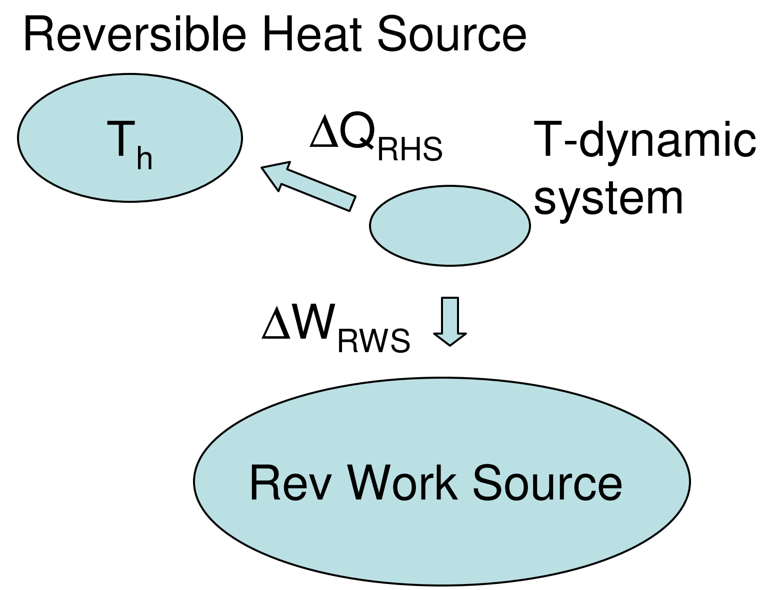
Summary of heat engines so far

- A thermodynamic system in a process connecting state A to state B

- In this process, the system can do work and emit/absorb heat

- What processes maximize work done by the system?

- We have proven that reversible processes (processes that do not change the total entropy of the system and its environment) maximize work done by the system in the process $A \rightarrow B$



Process $A \rightarrow B$ of the system

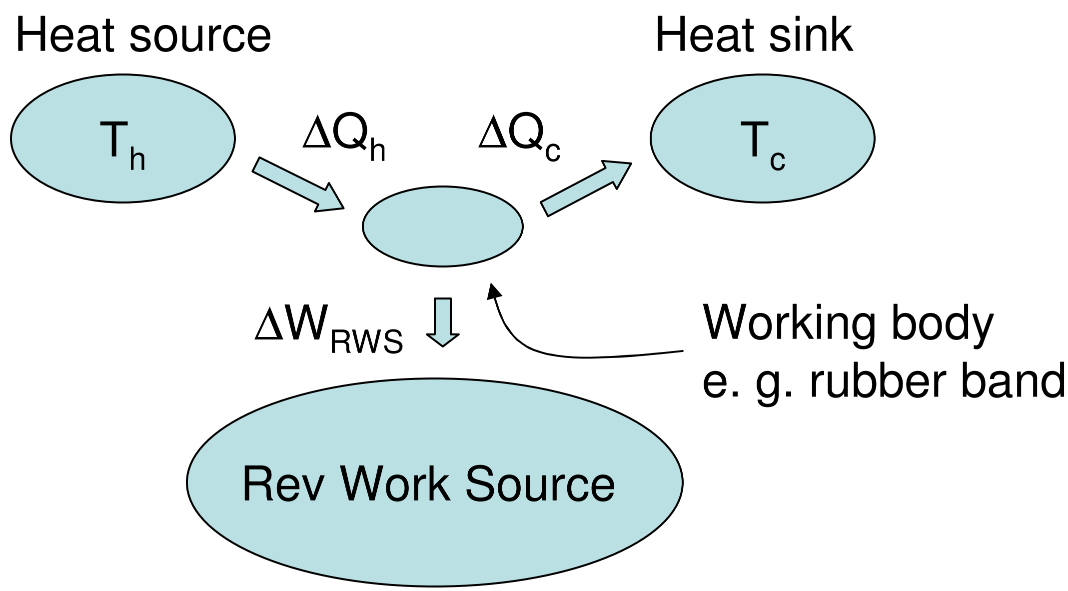
Notes

Cyclic Engine

-To continuously generate work, a heat engine should go through a cyclic process.

- Such an engine needs to have a heat source and a heat sink at different temperatures.

- The net result of each cycle is to extract heat from the heat source and distribute it between the heat transferred to the sink and useful work.



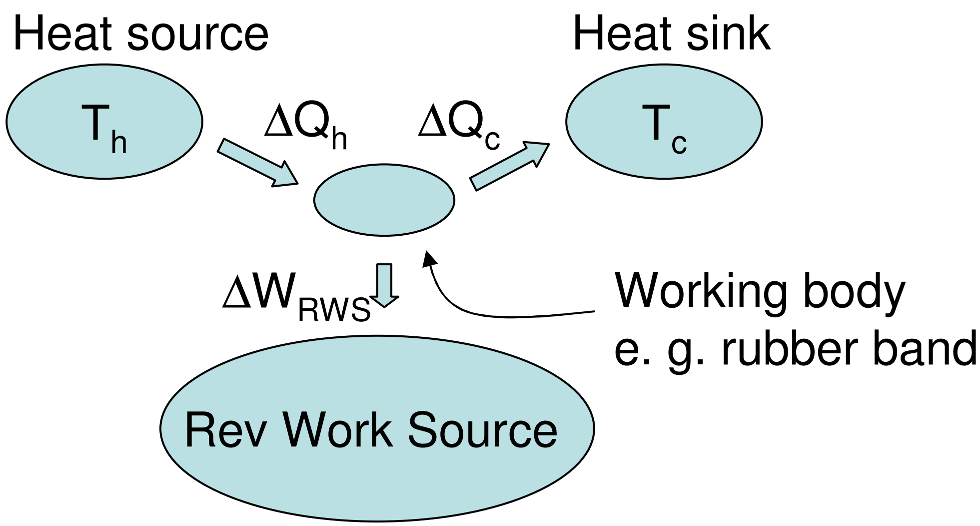
Notes

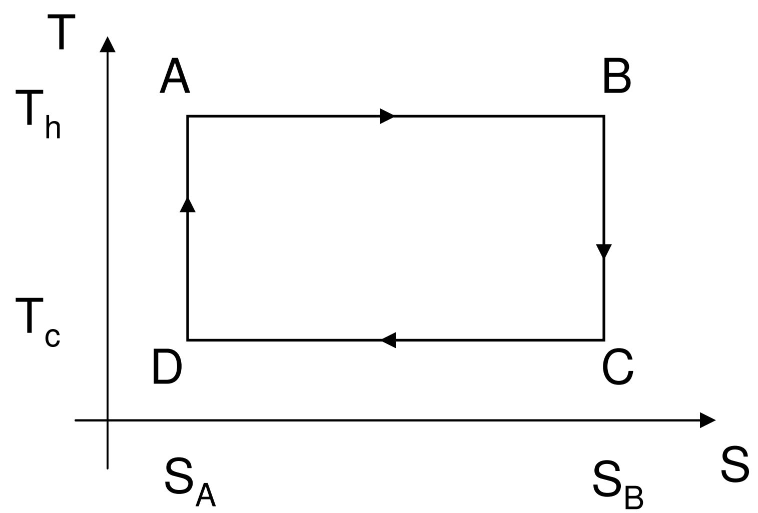
Carnot cycle

As an example, we will consider a particular cycle consisting of two isotherms and two adiabats called the Carnot cycle applied to a rubber band engine.

As we will see, Carnot cycle is **reversible**

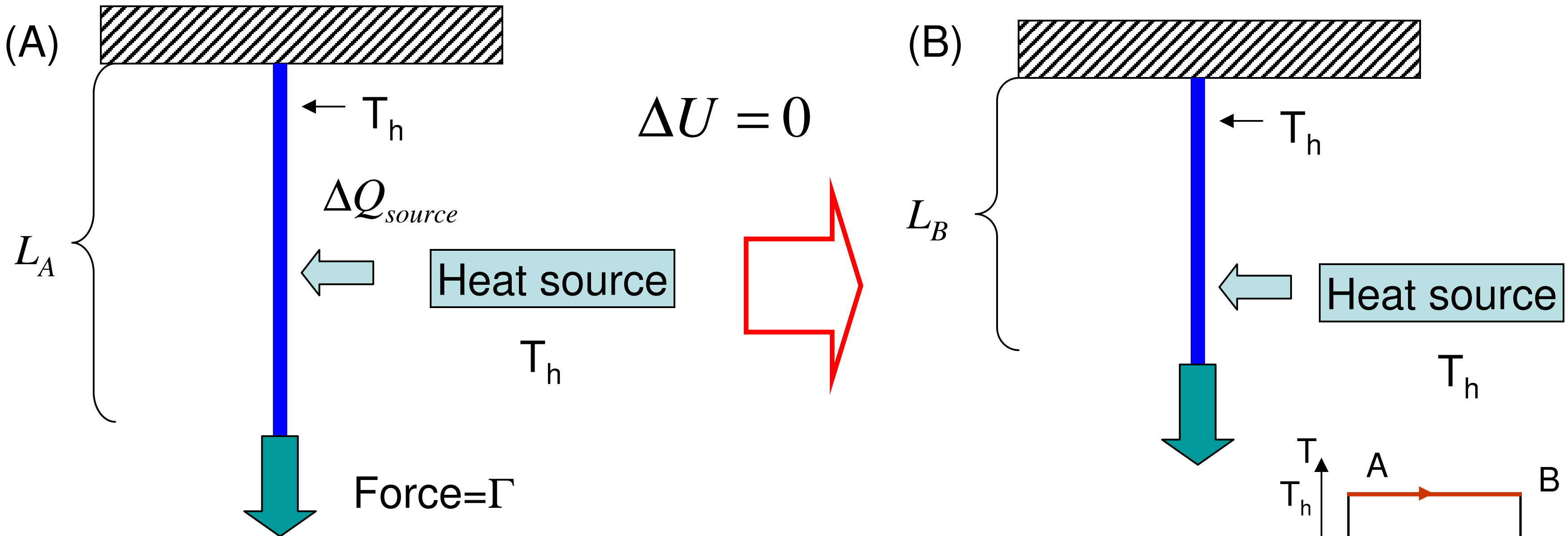
Here T and S are temperature and entropy of the working body



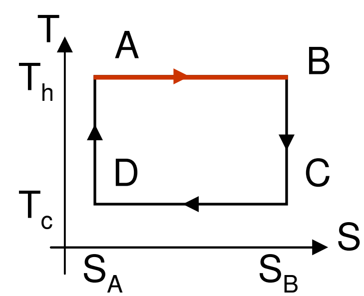


Notes

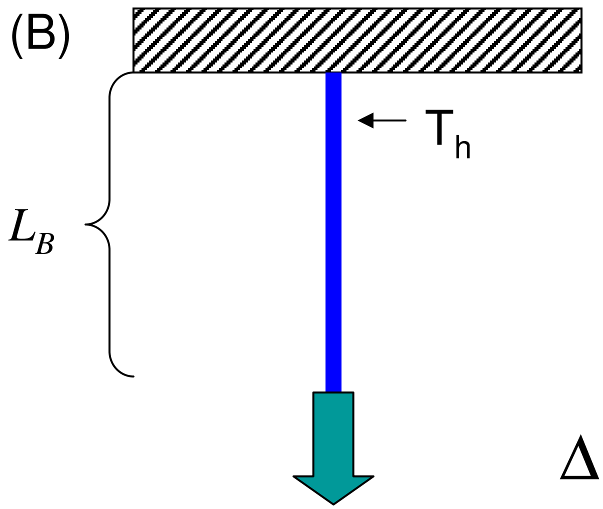
1. Carnot Cycle: Isothermal Contraction

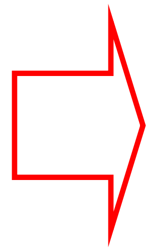


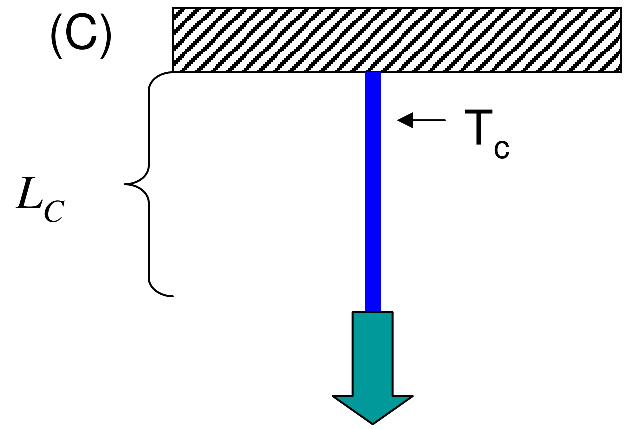
$$\Delta W_{RWS}^{A\to B} = \Delta Q_{source} = T_h \Delta S = \frac{cBT_h}{2}\left[\frac{(L_A - L_0)^2}{L_0} - \frac{(L_B - L_0)^2}{L_0}\right]$$



2. Carnot Cycle: Adiabatic Contraction





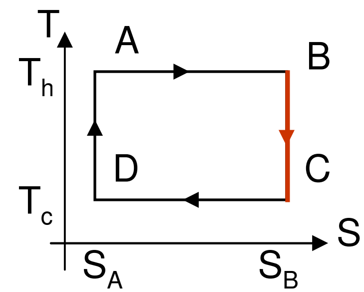


$$\Delta Q = 0$$

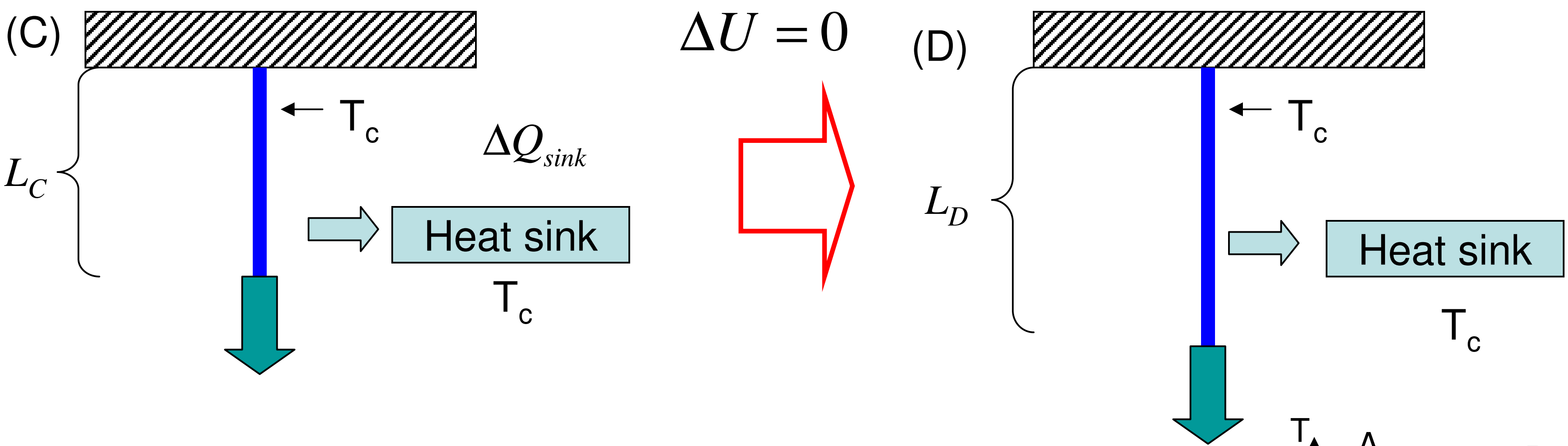
$$\Delta W_{RWS}^{B\rightarrow C} = -\Delta U$$

Quasi-static adiabatic process

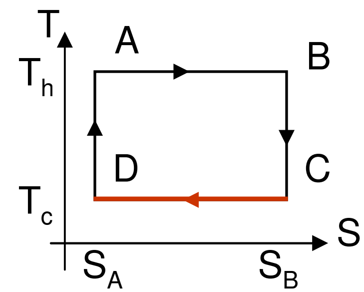
$$\Delta W_{RWS}^{B\rightarrow C} = U_B - U_C = cL_0\left(T_h - T_c\right)$$



3. Carnot Cycle: Isothermal Expansion

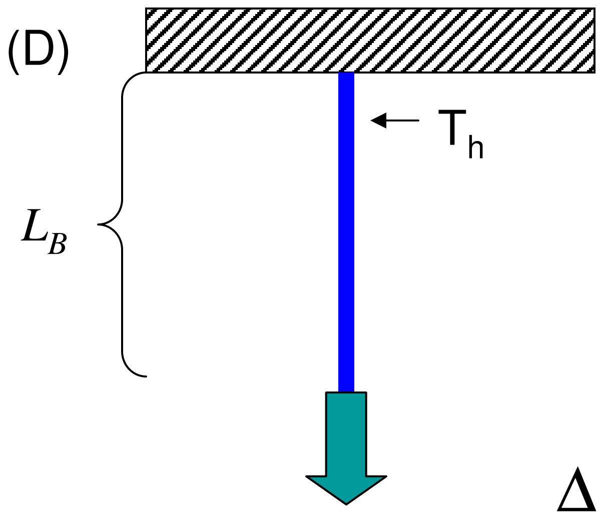


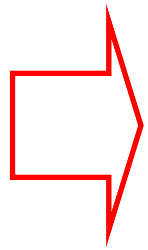
$$\Delta W_{RWS}^{C\to D} = \Delta Q_{sink} = T_c \Delta S = \frac{cBT_c}{2}\left[\frac{(L_C - L_0)^2}{L_0} - \frac{(L_D - L_0)^2}{L_0}\right]$$

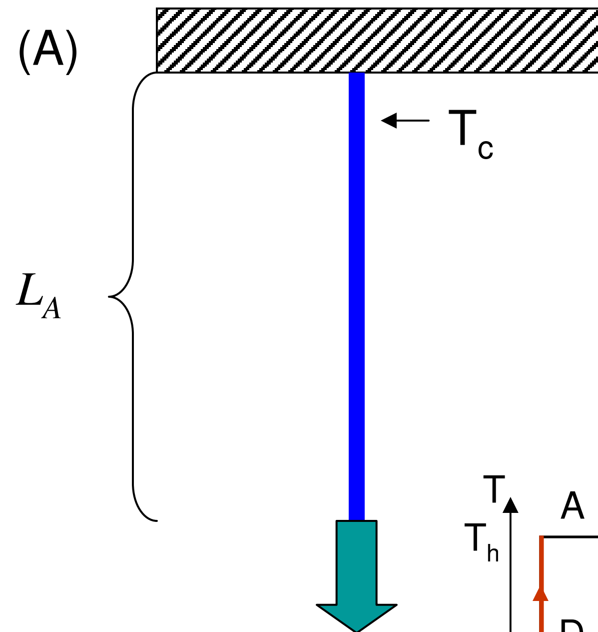


Notes

4. Carnot Cycle: Adiabatic Expansion



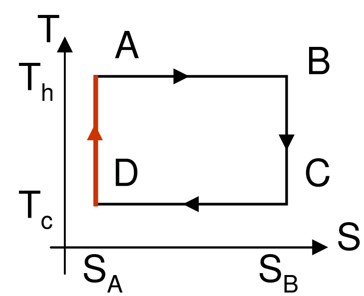




$$\Delta Q = 0$$

$$\Delta W_{RWS}^{D\to A} = \Delta U$$

$$\Delta W_{RWS}^{D\to A} = U_D - U_A = cL_0\left(T_c - T_h\right)$$



Notes

Carnot Cycle: Summary

Total work done in one cycle:

$$\Delta W_{RWS} = \Delta Q_{source} + \Delta Q_{sink} + cL_0(T_h - T_c) + cL_0(T_c - T_h) = \Delta Q_{source} + \Delta Q_{sink}$$

where

$$\Delta Q_{source} = \frac{cBT_h}{2}\left[\frac{(L_A - L_0)^2}{L_0} - \frac{(L_B - L_0)^2}{L_0}\right] \qquad \Delta Q_{sink} = \frac{cBT_c}{2}\left[\frac{(L_C - L_0)^2}{L_0} - \frac{(L_D - L_0)^2}{L_0}\right]$$

Carnot engine is reversible because each of the four parts of the cycle is reversible!

Exercise: show that adiabatic expansion of a rubber band where all internal energy goes into work is a reversible process. Note that spontaneous adiabatic contraction (non-quasistatic) where internal energy does not change is irreversible!

Notes

Heat Engine Efficiency

Efficiency can be defined for a cyclic engine

Efficiency of a cyclic engine is defined as the ratio of the net work done in a cycle to the net amount of heat extracted from the hot body in one cycle

$$\varepsilon = \frac{\Delta W_{RWS}}{\Delta Q_{source}}$$

For Carnot engine:

$$\frac{\Delta W_{RWS}}{\Delta Q_{source}} = 1 + \frac{\Delta Q_{sink}}{\Delta Q_{source}} = 1 - \frac{T_c}{T_h}\frac{(L_A - L_0)^2 - (L_B - L_0)^2}{(L_D - L_0)^2 - (L_C - L_0)^2} = 1 - \frac{T_c}{T_h}$$

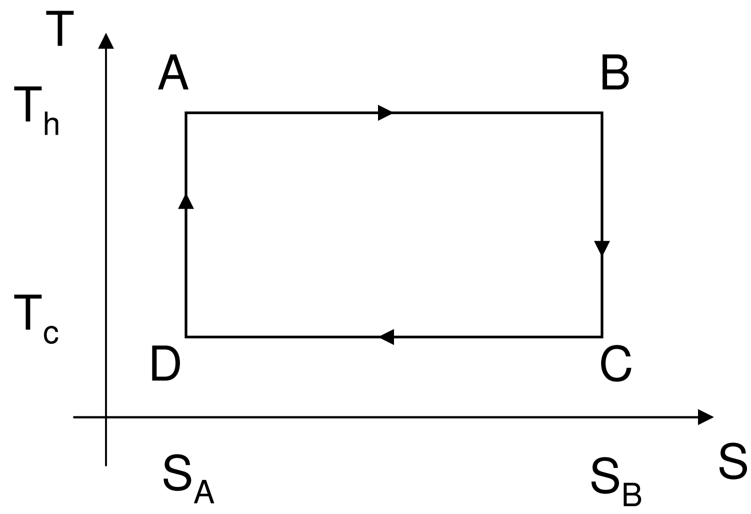
Exercise: show that this is correct

Note that since all reversible engines do the same work (maximum), they have the same efficiency (e. g. same as Carnot engine).

Notes

Heat Engine Efficiency

There is a simpler way to calculate Carnot engine efficiency using the (S,T) plot of the Carnot cycle.



$$\Delta W_{RWS} = \Delta Q_{source} + \Delta Q_{sink}$$

Since $$\Delta Q_{source} = T_h(S_B - S_A) = T_h \Delta S$$

$$\Delta Q_{source} = T_c(S_A - S_B) = -T_c \Delta S$$

We obtain $$\Delta W_{RWS} = (T_h - T_c)\Delta S$$ ⇨ $$\varepsilon = \frac{\Delta W_{RWS}}{\Delta Q_{source}} = \frac{(T_h - T_c)\Delta S}{T_h \Delta S} = 1 - \frac{T_c}{T_h}$$

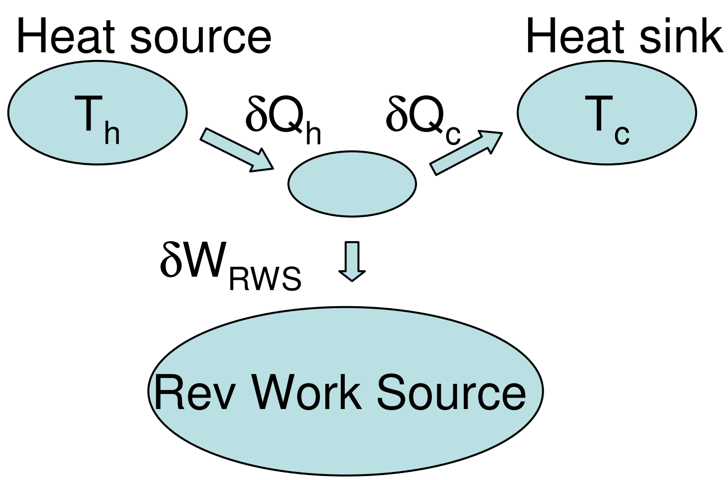
Notes

Engine, refrigerator, heat pump

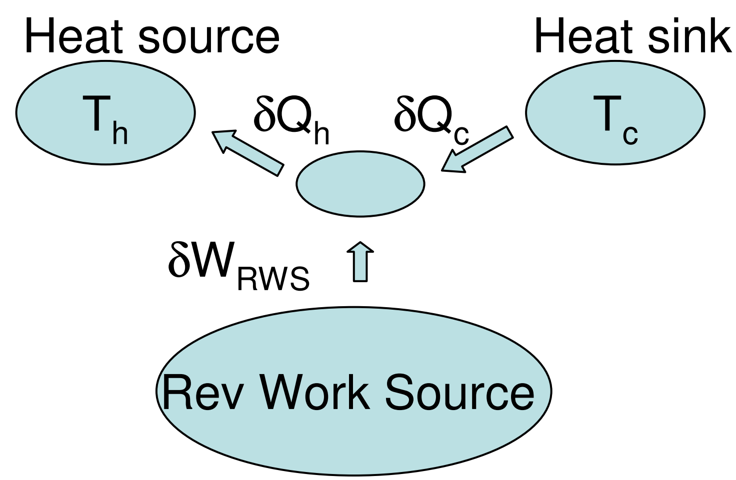
Heat engine can run in direct and reverse cycles.

If it runs in the reverse cycle, work is done to transfer heat from the cold to the hot system (refrigerator, heat pump).

Engine



Refrigerator or heat pump



Notes

Real versus Ideal Engines

Completely reversible engines are impossible to realize due to friction and other loss mechanisms.

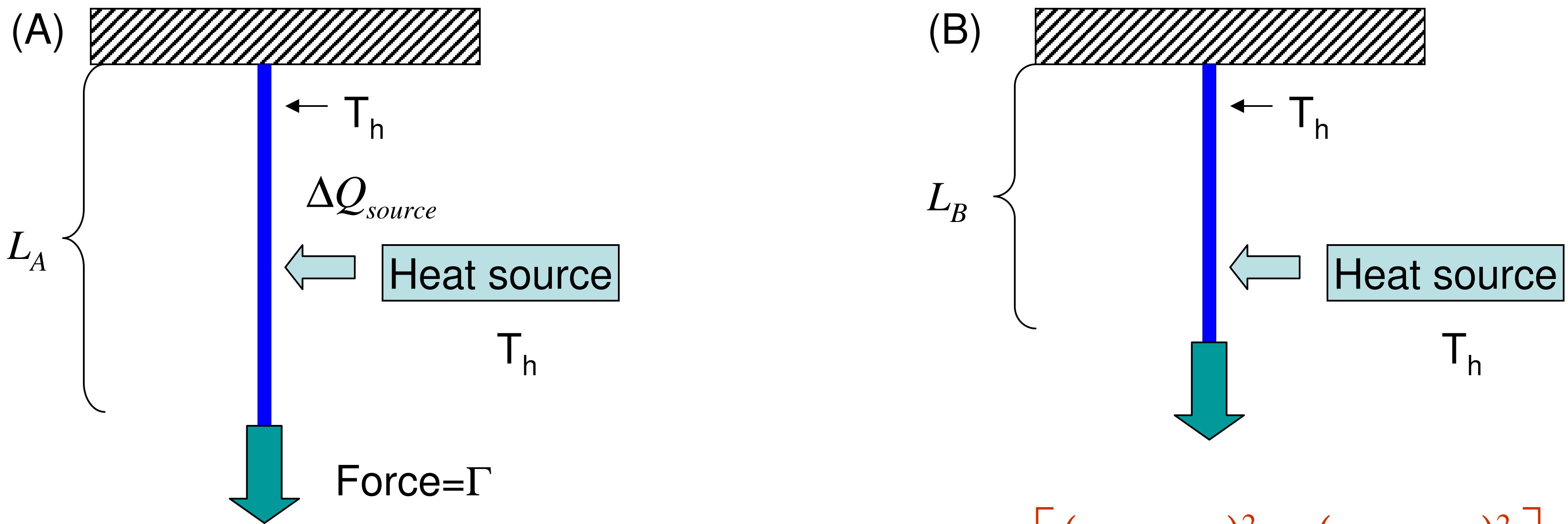
Even if such engines were realizable, they would generate very little power since they have to employ very slow quasi-static processes in order to be reversible.

The reason why reversible engines are slow is because they employ heat transfer between bodies of the same temperature. Since heat flow is proportional to the gradient of temperature (non-equilibrium thermodynamics result), the flow of heat in the reversible engine has to be infinitely slow.

Real engines are always irreversible and thus have lower efficiency compared to the reversible engines.

Notes

Carnot Cycle: Isothermal Contraction – a very slow process



$$\Delta W_{RWS}^{A\rightarrow B} = \Delta Q_{source} = T_h \Delta S = \frac{cBT_h}{2}\left[\frac{(L_A - L_0)^2}{L_0} - \frac{(L_B - L_0)^2}{L_0}\right]$$

Notes

Endoreversible Engine

To transfer heat fast in real engines, we need to employ irreversible processes.

An example of such an engine is endoreversible engine.

For endoreversible Carnot engine, the isothermal processes are done at a temperatures T_w and T_t that are higher than the temperature of the heat sink T_c but lower than the temperature of the heat source T_h.

Since for both isothermal processes, the temperature difference between the heat source/sink and the working body of the engine is non-zero, the isothermal heat transfer processes are not infinitely slow.

Notes

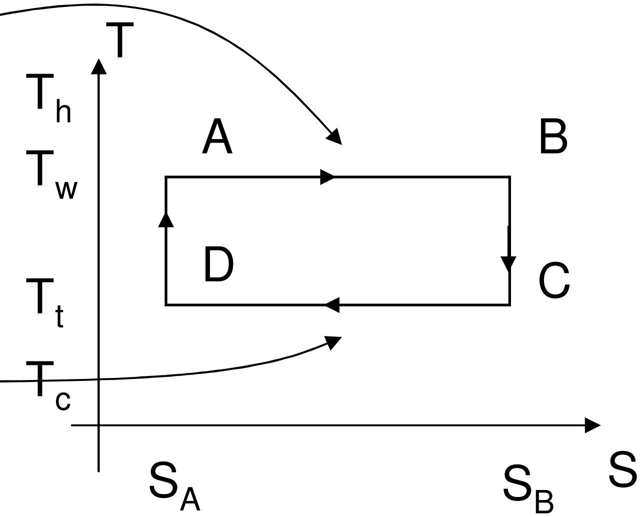
Endoreversible Engine

For endoreversible Carnot engine, the isothermal processes are done at a temperatures T_w and T_t that are higher than the temperature of the heat sink T_c but lower than the temperature of the heat source T_h.

Rubber band is at T_w here

$T_h > T_w > T_t > T_c$

Rubber band is at T_t here



A→B and C→D are irreversible now while
B→C and D→A are still reversible

Notes

Endoreversible Engine Analysis

Non equilibrium thermodynamics results for heat flow rate:

$$\frac{\partial Q}{\partial t} = \sigma(T_1 - T_2)$$

where σ is heat conductivity

From this equation we can calculate time of the isothermal processes of the endoreversible engine

$$t_{source} = \frac{\Delta Q_{source}}{\sigma_h (T_h - T_w)}$$

- time of isothermal process in contact with heat source

$$t_{sink} = \frac{\Delta Q_{sink}}{\sigma_c (T_t - T_c)}$$

- time of isothermal process in contact with heat sink

$$t_{cycle} \approx t_{source} + t_{sink}$$

Because adiabatic processes can be very fast

Notes

Endoreversible Engine Analysis

$$t_{cycle} \approx \frac{\Delta Q_{source}}{\sigma_h (T_h - T_w)} + \frac{\Delta Q_{sink}}{\sigma_c (T_t - T_c)}$$

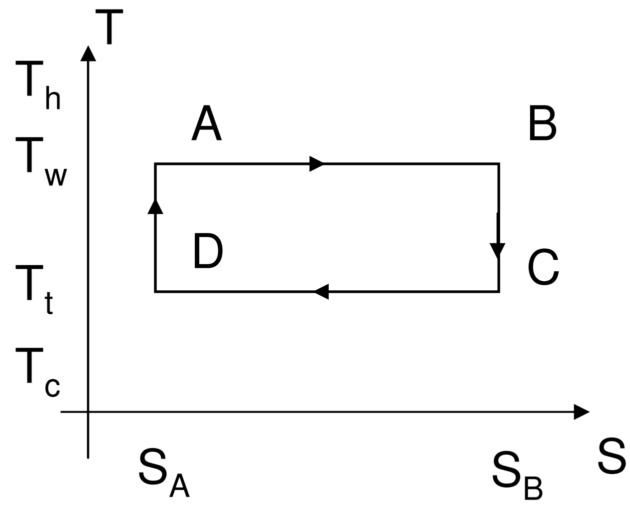
$$\Delta Q_{source} = T_w \Delta S$$

$$\Delta Q_{\sin k} = T_t \Delta S$$

$$W = (T_w - T_t)\Delta S$$

$$\Delta Q_{source} = \frac{T_w}{T_w - T_t} W$$

$$\Delta Q_{\sin k} = \frac{T_t}{T_w - T_t} W$$



$$t_{cycle} \approx \left(\frac{1}{\sigma_h} \frac{1}{T_h - T_w} \frac{T_w}{T_w - T_t} + \frac{1}{\sigma_c} \frac{1}{T_t - T_c} \frac{T_t}{T_w - T_t} \right) W$$

Notes

Maximum Endoreversible Engine Power

$$P = \frac{W}{t_{cycle}} = \left(\frac{1}{\sigma_h} \frac{1}{T_h - T_w} \frac{T_w}{T_w - T_t} + \frac{1}{\sigma_c} \frac{1}{T_t - T_c} \frac{T_t}{T_w - T_t} \right)^{-1}$$

To find maximum power delivered, we need to maximize P with respect to T_w and T_c

$$\frac{\partial P}{\partial T_w} = 0$$

$$\frac{\partial P}{\partial T_t} = 0$$

⇒

$$T_w = c\sqrt{T_h}$$

$$T_t = c\sqrt{T_c}$$

where

$$c = \frac{\sqrt{\sigma_h T_h} + \sqrt{\sigma_c T_c}}{\sqrt{\sigma_h} + \sqrt{\sigma_c}}$$

Exercise: show this

Notes

Maximum Endoreversible Engine Power

Substituting $T_w = c\sqrt{T_h}$ $T_t = c\sqrt{T_c}$

into

$$P = \frac{W}{t_{cycle}} = \left(\frac{1}{\sigma_h} \frac{1}{T_h - T_w} \frac{T_w}{T_w - T_t} + \frac{1}{\sigma_c} \frac{1}{T_t - T_c} \frac{T_t}{T_w - T_t} \right)^{-1}$$

we obtain the maximum power delivered by endoreversible engine

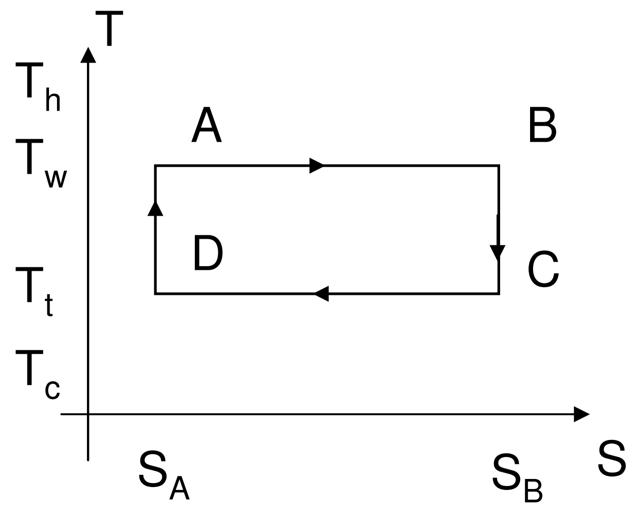
$$P_{\max} = \frac{W}{t_{cycle}} = \sigma_h \sigma_c \left(\frac{\sqrt{T_h} - \sqrt{T_c}}{\sqrt{\sigma_h} + \sqrt{\sigma_c}} \right)^2$$

Notes

Endoreversible Engine Efficiency

Endoreversible engine efficiency at max power:

$$\varepsilon = \frac{W}{\Delta Q_{source}} = \frac{T_w - T_t}{T_h}$$



Substituting $T_w = c\sqrt{T_h}$ $\quad T_t = c\sqrt{T_c}$

$$c = \frac{\sqrt{\sigma_h T_h} + \sqrt{\sigma_c T_c}}{\sqrt{\sigma_h} + \sqrt{\sigma_c}}$$

We obtain: $\varepsilon = 1 - \sqrt{\frac{T_c}{T_h}}$

Not how this expression does not depend on the values of thermal conductivities!

Notes