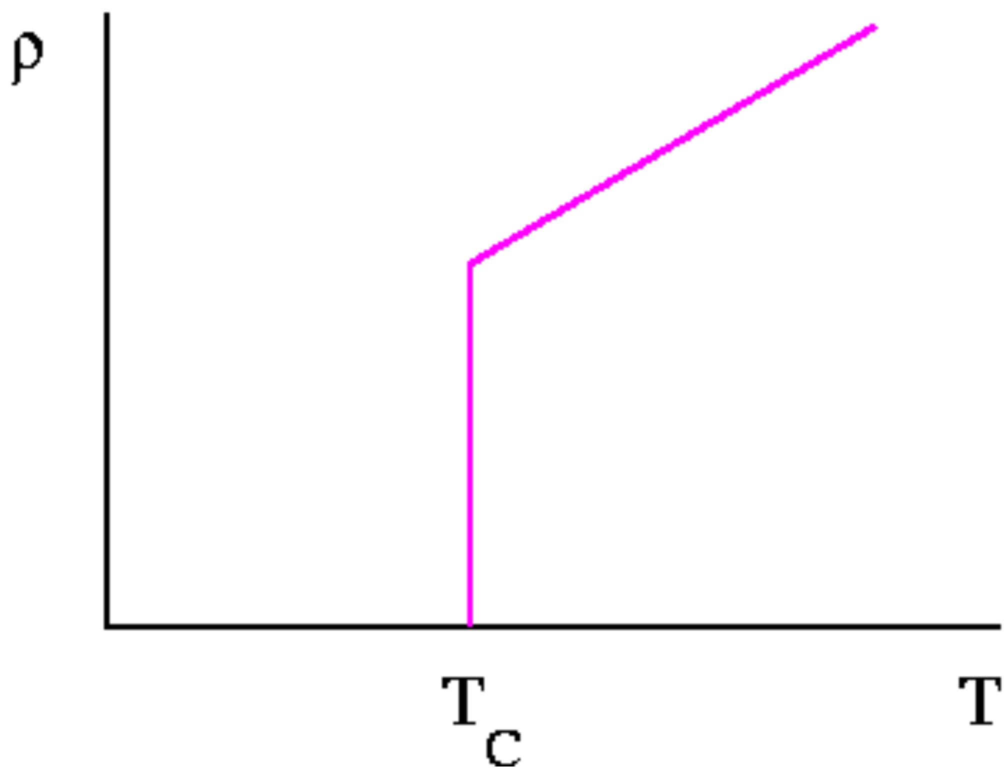
Introduction to Superconductivity

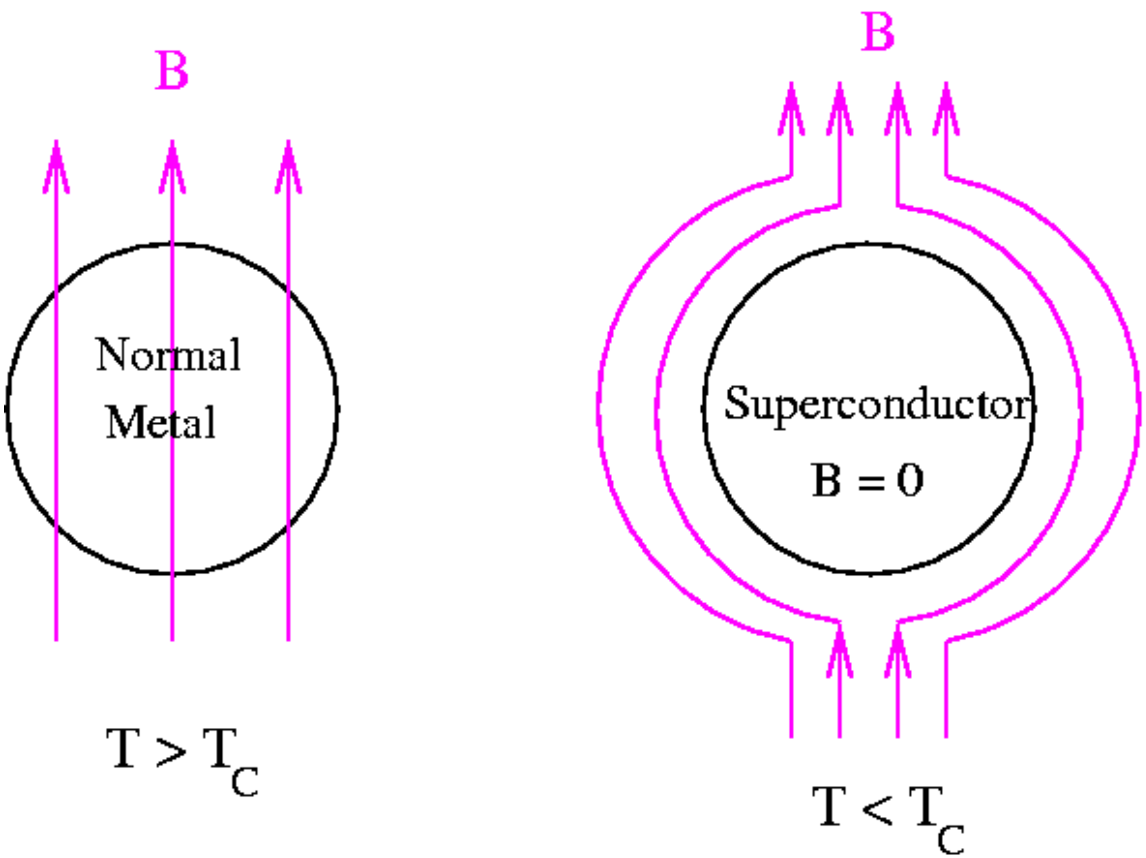
Superconductivity was discovered in 1911 by Kamerlingh Onnes.

- Zero electrical resistance



Meissner Effect

- Magnetic field expelled. Superconducting surface current ensures B=0 inside the superconductor.



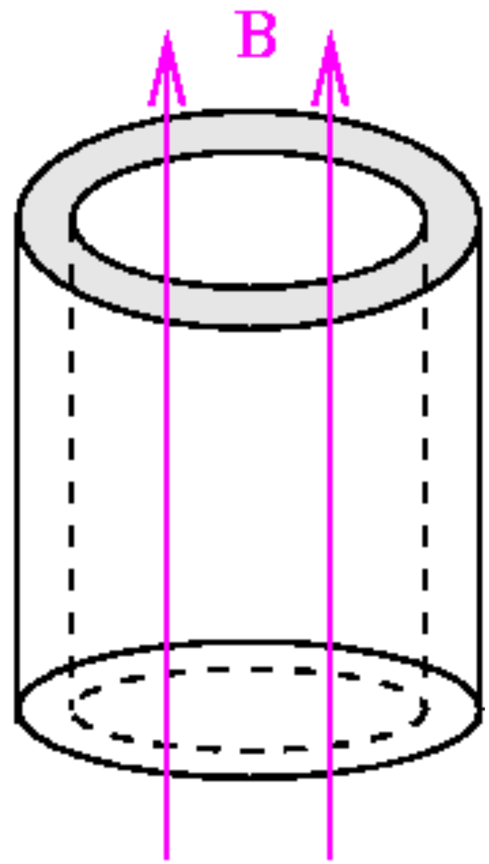
Flux Quantization

$$\Phi = \int \vec{B} \bullet d\vec{A} = n\Phi_o$$

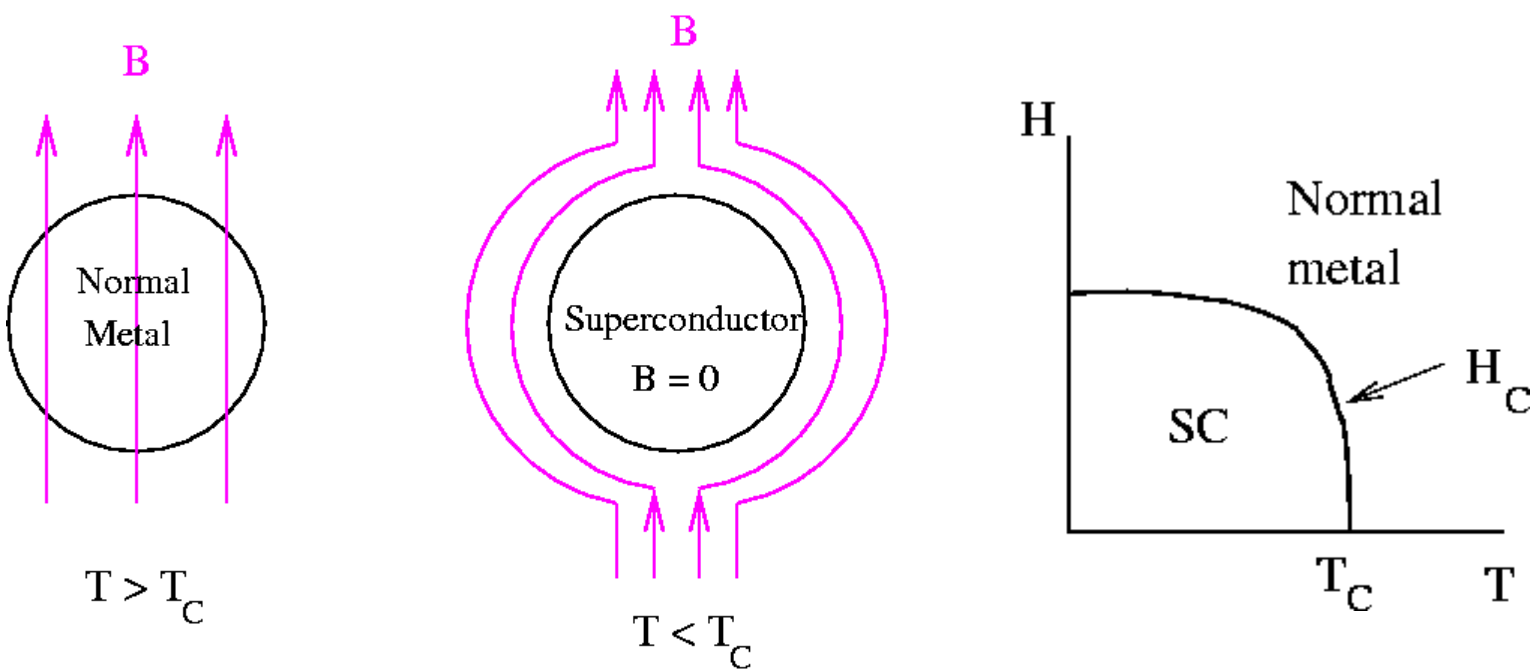
where the "flux quantum" Φ_o is given by

$$\Phi_o = \frac{hc}{2e}$$

$$= 2 \times 10^{-7} \text{ gauss cm}^2$$



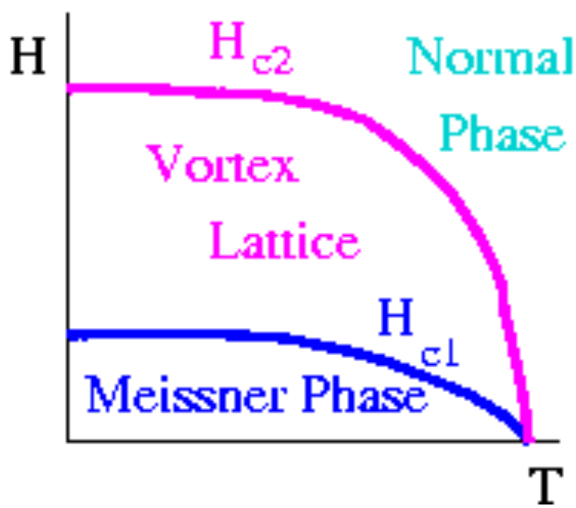
Type I Superconductors

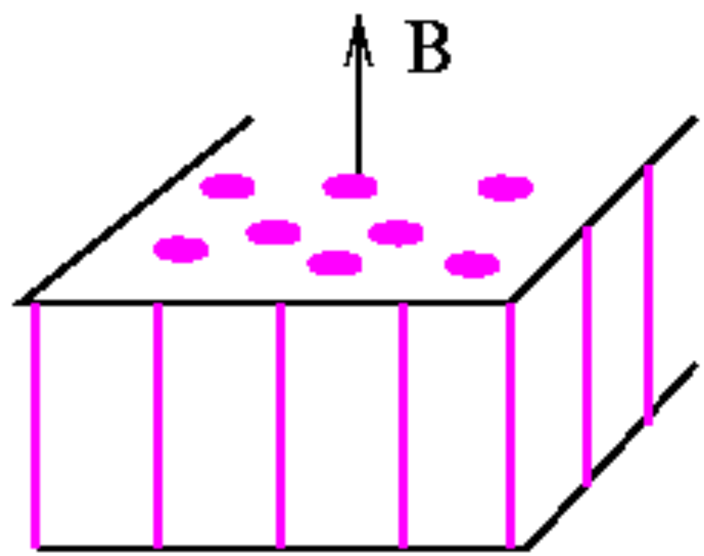


Type I superconductors expel the magnetic field totally, but if the field is too big, the superconductivity is destroyed.

Type II Superconductors

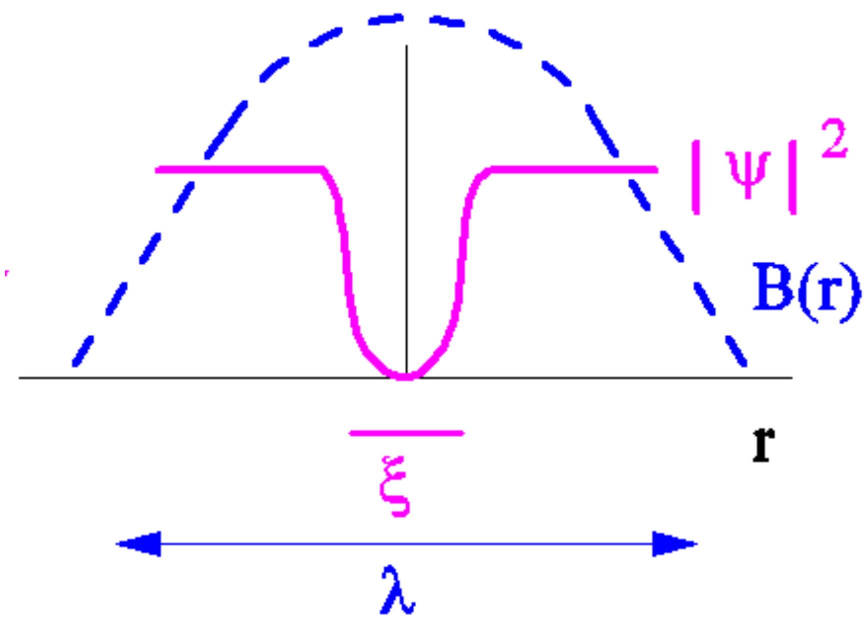
For intermediate field strengths, there is partial field penetration in the form of vortex lines of magnetic flux.

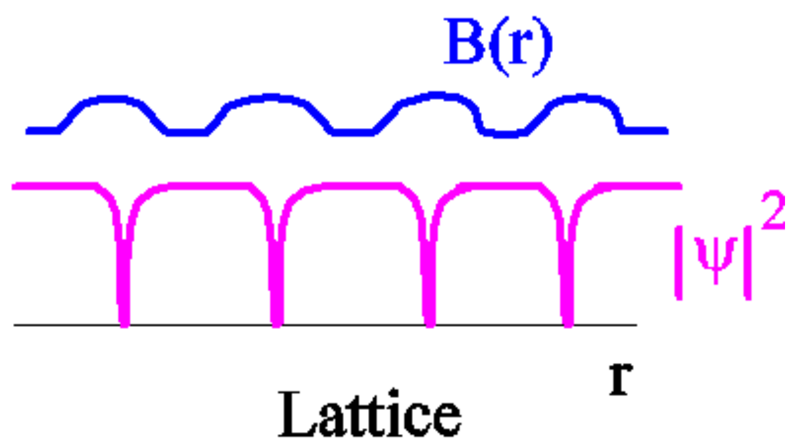




Vortices

Each vortex contains 1 flux quantum Φ_o=hc/2e. The superconducting order parameter goes to zero at the center of a flux quantum. The core of the vortex has normal electrons.



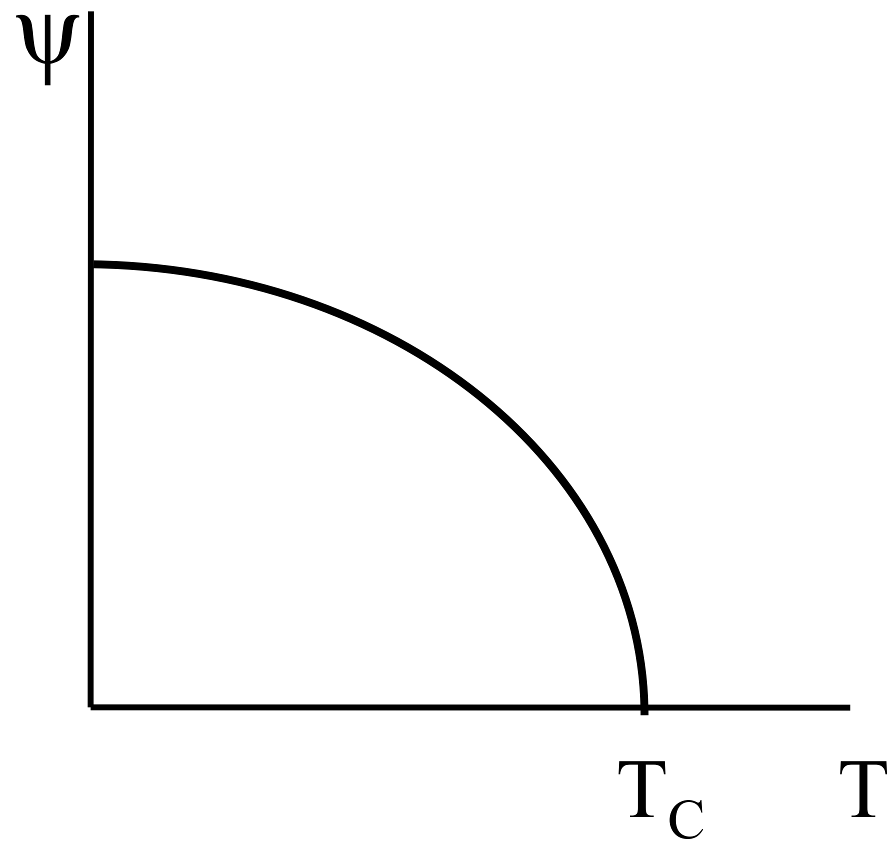


Explanation of Superconductivity

- Ginzburg-Landau Order Parameter

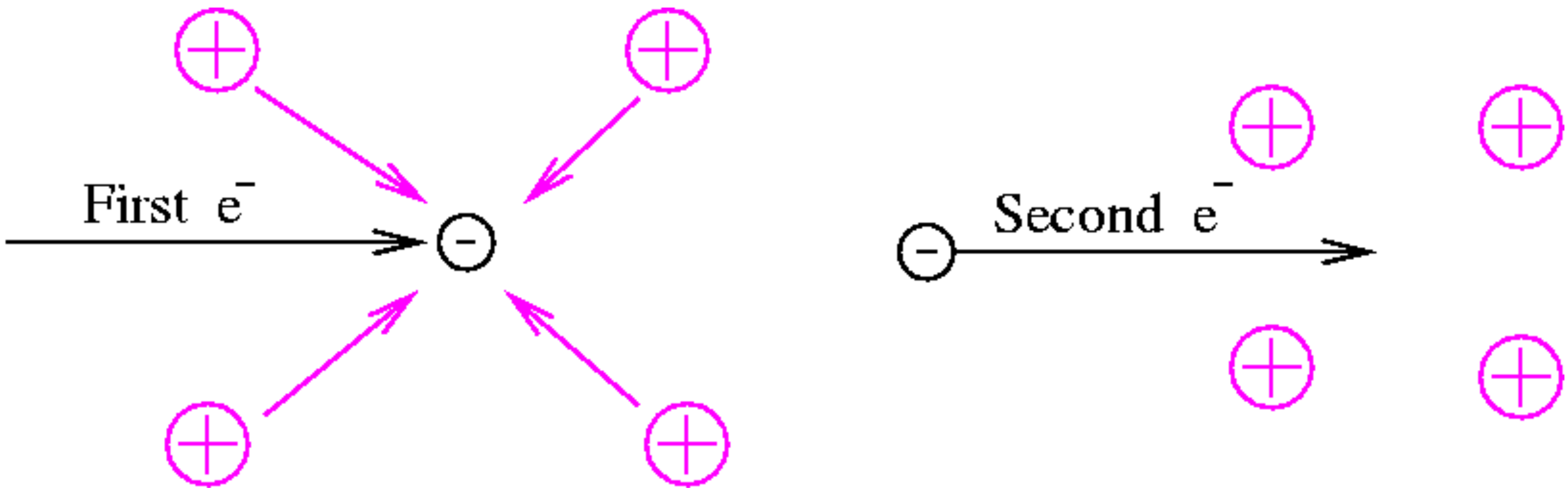
$$\psi = \left|\psi\right| e^{i\theta}$$

Think of this as a wavefunction describing all the electrons. Phase θ wants to be spatially uniform ("phase rigidity").

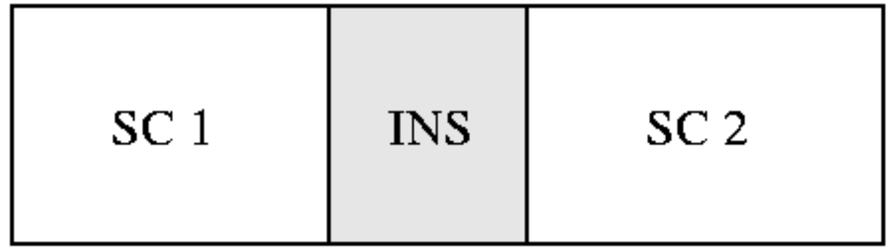


BCS Theory
(Bardeen-Cooper-Schrieffer)

- Electrons are paired into Cooper pairs.

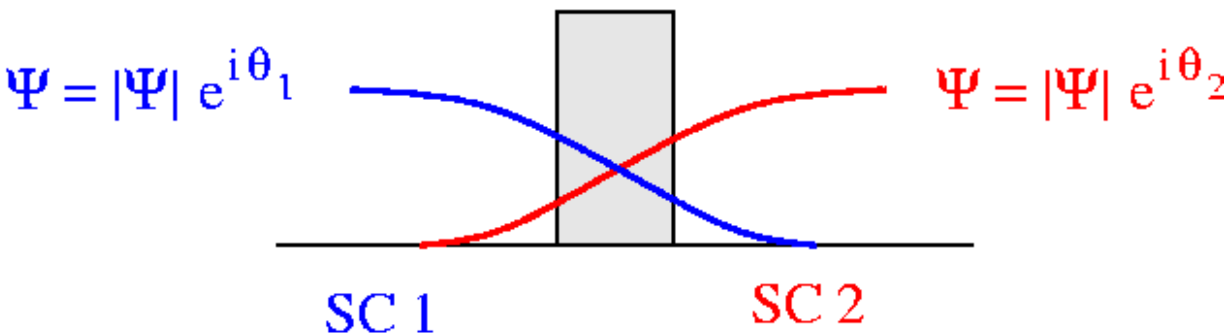


Josephson Effect



Josephson Tunnel Junction

If we put 2 superconductors next to each other separated by a thin insulating layer, the phase difference (θ_2-θ_1) between the 2 superconductors will cause a current of superconducting Cooper pairs to flow between the superconductors. Current flow without batteries! This is the Josephson effect.



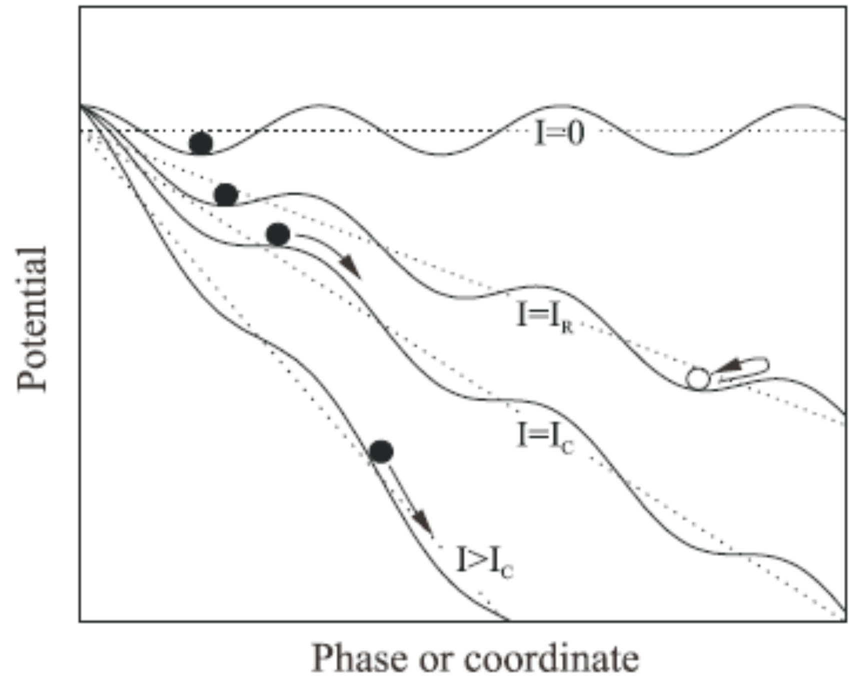
$J = J_o \sin\left(\theta_2 - \theta_1\right) = J_o \sin\delta$ where J_o is the critical current density and δ is the phase difference.

Josephson Junction Washboard Potential

$$J = \frac{2e}{\hbar}\frac{\partial U}{\partial \delta} = J_o \sin\delta - J_{ext}$$

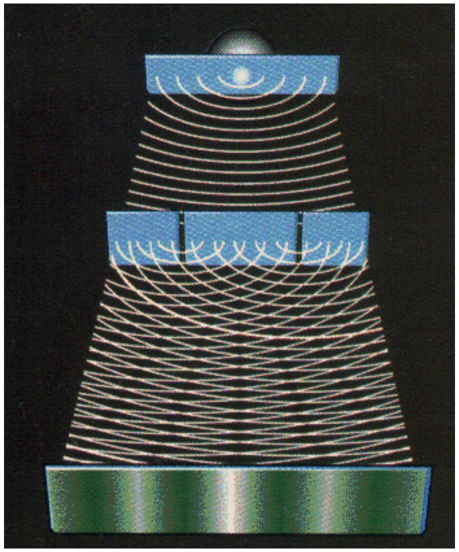
$$U = \frac{\hbar J_o}{2e} - \frac{\hbar J_o}{2e}\cos\delta - \frac{\hbar J_{ext}}{2e}\delta$$

Washboard potential tilts with application of external current.

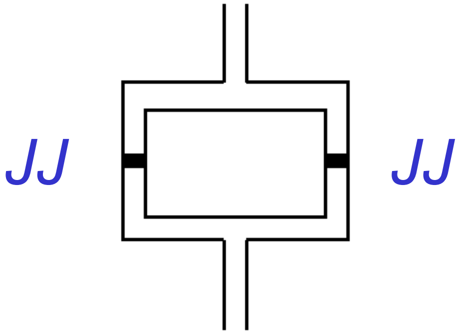


SQUIDs

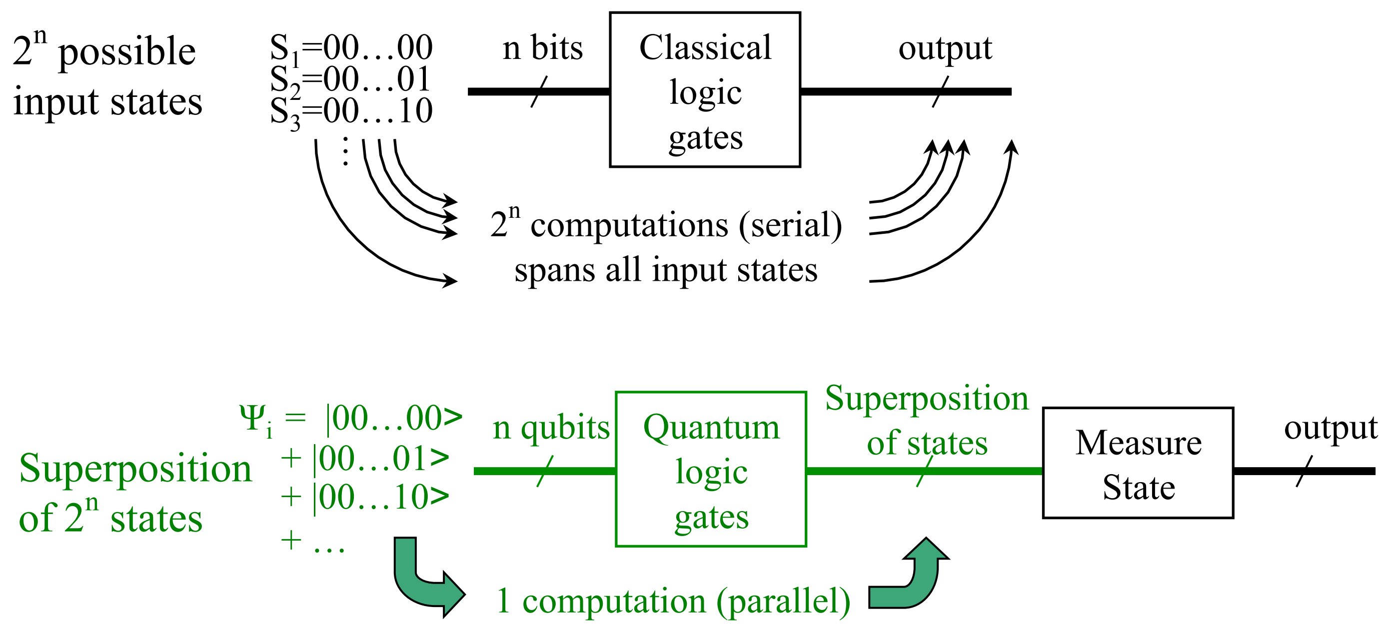
(~ 2 slit device for superconducting wave functions)



- SQUID is a Superconducting QUantum Interference Device.
- DC SQUID is a loop with 2 Josephson junctions.
- Phase difference around the loop proportional to magnetic flux through loop.
- Current through the SQUID is modulated by the magnetic flux through loop.
- SQUIDs are sensitive detectors of the amount of magnetic flux Φ through the loop.
- SQUIDs can be used as qubits (quantum bits).

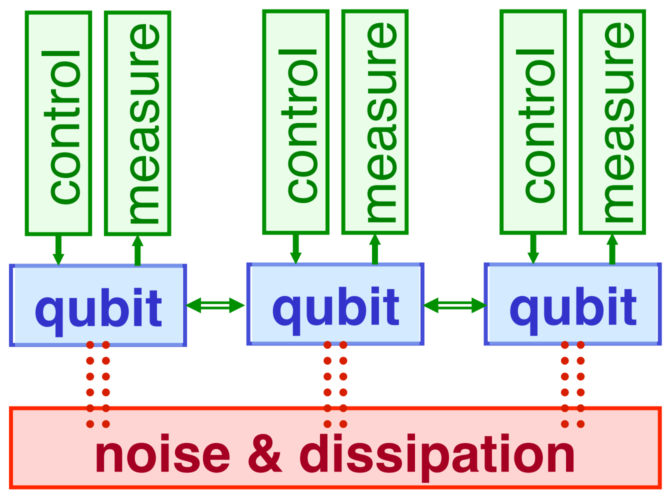


Why is Quantum Computing Useful?



- Parallel computation of exponentially-large states
- Factorization of large numbers into prime numbers (Shor) (cryptography)
  Exponential speedup of algorithm
- Fast search algorithms (Grover) { $n^{1/2}$ vs. n }
- Adiabatic algorithms for minimization (Farhi)
- Simulation of quantum systems (Feynman)
- Other? (Quantum Information Theory)

Challenge: Coupling vs. Decoherence



Experimental challenge:
Couple qubits to each other, and control, & measure,
Avoid coupling qubits to noise and dissipation

Qubit is a quantum bit

$$\Psi = \cos\frac{\theta}{2}\left|0\right\rangle + \sin\frac{\theta}{2}e^{i\phi}\left|1\right\rangle$$

Qubit wavefunction

Experimental Systems

Atoms **EM modes**

Feynmann (1985):
"it seems that the laws of physics present no barrier to reducing the size of computers until bits are the size of atoms, and quantum behavior holds sway."

atomic

- Photons
- Ions
- Neutral Atoms
- NMR

mesoscopic

- Spin
- Semiconductor spin
- Quantum Dot

microscopic

- Superconductor SET
- 2-Degenerate SSET
- 3 Junction SQUID
- RF SQUID
- Josephson Junctions

Coherence easier

Coupling easier

Charge
(NEC, Chalmers)
(Saclay)

(Delft)
Flux
(MIT)

Phase
(NIST, Kansas)

Josephson Junction Qubit Taxonomy

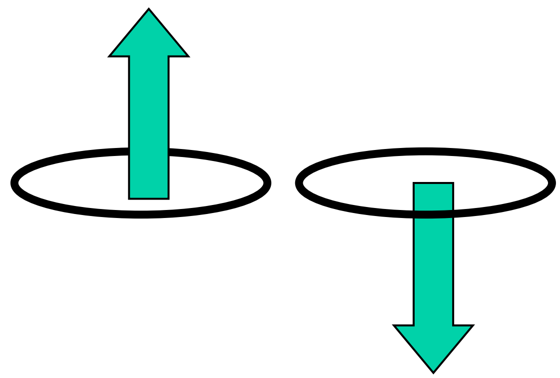
States are a linear combination of

Phase

2 different energy states in one well of JJ potential.

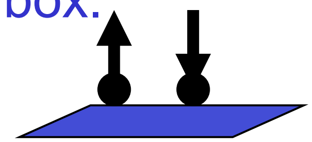
Flux

2 different flux states, e.g., up and down states of a SQUID.

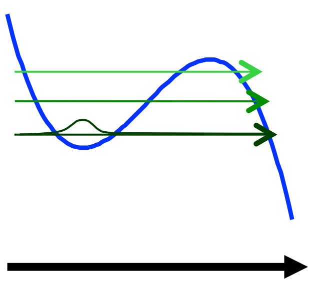


Charge

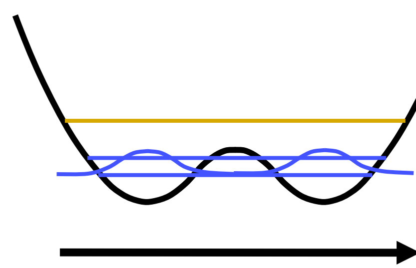
2 different charge states, e.g., n and (n +1) Cooper pairs in a Cooper pair box.



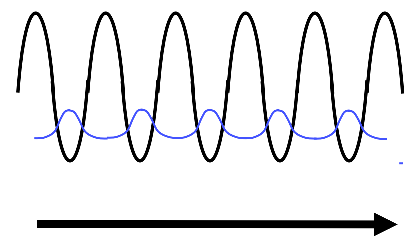
Potential & wavefunction



Phase difference δ



Flux Φ



Charge Q

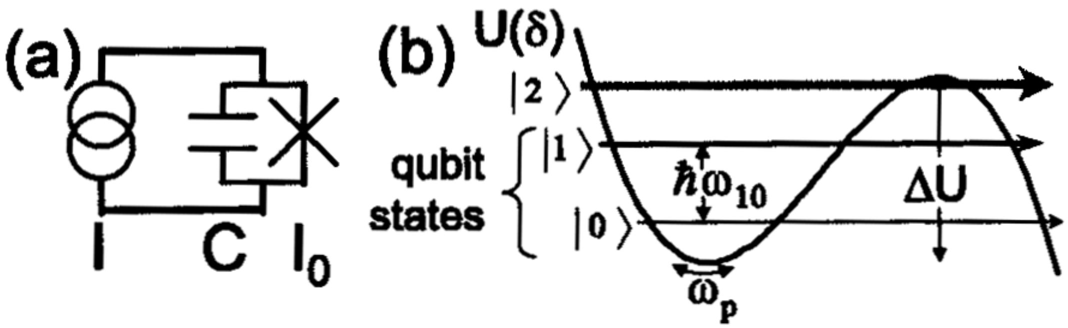
Quantum Computing and Qubits

Josephson junctions can be used to construct qubits.

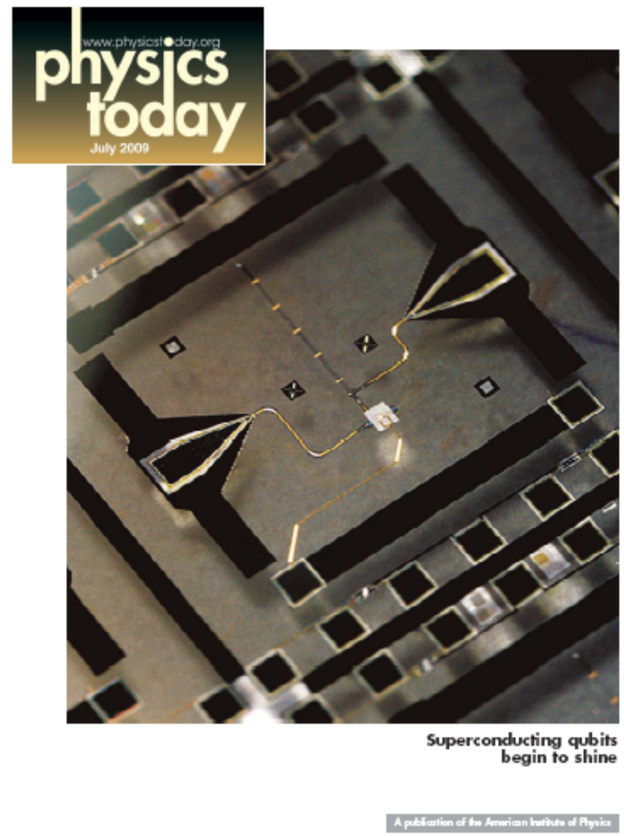
- Major Advantage: scalability using integrated circuit (IC) fabrication technology.
- Major Obstacle: Noise and Decoherence

$$\Psi = \cos\frac{\theta}{2}\left|0\right\rangle + \sin\frac{\theta}{2}e^{i\phi}\left|1\right\rangle$$

Qubit wavefunction



J. M. Martinis *et al.*, PRL 89, 117901 (2002).

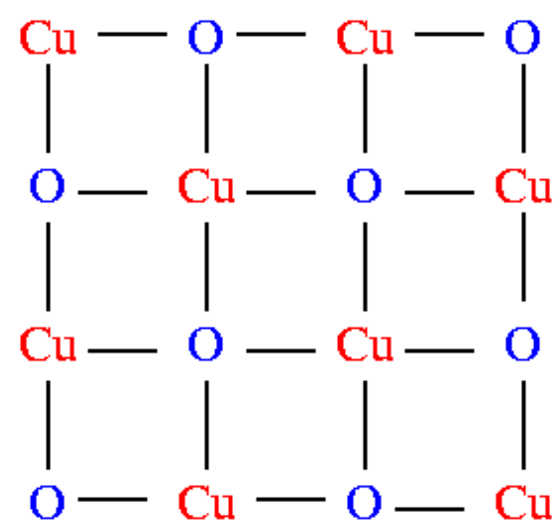


High Temperature Superconductors

The high temperature superconductors have high transition temperatures.

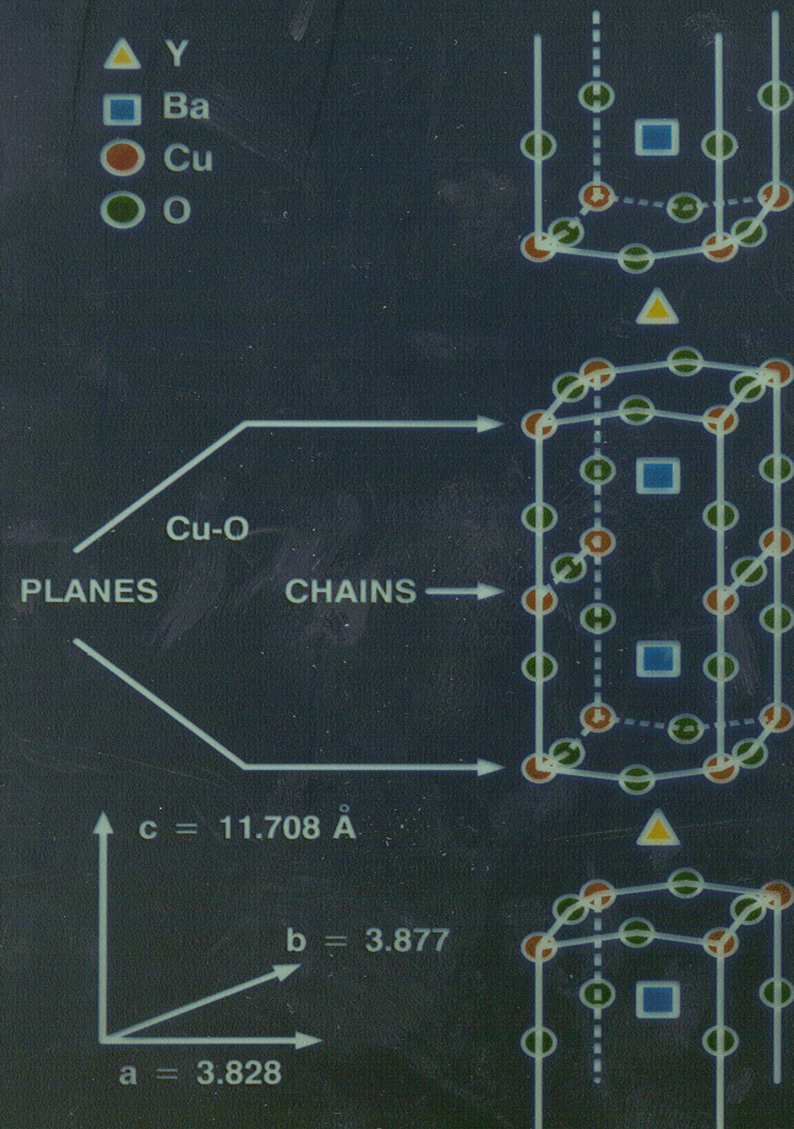
$YBa_2Cu_3O_{7-\delta}$ (YBCO)	T_C=92 K
$Bi_2Sr_2CaCu_2O_8$ (BSCCO)	T_C=90 K

The basis of the high temperature superconductors are copper-oxygen planes. These planes are separated from other copper-oxygen planes by junk. The superconducting current flows more easily in the planes than between the planes.



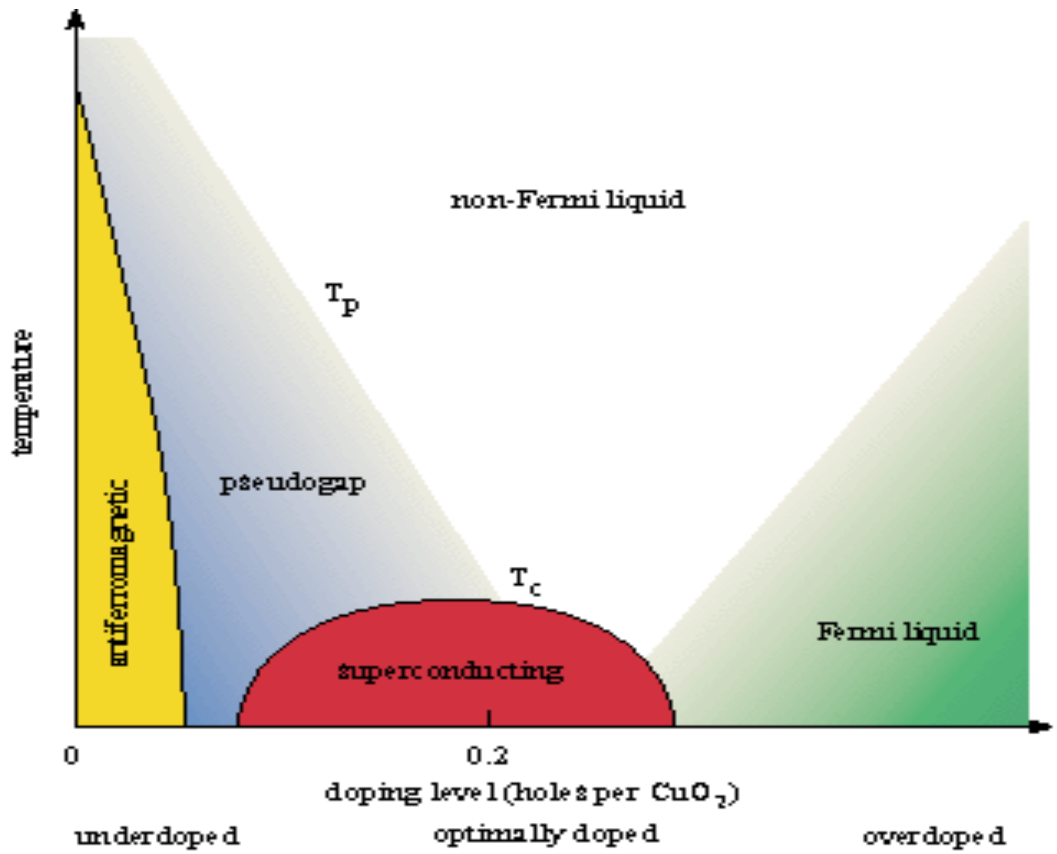
Copper Oxide Plane

CRYSTAL STRUCTURE
Y $Ba_2Cu_3O_{7-X}$



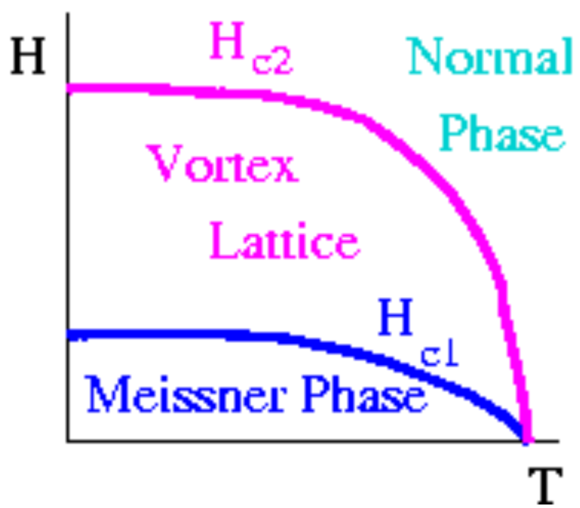
High Temperature Superconductivity Phase Diagram

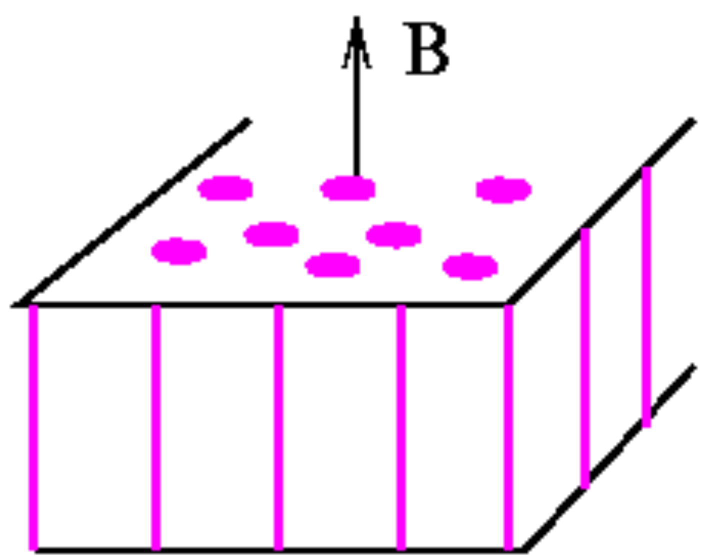
Cuprates



Type II Superconductors

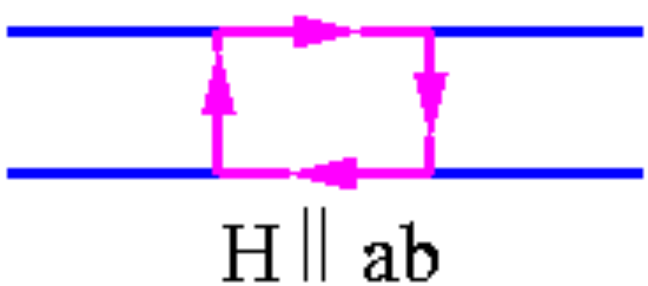
For intermediate field strengths, there is partial field penetration in the form of vortex lines of magnetic flux.

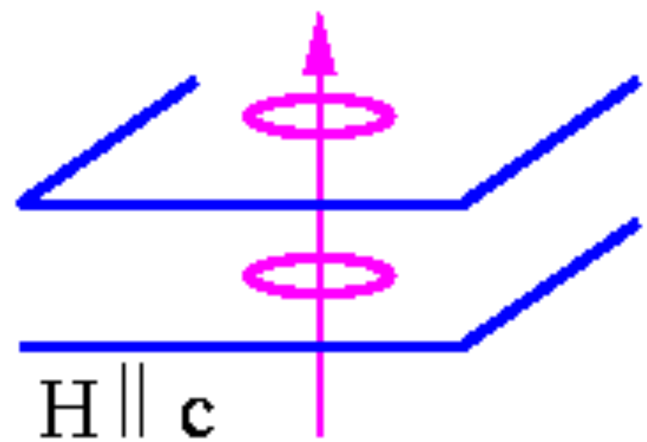




Vortices in Layered Superconductors

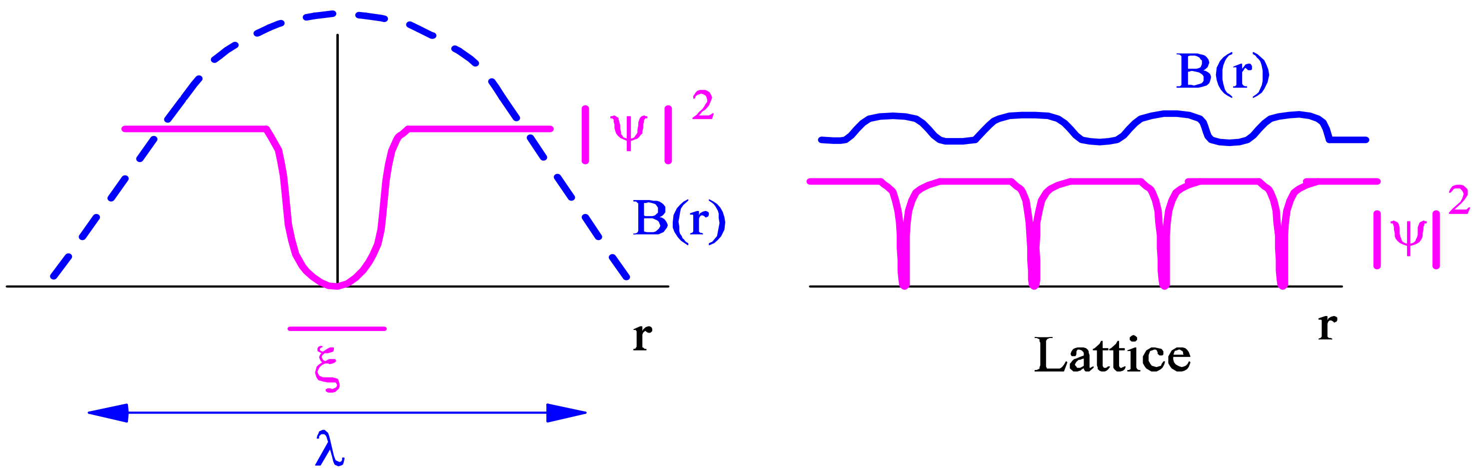
In a layered superconductor the planes are superconducting and the vortex lines are correlated stacks of pancakes (**H** || ĉ). A pancake is a vortex in a plane. Josephson tunneling occurs between planes. So if **H** || *ab*, we get Josephson vortices.





Vortices

Each vortex contains 1 flux quantum Φ_o=hc/2e. The superconducting order parameter goes to zero at the center of a flux quantum. The core of the vortex has normal electrons.



Lorentz Force on Vortices

When a transport current flows, vortices experience a Lorentz force per unit length

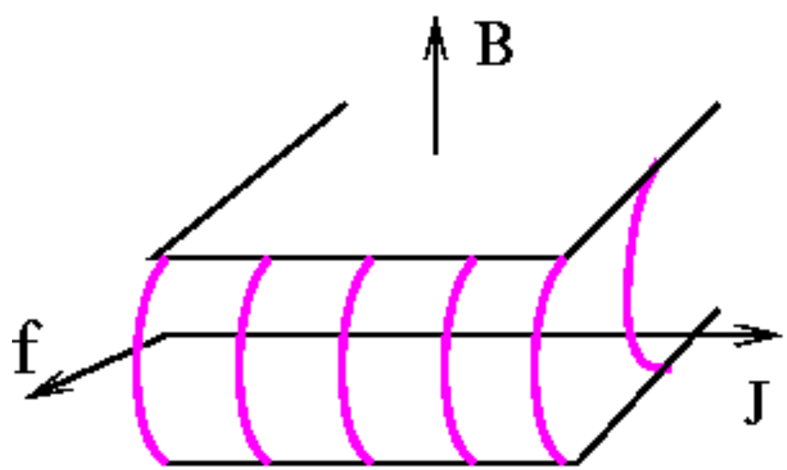
$$\mathbf{f} = \mathbf{J} \times \frac{\mathbf{\Phi}_\mathbf{o}}{\mathbf{c}}$$

where the flux quantum Φ_o=hc/2e and its direction is parallel to **B** locally. This force is analogous to the force density

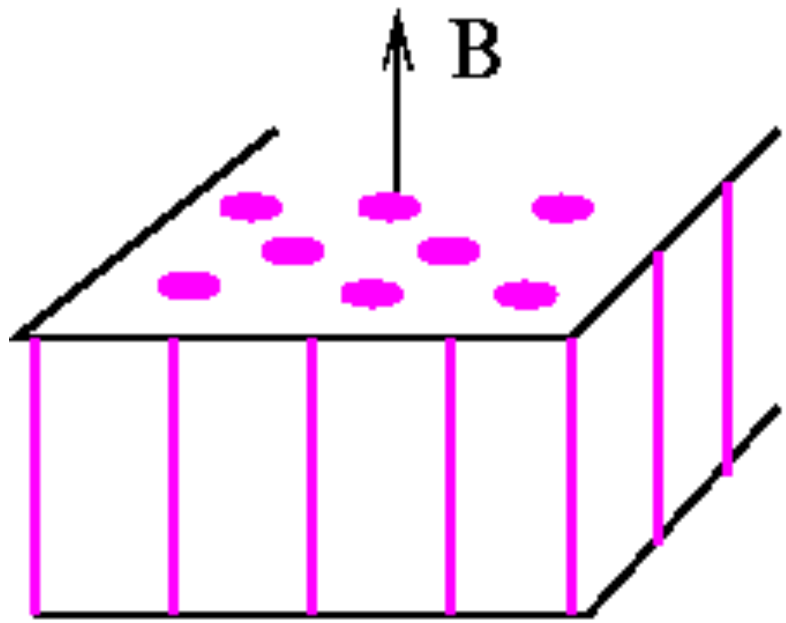
$$\frac{\mathbf{F}}{V} = \mathbf{J} \times \frac{\mathbf{B}}{c} \qquad \left(\mathbf{F} = \frac{q}{c} \mathbf{v} \times \mathbf{B} \right)$$

Vortex Motion

If the vortices move, they produce resistance because their cores have normal electrons. Vortices are often pinned by inhomogeneities which prevent vortex motion until the critical current density J_C is reached. The vortices break free for $J > J_C$.



Abrikosov Flux Lattice

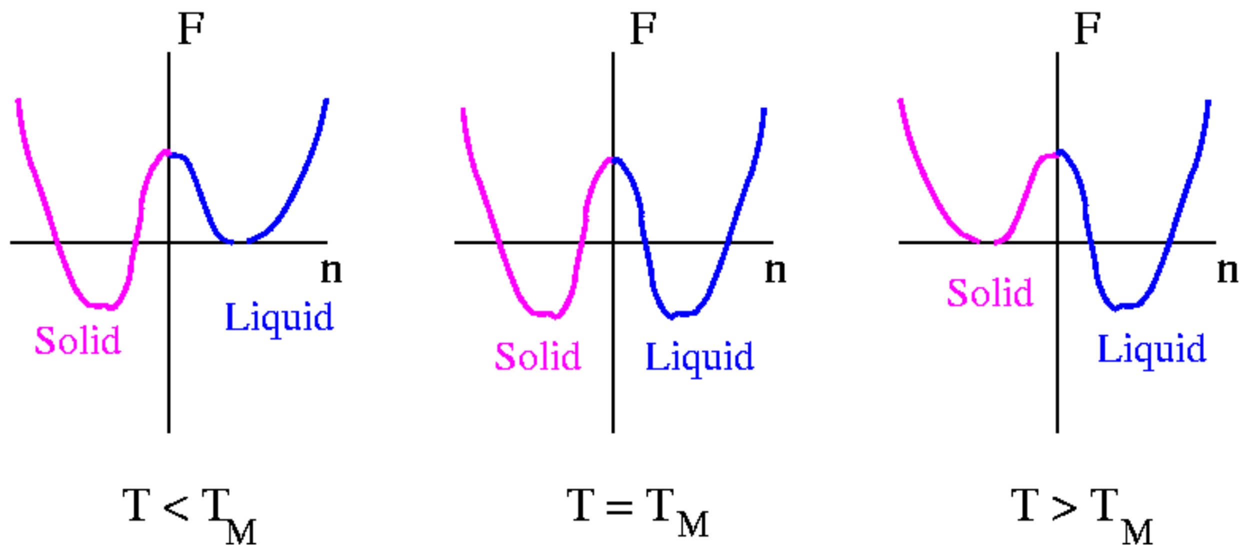


In a clean type II superconductor, repulsive interactions between vortices lead to an Abrikosov flux lattice. Traditionally this is a triangular lattice. At higher T and H, the flux lattice can melt into a flux liquid.

First Order Phase Transitions

A "phase transition" occurs when a system undergoes a transformation from one phase to another. Going from water to ice (or liquid to crystalline solid) is an example of a "first order phase transition." Typically a first order phase transition is associated with a volume change; ice expands. First order phase transitions are also associated with a discontinuity ΔS in the entropy. The entropy of the liquid S_L is greater than the entropy of the solid S_S and $\Delta S = S_L - S_S$. The latent heat *L* is given by

$$L = T\Delta S$$



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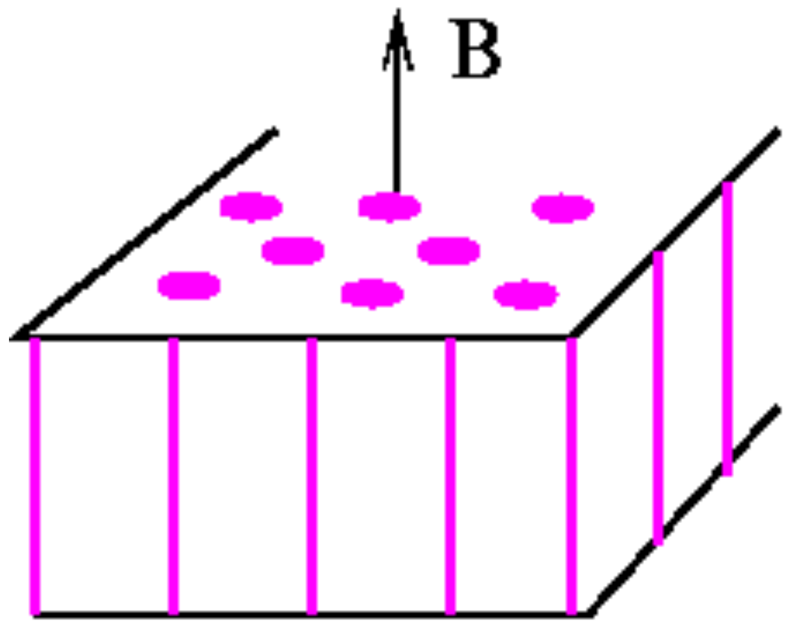

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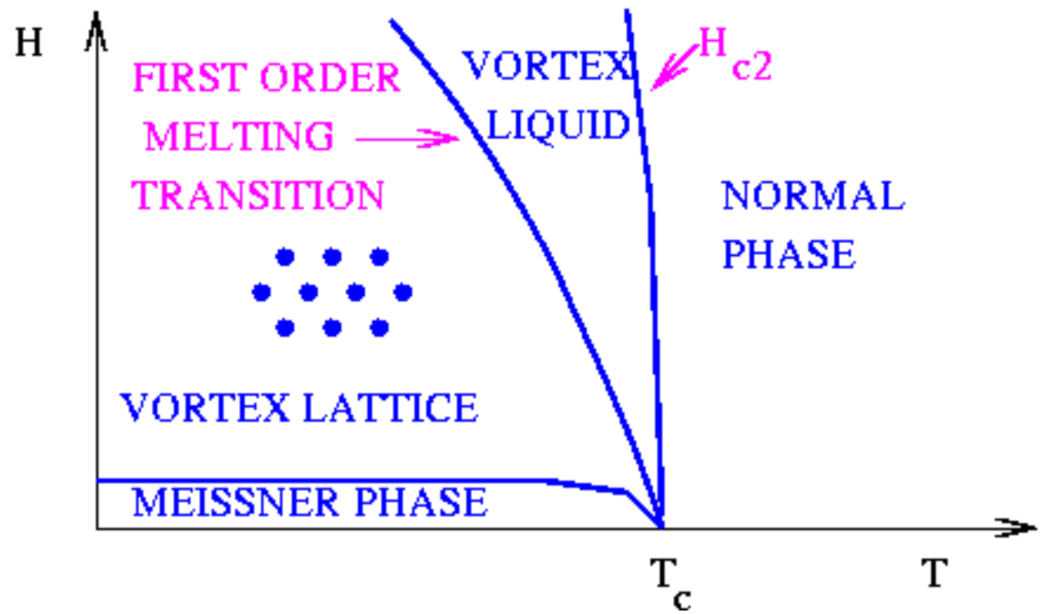
Phase Diagram



In a clean type II superconductor, there is an Abrikosov flux lattice at low T and H. At higher T and H, the flux lattice can melt into a flux liquid. The melting is like ice melting into water. Just as ice expands upon freezing, so the vortex lattice expands upon freezing. This produces a jump in the magnetization as well as other effects.

THE END