Formula Sheet

$$\ln n! \approx n \ln n - n$$  \hspace{1cm}  \sum_{n=0}^{\infty} r^n = \frac{1}{1 - r} \hspace{1cm} (1)$$

$$\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \left(\frac{\pi}{\alpha}\right)^{1/2} \hspace{1cm} \int_{-\infty}^{\infty} e^{-\alpha x^2} x^2 \, dx = \frac{\sqrt{\pi}}{2} \alpha^{-3/2} \hspace{1cm} (2)$$

Distributions

Binomial distribution  \hspace{1cm}  \begin{align*}
P(n, N) &= \frac{N!}{n!(N-n)!} \, p^n \, q^{N-n} \hspace{1cm} (3)
\end{align*}

Gaussian Distribution  \hspace{1cm}  P(x) dx = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2} \, dx \hspace{1cm} (4)

Poisson Distribution  \hspace{1cm}  P(n) = \frac{\mu^n}{n!} e^{-\mu} \hspace{1cm} \mu = Np \hspace{1cm} (5)

Thermodynamics

$$dW = pdV \quad \Delta E = Q - W + \mu \Delta N \hspace{1cm} (6)$$

$$\beta = \frac{1}{k_B T} = \frac{\partial \ln \Omega(E)}{\partial E} \hspace{1cm} S \equiv \text{entropy} \equiv k_B \ln \Omega \hspace{1cm} (7)$$

$$\frac{1}{T} = \frac{\partial S}{\partial E} \bigg|_V \hspace{1cm} (8)$$

$$\overline{X}_\alpha \equiv -\frac{\partial E_r}{\partial x_\alpha} \hspace{1cm} \overline{S} \frac{\partial S}{\partial x_\alpha} = \frac{\overline{X}_\alpha}{T} \hspace{1cm} (9)$$

$$dQ = TdS = dE + dW - \mu dN \hspace{1cm} dW = \sum_{\alpha=1}^{n} \overline{X}_\alpha \, dx_\alpha \hspace{1cm} (10)$$

$$\Delta S \geq 0 \hspace{1cm} \text{For an isolated system} \hspace{1cm} (11)$$

Numbers

$$T_C = T_K - 273.15 \hspace{1cm} \text{degrees Celsius} \hspace{1cm} (12)$$

$$R = (8.3143 \pm 0.0012) \text{ joules mole}^{-1} \text{ deg}^{-1} \hspace{1cm} (13)$$

$$k_B = (1.38054 \pm 0.00018) \times 10^{-16} \text{ ergs degree}^{-1} \hspace{1cm} (14)$$

1 calorie = 4.1840 joules  \hspace{1cm} 1 Joule = 10^7 ergs \hspace{1cm} (15)$$

$$N_a = 6.023 \times 10^{23} \text{ molecules mole}^{-1} \hspace{1cm} (16)$$

1 atm = 1.01325 \times 10^6 \text{ dynes/cm}^2 = 1.013 \times 10^5 \text{ N/m}^2 \hspace{1cm} (17)$$
\[ C_y = \left. \frac{dQ}{dT} \right|_y = T \left. \frac{\partial S}{\partial T} \right|_y \]  

\[ \Delta S = S(T_f) - S(T_i) = \int_{T_i}^{T_f} \frac{C_v(T')}{T'} dT' \quad \text{For fixed volume} \]  

\[ \tau_i = \frac{1}{2} k_B T \quad \text{Equipartition Theorem} \]  

**Ideal Gas**

\[ pV = N k_B T = \nu RT \quad c_p = c_v + R \quad E = \frac{3}{2} N k_B T \]  

\[ Z = \frac{Z'}{N!} = \frac{\xi^N}{N!} \quad S = k_B N \left[ \frac{V}{N} + \frac{3}{2} \ln T + \sigma_o \right] \]  

\[ \sigma_o = \sigma + 1 = \frac{3}{2} \ln \left( \frac{2\pi mk_B}{h^2} \right) + \frac{5}{2} \]  

\[ pV^\gamma = \text{constant} \quad \gamma = \frac{c_p}{c_v} \quad V^{\gamma - 1} T = \text{constant} \]  

**Maxwell Relations and Thermodynamic Functions**

\[
\begin{align*}
\left( \frac{\partial T}{\partial V} \right)_S &= - \left( \frac{\partial p}{\partial S} \right)_V \quad \left( \frac{\partial T}{\partial p} \right)_S = \left( \frac{\partial V}{\partial S} \right)_P \\
\left( \frac{\partial S}{\partial V} \right)_T &= \left( \frac{\partial p}{\partial T} \right)_V \quad - \left( \frac{\partial S}{\partial p} \right)_T = \left( \frac{\partial V}{\partial T} \right)_P 
\end{align*}
\]  

Energy \quad \begin{align*} E &= E(S, V) \quad dE = T dS - pdV \\
\text{Enthalpy} \quad H &= H(S, p) = E + pV \quad dH = T dS + V dp \\
\text{Helmholtz Free Energy} \quad F &= F(T, V) = E - TS \quad dF = -SdT - pdV \\
\text{Gibbs Free Energy} \quad G &= G(T, p) = E - TS + pV \quad dG = -SdT + V dp 
\end{align*}  

\[
\begin{align*}
T &= \left( \frac{\partial E}{\partial S} \right)_V = \left( \frac{\partial H}{\partial S} \right)_p \quad p = - \left( \frac{\partial E}{\partial V} \right)_S = - \left( \frac{\partial F}{\partial V} \right)_T \\
V &= \left( \frac{\partial H}{\partial p} \right)_S = \left( \frac{\partial G}{\partial p} \right)_T \quad S = - \left( \frac{\partial F}{\partial T} \right)_V = - \left( \frac{\partial G}{\partial T} \right)_p 
\end{align*}
\]  

Clausius – Clapeyron Equation \quad \frac{dp}{dT} = \frac{\Delta s}{\Delta v} \]  

**Engines and Refrigerators**

\[ q_1 = w + q_2 \quad \frac{q_2}{q_1} \leq \frac{T_2}{T_1} \]
Efficiency: \[ \eta \equiv \frac{w}{q_1} = \frac{q_1 - q_2}{q_1} < 1 \quad \eta_{\text{max}} = \frac{T_1 - T_2}{T_1} \] (35)

Statistical Mechanics

\[ \mathcal{A} = \frac{1}{Z} \sum_R A_re^{-\beta E_r} \quad Z = \sum_re^{-\beta E_r} \] (36)

\[ \mathcal{E} = -\frac{\partial \ln Z}{\partial \beta} \quad S = k_B(\ln Z + \beta \mathcal{E}) \quad X = \frac{1}{\beta} \frac{\partial \ln Z}{\partial \beta} \] (37)

\[ \mathcal{p} = \frac{1}{\beta} \frac{\partial \ln Z}{\partial V} \quad \mathcal{M} = \frac{1}{\beta} \frac{\partial \ln Z}{\partial H} \] (38)

\[ F = -k_B T \ln Z \quad C_V = k_B \beta^2 \frac{\partial^2 \ln Z}{\partial \beta^2} = -T \frac{\partial^2 F}{\partial T^2} \] (39)

\[ \mathcal{Z} = \sum_re^{-\beta(E_r-\mu N_r)} \quad \mu = -\frac{\partial E'}{\partial N'} \] (40)

\[ \mathcal{N} = \frac{1}{\beta} \frac{\partial \ln Z}{\partial \mu} \quad \mathcal{E} = \mu \mathcal{N} - \frac{\partial \mathcal{E}}{\partial \beta} \ln Z \] (41)

\[ \ln Z = -\beta \mu N + \ln \mathcal{Z} \quad \mu = -\frac{1}{\beta} \frac{\partial \ln Z(N)}{\partial N} = \frac{\partial F}{\partial N} \] (42)

\[ f(\vec{v})d^3rd^3v = n \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT} d^3rd^3v \quad \text{Maxwell velocity distribution} \] (43)

\[ \mathcal{\pi}_s = -\frac{1}{\beta} \frac{\partial \ln Z}{\partial \varepsilon_s} \quad \mathcal{N} = \sum_s \mathcal{\pi}_s \] (44)

\[ \mathcal{\pi}_s = N \frac{e^{-\beta \varepsilon_s}}{\sum_r e^{-\beta \varepsilon_r}} \quad \text{Maxwell Boltzmann Distribution} \] (45)

\[ \mathcal{\pi}_s = \frac{1}{e^{\beta \varepsilon_s} - 1} \quad \text{Planck distribution} \] (46)

\[ \mathcal{\pi}_s = e^{\beta \varepsilon_s - \mu} - 1 \quad \text{bosons} \quad \mathcal{\pi}_s = \frac{1}{e^{\beta \varepsilon_s - \mu} + 1} \quad \text{fermions} \] (47)

Black Body Radiation: \[ \pi(\omega, T)d\omega = \frac{\hbar \omega^3}{\pi^2 c^3 (e^{\beta \hbar \omega} - 1)} d\omega \quad \pi_0(T) = \frac{\pi^2 (k_B T)^4}{15 (hc)^3} \] (48)

\[ P_{\text{e}}^{\text{tot}} = \frac{1}{4} c u_0 = \sigma T^4 \quad \sigma \equiv \frac{\pi^2}{60} \frac{k_B^4}{c^2 h^3} \] (49)

Free Electron Gas: \[ k_F = \left(3\pi^2 N/V\right)^{1/3} \quad C_V = \frac{\pi^2}{2} R \left(\frac{T}{T_F}\right) \] (50)

Einstein Heat Capacity: \[ C_V = 3Nk_B \left(\frac{\theta_E}{T}\right)^2 \frac{e^{\theta_E/T}}{(e^{\theta_E/T} - 1)^2} \] (51)
Debye Heat Capacity: \[ C_V = \frac{12\pi^4}{5}Nk_B \left( \frac{T}{\Theta_D} \right)^3 \] (52)

Bose Einstein Condensation: \[ n_1 = N \left[ 1 - \left( \frac{T}{T_C} \right)^{3/2} \right] \quad \mu = -k_BT\ln \left( \frac{1}{n_1} + 1 \right) \] (53)

Spin Systems

\[ \bar{\mu} = g\mu_oS \] (54)

\[ \mathcal{H}_o = -g\mu_o \sum_{j=1}^{N} \mathbf{S}_j \cdot \mathbf{H}_o \] (55)

Heisenberg Hamiltonian: \[ \mathcal{H} = -2J \sum_{i>j} \mathbf{S}_i \cdot \mathbf{S}_j \] (56)

If we take \( J_{ij} = J \), then

\[ \mathcal{H} = -2J \sum_{i>j} \mathbf{S}_i \cdot \mathbf{S}_j \] (57)

Ising Model: \[ \mathcal{H} = -2J \sum_{i>j} S_{iz}S_{jz} \] (58)