephson junction, an AC current results. To see how this comes about, notice that an electron pair experiences a potential energy difference qV on passing across the junction, where $q = -2e$. We can say that a pair on one side is at potential $-eV$ and a pair on the other side is at $+eV$. Thus the equations of motion become

$$i\hbar\frac{\partial\psi_1}{\partial t} = \hbar T\psi_2 - eV\psi_1 \qquad i\hbar\frac{\partial\psi_2}{\partial t} = \hbar T\psi_1 + eV\psi_2 \tag{4}$$

where ψ_1 is the superconducting order parameter on side 1:

$$\psi_1 = \sqrt{n_1}e^{i\theta_1} \tag{5}$$

n_1 is the density of superconducting pairs on side 1. Similarly

$$\psi_2 = \sqrt{n_2}e^{i\theta_2} \tag{6}$$

Assume that the superconductors are identical. Find the current density J as a function of time and of the phase difference $\delta(0)$. $\delta(0) = \theta_2 - \theta_1$ is the phase difference at $V = 0$. What is the angular frequency ω at which the current oscillates when a voltage V is applied?