Formula Sheet

$$\ln n! \approx n \ln n - n \qquad \sum_{n=0}^{\infty} r^n = \frac{1}{1-r} \tag{1}$$

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \left(\frac{\pi}{\alpha}\right)^{1/2} \qquad \int_{-\infty}^{\infty} e^{-\alpha x^2}\ x^2\ dx = \frac{\sqrt{\pi}}{2}\alpha^{-3/2} \tag{2}$$

Distributions

Binomial distribution $$P(n,N) = \frac{N!}{n!(N-n)!}\ p^n\ q^{N-n} \tag{3}$$

Gaussian Distribution $$P(x)dx = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} dx \tag{4}$$

Poisson Distribution $$P(n) = \frac{\mu^n}{n!} e^{-\mu} \qquad \mu = Np \tag{5}$$

Thermodynamics

$$dW = \overline{p} dV \qquad \Delta\overline{E} = Q - W + \mu\Delta\overline{N} \tag{6}$$

$$\beta = \frac{1}{k_B T} = \frac{\partial \ln \Omega(E)}{\partial E} \qquad S \equiv \text{entropy} \equiv k_B \ln \Omega \tag{7}$$

$$\frac{1}{T} = \left.\frac{\partial S}{\partial E}\right|_V \tag{8}$$

$$\overline{X}_\alpha \equiv -\overline{\frac{\partial E_r}{\partial x_\alpha}} \qquad \frac{\partial S}{\partial x_\alpha} = \frac{\overline{X}_\alpha}{T} \tag{9}$$

$$dQ = TdS = d\overline{E} + dW - \mu d\overline{N} \qquad dW = \sum_{\alpha=1}^{n} \overline{X}_\alpha dx_\alpha \tag{10}$$

$$\Delta S \geq 0 \quad \text{For an isolated system} \tag{11}$$

Numbers

$$T_C = T_K - 273.15 \qquad \text{degrees Celsius} \tag{12}$$

$$R = (8.3143 \pm 0.0012)\ \text{joules mole}^{-1}\ \text{deg}^{-1} \tag{13}$$

$$k_B = (1.38054 \pm 0.00018) \times\ 10^{-16}\ \text{ergs degree}^{-1} \tag{14}$$

$$1\ \text{calorie} = 4.1840\ \text{joules} \qquad 1\ \text{Joule} = 10^7\ \text{ergs} \tag{15}$$

$$N_a = 6.023 \times 10^2 3\ \frac{\text{molecules}}{\text{mole}} \tag{16}$$

$$1\ \text{atm} = 1.01325 \times 10^6\ \text{dynes/cm}^2 = 1.013 \times 10^5\ \text{N/m}^2 \tag{17}$$

$$C_y = \left.\frac{dQ}{dT}\right|_y = T\left.\frac{\partial S}{\partial T}\right|_y \tag{18}$$

$$\Delta S = S(T_f) - S(T_i) = \int_{T_i}^{T_f} \frac{C_V(T')}{T'}dT' \text{ For fixed volume} \tag{19}$$

$$\bar{\epsilon}_i = \frac{1}{2}k_B T \quad \text{Equipartition Theorem} \tag{20}$$

Ideal Gas

$$pV = Nk_BT = \nu RT \qquad c_p = c_V + R \qquad \overline{E} = \frac{3}{2}Nk_BT \tag{21}$$

$$Z = \frac{Z'}{N!} = \frac{\zeta^N}{N!} \qquad S = k_BN[\ln\frac{V}{N} + \frac{3}{2}\ln T + \sigma_o] \tag{22}$$

$$\sigma_o = \sigma + 1 = \frac{3}{2}\ln\left(\frac{2\pi mk_B}{h^2}\right) + \frac{5}{2} \tag{23}$$

$$pV^\gamma = \text{ constant} \qquad \gamma = \frac{c_p}{c_V} \qquad V^{\gamma-1}T = \text{ constant} \tag{24}$$

Maxwell Relations and Thermodynamic Functions

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V \qquad \left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p \tag{25}$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V \qquad -\left(\frac{\partial S}{\partial p}\right)_T = \left(\frac{\partial V}{\partial T}\right)_p \tag{26}$$

$$\text{Energy} \quad E = E(S,V) \qquad dE = TdS - pdV \tag{27}$$

$$\text{Enthalpy} \quad H = H(S,p) = E + pV \qquad dH = TdS + Vdp \tag{28}$$

$$\text{Helmholtz Free Energy} \quad F = F(T,V) = E - TS \qquad dF = -SdT - pdV \tag{29}$$

$$\text{Gibbs Free Energy} \quad G = G(T,p) = E - TS + pV \quad dG = -SdT + Vdp \tag{30}$$

$$T = \left(\frac{\partial E}{\partial S}\right)_V = \left(\frac{\partial H}{\partial S}\right)_p \qquad p = -\left(\frac{\partial E}{\partial V}\right)_S = -\left(\frac{\partial F}{\partial V}\right)_T \tag{31}$$

$$V = \left(\frac{\partial H}{\partial p}\right)_S = \left(\frac{\partial G}{\partial p}\right)_T \qquad S = -\left(\frac{\partial F}{\partial T}\right)_V = -\left(\frac{\partial G}{\partial T}\right)_p \tag{32}$$

$$\text{Clausius} - \text{Clapeyron Equation} \quad \frac{dp}{dT} = \frac{\Delta s}{\Delta v} \tag{33}$$

Engines and Refrigerators

$$q_1 = w + q_2 \qquad \frac{q_2}{q_1} \leq \frac{T_2}{T_1} \tag{34}$$

$$\text{Efficiency}: \qquad \eta \equiv \frac{w}{q_1} = \frac{q_1 - q_2}{q_1} < 1 \qquad \eta_{max} = \frac{T_1 - T_2}{T_1} \tag{35}$$

Statistical Mechanics

$$\overline{A} = \frac{1}{Z} \sum_R A_R e^{-\beta E_R} \qquad Z = \sum_r e^{-\beta E_r} \tag{36}$$

$$\overline{E} = -\frac{\partial \ln Z}{\partial \beta} \qquad S = k_B(\ln Z + \beta \overline{E}) \qquad \overline{X} = \frac{1}{\beta} \frac{\partial \ln Z}{\partial x} \tag{37}$$

$$\overline{p} = \frac{1}{\beta} \frac{\partial \ln Z}{\partial V} \qquad \overline{M} = \frac{1}{\beta} \frac{\partial \ln Z}{\partial H} \tag{38}$$

$$F = -k_B T \ln Z \qquad C_V = k_B \beta^2 \frac{\partial^2 \ln Z}{\partial \beta^2} = -T \frac{\partial^2 F}{\partial T^2}\bigg|_V \tag{39}$$

$$\mathcal{Z} = \sum_r e^{-\beta(E_r - \mu N_r)} \qquad \mu = -\frac{\partial E'}{\partial N'} \tag{40}$$

$$\overline{N} = \frac{1}{\beta} \frac{\partial \ln \mathcal{Z}}{\partial \mu} \qquad \overline{E} = \mu \overline{N} - \frac{\partial}{\partial \beta} \ln \mathcal{Z} \tag{41}$$

$$\ln Z = -\beta \mu N + \ln \mathcal{Z} \qquad \mu = -\frac{1}{\beta} \frac{\partial \ln Z(N)}{\partial N} = \frac{\partial F}{\partial N} \tag{42}$$

$$f(\vec{v}) d^3r d^3v = n \left(\frac{m}{2\pi k T} \right)^{3/2} e^{-mv^2/2kT} d^3r d^3v \quad \text{Maxwell velocity distribution} \tag{43}$$

$$\overline{n}_s = -\frac{1}{\beta} \frac{\partial \ln Z}{\partial \varepsilon_s} \qquad \overline{N} = \sum_s \overline{n}_s \tag{44}$$

$$\overline{n}_s = N \frac{e^{-\beta \varepsilon_s}}{\sum_r e^{-\beta \varepsilon_r}} \qquad \text{Maxwell Boltzmann Distribution} \tag{45}$$

$$\overline{n}_s = \frac{1}{e^{\beta \varepsilon_s} - 1} \qquad \text{Planck distribution} \tag{46}$$

$$\overline{n}_s = \frac{1}{e^{\beta(\varepsilon_s - \mu)} - 1} \quad \text{bosons} \qquad \overline{n}_s = \frac{1}{e^{\beta(\varepsilon_s - \mu)} + 1} \quad \text{fermions} \tag{47}$$

$$\text{Black Body Radiation}: \quad \overline{u}(\omega, T) d\omega = \frac{\hbar \omega^3}{\pi^2 c^3} \frac{d\omega}{(e^{\beta \hbar \omega} - 1)} \qquad \overline{u}_o(T) = \frac{\pi^2}{15} \frac{(k_B T)^4}{(\hbar c)^3} \tag{48}$$

$$\mathcal{P}_e^{tot} = \frac{1}{4} c \overline{u}_o = \sigma T^4 \qquad \sigma \equiv \frac{\pi^2}{60} \frac{k_B^4}{c^2 \hbar^3} \tag{49}$$

$$\text{Free Electron Gas}: \quad k_F = \left(3\pi^2 \frac{N}{V} \right)^{1/3} \qquad C_V = \frac{\pi^2}{2} R \cdot \frac{T}{T_F} \tag{50}$$

$$\text{Einstein Heat Capacity}: \quad C_V = 3N k_B \left(\frac{\theta_E}{T} \right)^2 \frac{e^{\theta_E/T}}{\left(e^{\theta_E/T} - 1\right)^2} \tag{51}$$

$$\text{Debye Heat Capacity :} \quad C_V = \frac{12\pi^4}{5} N k_B \left(\frac{T}{\Theta_D}\right)^3 \tag{52}$$

$$\text{Bose Einstein Condensation :} \quad n_1 = N\left[1 - \left(\frac{T}{T_C}\right)^{3/2}\right] \quad \mu = -k_B T \ln\left(\frac{1}{n_1} + 1\right) \tag{53}$$

Spin Systems

$$\vec{\mu} = g\mu_o \mathbf{S} \tag{54}$$

$$\mathcal{H}_o = -g\mu_o \sum_{j=1}^{N} \mathbf{S}_j \cdot \mathbf{H}_o \tag{55}$$

$$\text{Heisenberg Hamiltonian :} \quad \mathcal{H} = -2J \sum_{i>j} \mathbf{S}_i \cdot \mathbf{S}_j \tag{56}$$

If we take $J_{ij} = J$, then

$$\mathcal{H} = -2J \sum_{i>j} \mathbf{S}_i \cdot \mathbf{S}_j \tag{57}$$

$$\text{Ising Model :} \quad \mathcal{H} = -2J \sum_{i>j} S_{iz} S_{jz} \tag{58}$$